PHYSICAL REVIEW E **86**, 045101(R) (2012)

Optimizing cooperation on complex networks in the presence of failure

Yu-Zhong Chen[1] and Ying-Cheng Lai[1,2]
[1]*School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, Arizona 85287, USA*
[2]*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA*

(Received 26 July 2012; published 11 October 2012)

Cooperation has been recognized as a fundamental driving force in many natural, social, and economic systems. We investigate whether, given a complex-networked system in which agents (nodes) interact with one another according to the rules of evolutionary games and are subject to failure or death, cooperation can prevail and be optimized. We articulate a control scheme to maximize cooperation by introducing a time tolerance, a time duration that sustains an agent even if its payoff falls below a threshold. Strikingly, we find that a significant cooperation cluster can emerge when the time tolerance is approximately uniformly distributed over the network. A heuristic theory is derived to understand the optimization mechanism, which emphasizes the role played by medium-degree nodes. Implications for policy making to prevent or mitigate large-scale cascading breakdown are pointed out.

DOI: 10.1103/PhysRevE.86.045101 PACS number(s): 89.75.Hc, 02.50.Le, 87.23.Ge, 87.23.Kg

Natural selection favors the survival and prevalence of species with a competitive edge, yet the phenomenon of cooperation is ubiquitous in many biological, economic, and social systems [1,2]. Understanding the emergence and evolution of cooperation has thus become a field of significant interdisciplinary interest, where evolutionary-game theory [3] has served as a powerful mathematical paradigm [1,4,5]. In a typical setting, a number of agents on a network interact with one another, where the network topology can be regular or complex and each agent can take on one of two strategies at any given time: cooperation or defection. The defection strategy is a selfish action that usually generates a higher payoff [3] temporally, as in paradigmatic games such as the prisoner's dilemma games (PDGs) [4], snowdrift games [6], and public good games [7]. A basic issue is then how cooperation can possibly survive when natural selection favors the defection strategy in order to gain higher individual fitness (at least temporally). In the past two decades, many cooperation-facilitating mechanisms have been uncovered, which include network reciprocity [8], reputation and punishment [9], random diffusion [10], success-driven migration [11], memory effect [12], benefit of noise [13], social diversity [14,15], asymmetric cost [16], and teaching ability [17].

In most previous works, no death mechanism was incorporated in the evolutionary-game model on networks, i.e., no agent can be removed from the system even if it gains no payoff in a substantial amount of time [18]. In real-world situations, an agent can go bankrupt and be eliminated immediately when its payoff falls below a critical threshold for a certain period of time. An example is the great economic recession in 2008, where a large number of financial institutions and corporations collapsed. In an ecological system, the death of individuals is a common phenomenon. In this regard, a recent work [19] has incorporated a simple elimination mechanism into the evolutionary-game rules. In particular, a tolerance parameter was assigned to each individual in the network, which is the lowest allowed payoff. An agent dies and is removed from the network when its payoff falls below this threshold. The threshold can be heterogeneously distributed among agents. It was shown that rapid, cascadelike elimination of agents can result from such a death mechanism, and a pure cooperation state can emerge afterwards, where all defectors are eliminated and the survivors are exclusively cooperators [19]. One implication is that defectors, despite their advantages in getting temporarily higher payoffs, may be particularly vulnerable to large-scale catastrophic failures. These findings thus suggest that in a complex system where agents are subject to failure or death, cooperation may be beneficial to mitigating large-scale breakdown.

In this paper we propose a control scheme to enhance cooperation and eliminate large-scale failures in complex-networked systems. Our key idea is that, due to the complex time evolution of the system, although the payoff of any agent can inevitably become arbitrarily low, the probability that the payoff remains low for an extended period of time will be small. We are thus led to introduce a *time tolerance* for each agent, where an agent will not die or be removed unless its payoff remains below a critical threshold for time longer than the tolerance. Since the degree distribution of the network is in general not uniform, it is reasonable that the time tolerance be degree dependent. A parameter β can then be introduced to characterize the heterogeneity of the distribution of the time tolerance, where $\beta = 0$ signifies a completely uniform distribution. Our main result is that properly chosen time delay can optimize cooperation and prevent large-scale death. A surprising finding is that optimal state of cooperation occurs near $\beta = 0$, indicating that making a time-tolerance distribution uniform is an effective strategy to enhance cooperation.

