1. (a). This is a direct network, the direction comes from species' prey to species.
Suppose the trophic level is \( X \) and adjacent matrix \( A \).

\[
\frac{1}{k_i \cdot \text{in} \sum_j A_{ij} X_j} \text{ is the mean trophic levels of species's prey.}
\]

So,

\[
X_i = 1 + \frac{1}{k_i \cdot \text{in} \sum_j A_{ij} X_j}
\]

(b). we write as the matrix form

\[
X = \vec{1} + AD^{-1}X,
\]

where \( D = \begin{pmatrix} k_1 & k_2 & \cdots & k_n \end{pmatrix} \)

\[
\vec{X}(I - AD^{-1})X = \vec{1}.
\]

\[
X = (I - AD^{-1})^{-1}\vec{1}.
\]

\[
X = (I - AD^{-1})^{-1}D^{-1}D\vec{1}
\]

\[
= (D - A)^{-1}D\vec{1}
\]

If \( k_i \cdot \text{in} = 0 \), then all \( A_{ij} = 0 \), we can assume in this case \( X_i = 1 \).
2. (a) The tree network suppose to have structure.

\[
\begin{pmatrix}
2 & -1 & -1 \\
-1 & 3 & -1 \\
-1 & -1 & 1 \\
\end{pmatrix}
\]

The Laplacian matrix is

\[
\frac{20}{20}
\]

The spectral is calculated, (by numerical)

0, 0.0968, 0.7679, 0.2679, 1, 1, 1, 1, 1, 2.1939, 3, 3.7321

3.7321, 4.7093, 5.

(b) The star network.

Suppose \( n = 2 \)

\[
L = \begin{pmatrix}
1 & -1 \\
-1 & 1
\end{pmatrix}
\]

\( p(\lambda) = (1 - \lambda)^2 - 1, \ \lambda = 0, 2. \)

\( n = 3 \)

\[
L = \begin{pmatrix}
2 & -1 & -1 \\
-1 & 1 & -1 \\
-1 & -1 & 1
\end{pmatrix}
\]

\( p(\lambda) = (1 - \lambda)^2 (2 - \lambda) - 2 (1 - \lambda), \ \lambda = 0, 1, 3. \)

\( n = n \)

\[
L_n = \begin{pmatrix}
(n-1) & -1 & -1 & \cdots & -1 \\
-1 & (n-1) & -1 & \cdots & -1 \\
-1 & -1 & (n-1) & \cdots & -1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-1 & -1 & -1 & \cdots & 1
\end{pmatrix}
\]

\( p(\lambda) = (n-1 - \lambda)(1 - \lambda)^{n-1} - (n-1)(1 - \lambda)^{n-2}. \)

\( \lambda = 0, 1, 1, \ldots, 1, n. \)
12(1) For each node, we select two vertices.

\[ \binom{n}{2} \]

the total chosen can be.

For each vertex (for example, \( i \)) we have average \( c \) links.

One already contact with \( i \), other \( c-1 \) node must have one to connect with \( k \). The probability is

\[
\frac{\binom{n-2}{c-2}}{\binom{n-1}{c-1}} \approx \frac{c^n}{n^{\frac{20}{20}}}
\]

when \( n \gg c \gg 1 \)

The degeneracy for triangle is 6.

So the total number of triangle are

\[
\frac{1}{6} \left[ n \binom{c}{2} \cdot \frac{c^n}{n} \right] = \frac{1}{6} C^3
\]

(2) The triples is defined as the first question, the total number is

\[ n \binom{c}{2} = \frac{1}{2} n c^2 \]

(3) \[ C = \frac{3 \# \text{triangle}}{\# \text{triple}} = \frac{\frac{1}{2} c^3}{\frac{1}{2} n c^2} = \frac{c}{n} \]

Under the limit \( n \gg c \gg 1 \), it equal with 12.11.
4. Consider the simple case where $m$ shortcut arrange in the circle with equal space.

Suppose there are total $L$ node, when $L$ is large treat $L$ as a continuous value, the path length along short cut is 0. Assume $\frac{L}{40} + \frac{L}{40} = \frac{L}{40} + \frac{L}{40} = \frac{L}{40}$

Now we exam the average length on the circle.

Put one point $P$, so the proba. Another point is $Q$.

1. $P, Q$ in the same section, the probability is $\frac{1}{m+1}$
   
   The largest length is $\frac{L}{m+1} \times \frac{1}{2}$.

   Shortest is 0.

   The average is $\frac{L}{m+1} \times \frac{1}{4}$.

2. $P, Q$ in the different section, the probability is $\frac{m}{m+1}$

   The largest is $\frac{L}{m+1}$.

   Shortest is 0.

   The average is $\frac{L}{m+1} \times \frac{1}{2}$.

So $d(p,Q) = \frac{1}{m+1} \cdot \frac{L}{m+1} + \frac{m}{m+1} \cdot \frac{L}{m+1} = \frac{2m+1}{4(m+1)} L$.

Consider each node have $k$ nearest neighbour, assume $\frac{L}{m+1} \gg k$, so average path length (normalized by $L$) is $\frac{2m+1}{4k(m+1)^2}$. 