To impose a time tolerance on a complex network, we conceive that a node or an agent's *debt capacity* depends on its relative importance in the network. We thus hypothesize the following relationship between the time tolerance of agent i and its degree k_i [15,20]:

$$T_i = N T_0 \frac{k_i^{\beta}}{\sum_l k_l^{\beta}}, \tag{1}$$

where N is the total number of agents, T_0 is the nominal time tolerance, and β is an externally control parameter. For $\beta < 0$, agents with higher (lower) degree have lower (higher) time tolerance; the situation is the opposite for $\beta > 0$ and $\beta = 0$

corresponds to a uniform time tolerance in the network. A large values of T_i means that the node is more resilient to failure or bankruptcy. A death mechanism can be introduced for a PDG by choosing the following payoff tolerance for agent i [19]: $P_i^T \equiv \alpha P_i^N = \alpha k_i$, where agent i dies and is removed from the network if its payoff is lower than P_i for consecutive T_i time steps, P_i^N is the normal payoff of agent i when the system is in a healthy state in which all agents are cooperators, and $0 < \alpha < 1$ is a tolerance parameter. Since an agent's degree may change when its neighbors die, k_i is the instantaneous degree of agent i. For $\alpha = 1$ agents have a zero payoff tolerance to breakdown, while for $\alpha = 0$ agents are completely tolerant.

In our evolutionary game model, each time step (iteration) thus consists of the following four dynamical processes.

(i) *Game playing and payoffs*. Each agent plays the classical prisoner's dilemma game with all its nearest neighbors and the total payoff is the sum of the payoffs gained in its two-player games with all other connected agents. The PDG parameters are chosen to be $R = 1$, $T = b > 1$, and $S = P = 0$ [4].

(ii) *Strategy updating*. At each time step, agent i randomly chooses a neighbor j and imitates j's strategy with the probability $W_{i\to j} = \{1 + \exp[-(P_j - P_i)/\kappa]\}^{-1}$, where P_i and P_j are the payoffs agents i and j, and κ is the level of the agents' rationality representing the uncertainties in assessing the best strategy. We set $\kappa = 0.1$.

(iii) *Failure and agent removal*. At each iteration, for agent i, the time in debt t_i increases by 1 if P_i falls below the payoff tolerance P_i^T during the prior t_i time steps. Otherwise, we set $t_i = 0$. Since k_i varies with time, P_i^T and T_i also change with time. If $t_i > T_i$ or if $k_i = 0$, agent i and all its links will be removed from the network.

(iv) *Random rewiring*. For agent i whose neighbor j has been removed in step (iii), a new connection is added between agent i and a randomly selected agent in the remaining agents outside i's current neighborhood, provided such an agent exists. This is motivated by the consideration that an agent in general will try to seek and engage new partners when the payoffs of some of its current partners become insignificant, and lack of global information leads to random selection.

Note that dynamical processes (i) and (ii) are conventional for typical evolutionary-game dynamics [21], but processes (iii) and (iv) are unique features of our model [22].

In our simulations we use the standard scale-free network model [23,24] with parameters $m_0 = 5$, $m = 5$, and average degree $\bar{k} = 10$. The total number of agents is $N = 1000$. The initial condition is that both cooperative and defective strategies populate the scale-free network with equal probability. A synchronous updating scheme is adopted. For the results that will be presented below, the transient time is chosen to be 10 000 iterations and each data point is obtained by averaging over 25 independent network realizations with 20 runs for each realization.

To characterize the behavior of the network dynamics in the presence of death, we use three quantities: the death rate S_d (the number of the dead agents in the final state normalized by the network size) and the asymptotic frequencies of cooperators ρ_C and defectors ρ_D, where $S_d + \rho_C + \rho_D = 1$. Figures 1(a)–1(f) show, for six combinations of three values of the control parameter β and two values of the nominal time

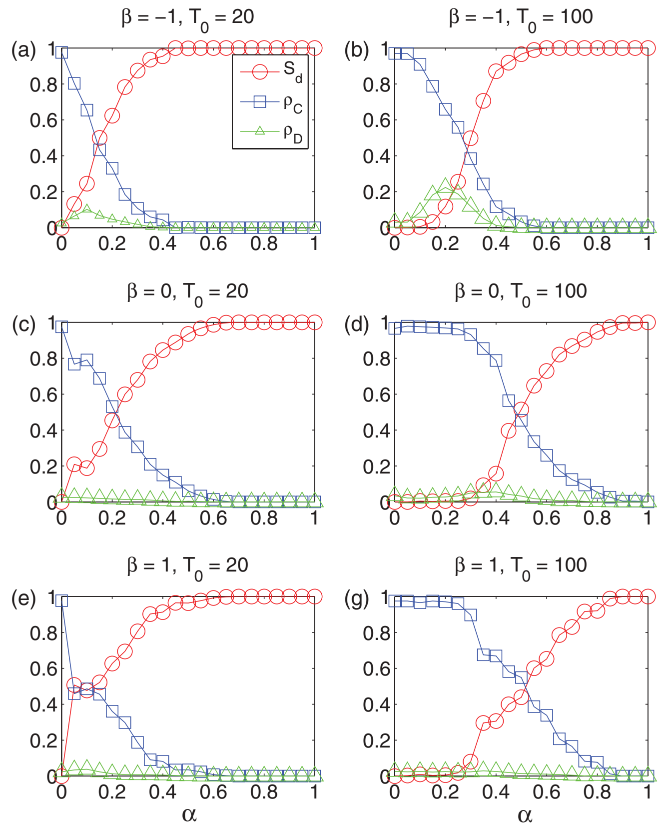

FIG. 1. (Color online) Death rate S_d (red circles) and frequencies of cooperation ρ_C (blue squares) and defection ρ_D (green triangles) versus the tolerance parameter α for six combinations of three values of the control parameter β (-1, 0, and 1) and two values of T_0 (20 and 100) for $b = 1.5$.

tolerance T_0, the asymptotic values of the three characteristic quantities versus the profit-tolerance parameter α. We observe that, while in all cases S_d increases and ρ_C decreases nearly monotonically with α, a large value of T_0 can delay the occurrence of total death. In fact, for $T_0 = 100$ there exists a region of small-α values in which the death rate is nearly zero and the vast majority of agents are cooperators. For $T_0 \to \infty$, cooperators rule the whole region of $0 \leqslant \alpha \leqslant 1$. Comparing the three cases of β values, we see that this interval of sustainable cooperation appears to be the largest for $\beta = 0$, implying that adjusting β can have the effect of significantly promoting cooperation. Further investigation shows that during the cascading failure and random rewiring process, the degree distribution of the network can evolve from power law to eventually being Poisson.

To demonstrate the ability of control scheme to optimize cooperation, we explore how the three characteristic quantities depend on the control parameter β. Figure 2 shows their behaviors for four combinations of two values of α and T_0. We observe that ρ_C exhibits a nonmonotonic behavior with β, indicating that ρ_C can be optimized by a proper choice of β and this optimization effect is stronger for larger value of T_0 [Figs. 2(b) and 2(d)]. For example, for $\alpha = 0.3$ and $T_0 = 100$ [Fig. 2(b)], for β near zero, the networked system becomes supercooperative in the sense that almost no agents die ($S_d \approx 0$) and nearly all agents survive as cooperators

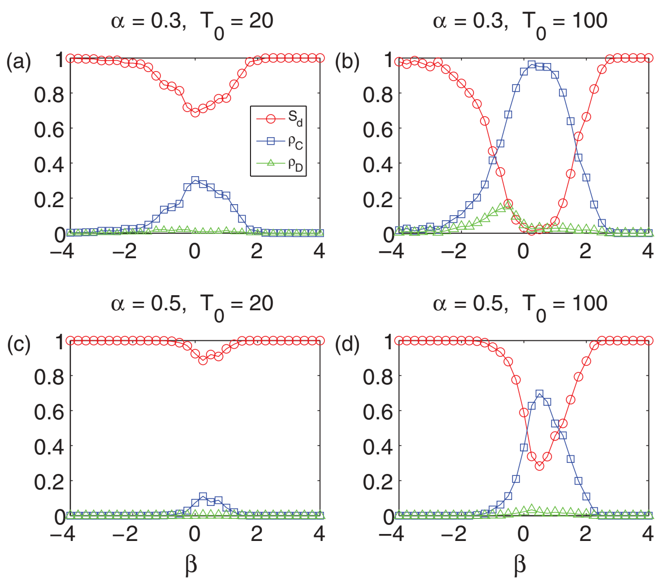

FIG. 2. (Color online) For four combinations of the parameter values of α and T_0, S_d (red circles), ρ_C (blue squares), and ρ_D (green triangles) versus the control parameter β for $b = 1.5$.

($\rho_C \approx 1$). The results do not depend on the choices of b and κ. The revelation that the optimal value of β occurs near zero is striking: It indicates that distributing the time tolerance uniformly can be tremendously advantageous to promoting and sustaining cooperation on the networked system, despite the highly heterogeneous character of the network topology. This finding has a certain implication for policy making. For example, an economic network constituting financial institutions and banks of various sizes as its nodes may be subject to catastrophic breakdown [19]. External intervention, e.g., government bailout, can prevent large-scale breakdown. Our result indicates that the intervention should be so as to make the time tolerance as uniform as possible to keep any agent "alive" after it becomes broke, regardless of the size of the agent. In particular, given a large and a small bank, both facing possible bankruptcy, the criterion to determine the amount of governmental bailout is that they can survive for approximately the same amount of time before turning themselves into profitable agents.

We now present a heuristic argument to understand the mechanism of cooperation optimization. It is useful to rank the nodes into three classes according to their degree values. From Eq. (1) and the degree distribution of the Barabási-Albert network $P(k) = 2m^2k^{-3}$, we obtain a power-law distribution of the time tolerance T as

$$P(T) = 2m^2\left[\left(\frac{\sum_l k_l^\beta}{NT_0}\right)^{-3/\beta}\right]T^{-3/\beta}. \tag{2}$$

From the expression of the degree distribution $P(k)$, we obtain

$$\bar{k^\beta} = \int_{k_S}^{k_L} 2m^2k^{-3}k^\beta dk = \frac{2m^2}{\beta-2}\left(k_L^{\beta-2} - m^{\beta-2}\right), \tag{3}$$

where k_L and $k_S = m$ denote the largest and smallest degree value in the network, respectively. From Eqs. (1) and (3) we get the dependence of an agent's time tolerance $T(k)$ on its degree value k as

$$T(k) = NT_0\frac{k^\beta}{\sum_l k_l^\beta} = T_0\frac{k^\beta}{\bar{k^\beta}} = \frac{T_0k^\beta}{\frac{2m^2}{\beta-2}\left(k_L^{\beta-2} - m^{\beta-2}\right)}$$

and hence

$$\frac{dT(k)}{d\beta} = \frac{2m^2T_0k^\beta}{\left[\frac{2m^2}{\beta-2}\left(k_L^{\beta-2} - m^{\beta-2}\right)\right]^2}[f(k_L) - f(m)], \tag{4}$$

where

$$f(x) \equiv \frac{x^{\beta-2}}{\beta-2}\left(\ln\frac{k}{x} + \frac{1}{\beta-2}\right) \quad (m \leqslant x \leqslant k_L). \tag{5}$$

We have $df/dx = x^{\beta-3}\ln(k/x)$. For the agent with the largest (or the smallest) degree value, i.e., $k = k_L$ (or m), the sign of df/dx is fixed in the open interval (m,k_L) of x and accordingly $f(x)$ is a monotonic function. For $k = k_L$ we have $k \geqslant x$, thus $df/dx \geqslant 0$ holds. The situation $df/dx = 0$ occurs if and only if $x = k_L$ and subsequently $f(x)$ monotonically increases with x. Since $k_L > m$ we have $f(k_L) - f(m) > 0$ and correspondingly $dT(k)/d\beta > 0$. This means that the time tolerance of the network's largest-degree node monotonically increases with β. By the same argument, the time tolerance of the smallest-degree agents decreases monotonically with β. This means that the agents with the largest-degree value, which play a critical role in maintaining cooperation [24], and the agents with the smallest-degree values, which account for the majority of the system population, do not play a significant role in optimizing cooperation. Note that the effects are in fact insignificant, as can be seen from Fig. 2, where ρ_C, ρ_D, and s_d vary nonmonotonically with the control parameter β. Since $T(m)$ and $T(k_L)$ exhibit a monotonic behavior (which can be argued analytically), the nonmonotonic behavior in ρ_C, ρ_D, and s_d is unlikely to be caused by the behavior of $T(m)$ and $T(k_L)$. In fact, it is caused by the nonmonotonic behavior of $T(k)$ for $m < k < k_L$.

The key to cooperation optimization thus lies in the nodes of medium degrees. In particular, for $m < k < k_L$ the sign of $df(x)/dx$ becomes indeterminate and accordingly the sign of $dT(k)/d\beta$ cannot be determined. The dependence of T on β for agents with different degrees can be studied numerically. We find that, except for the smallest- and highest-degree agents, T is typically a convex function of β. Since the time-tolerance values for the agents with medium degrees peak near $\beta = 0$, as shown in Fig. 3, the corresponding agents live longer than other agents. Since the peak values of T are not much larger than the nominal value T_0, a slight increase in the medium-degree agents' lifetimes can help stabilize cooperation. The lifetimes of the medium-degree agents are thus critically important for promoting cooperation [15].

In summary, we find that in complex-networked systems where agent interactions are governed by evolutionary-game dynamics but agents are subject to death, an effective control scheme can be introduced to optimize cooperation, which has the advantage of stabilizing the system by preventing large-scale cascading failures. The key to our control scheme is a time tolerance that prevents any agent from being removed from the system immediately when its payoff falls below a threshold. Our computation and heuristic analysis indicate that, despite the network's being highly heterogeneous, making

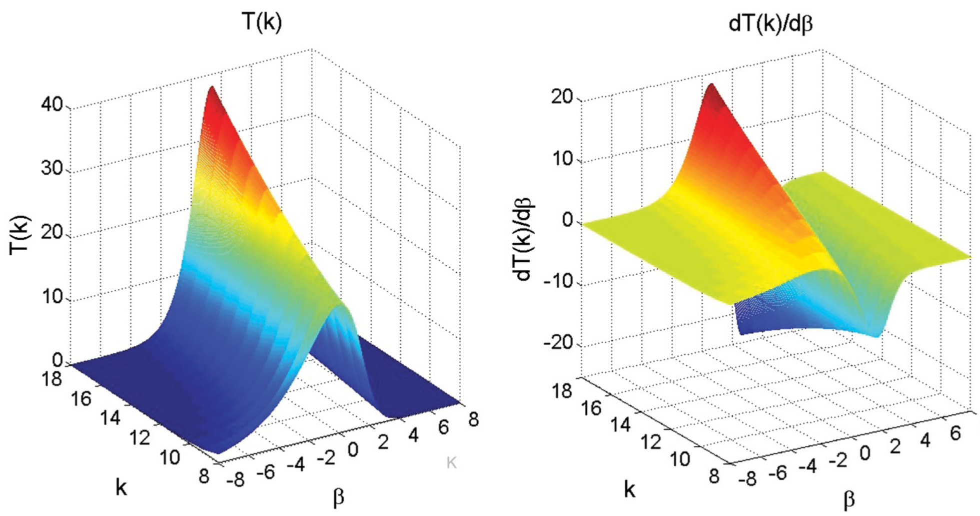

FIG. 3. (Color online) Time tolerance $T(k)$ (left) and its derivative $dT(k)/d\beta$ (right) versus the control parameter β for $T_0 = 20$.

the time tolerance as uniformly as possible across the network can lead to the emergence of a stable cooperation cluster that has recruited the majority of agents in the network. Simultaneously, the death of a substantial number of agents can be avoided. This finding may have implications in policy making to prevent, for example, large-scale breakdown of social and economic systems. The emergence and evolution of cooperation in complex systems have been recognized as fundamental issues in natural, social, and economic sciences [1] and our work provides insights into the control of complex dynamical systems in terms of critical issues such as stability, performance, and sustainability.

We thank Wenxu Wang and Xuan Ni for helpful discussions. This work was supported by Air Force Office of Scientific Research under Grant No. FA9550-10-1-0083 and by a seed grant from National Academies Keck Futures Initiative.

[1] M. A. Nowak, Science **314**, 1560 (2006); Sci. Am. **307**(1), 34 (2012); M. A. Nowak and R. Highfield, *Super Cooperators: Altruism, Evolution, and Why We Need Each Other to Succeed* (Free Press, New York, 2012).

[2] A. M. Colman, *Game Theory and its Applications in the Social and Biological Sciences* (Butterworth-Heinemann, Oxford, 1995).

[3] J. M. Smith, *Evolution and the Theory of Games* (Cambridge University Press, Cambridge, 1982); H. Gintis, *Game Theory Evolving* (Princeton University Press, Princeton, 2000).

[4] R. Axelrod and W. D. Hamilton, Science **211**, 1390 (1981).

[5] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984).

[6] R. Sugden, *The Economics of Rights, Co-operation and Welfare* (Blackwell, Oxford, 1986).

[7] G. Hardin, Science **162**, 1243 (1968).

[8] M. A. Nowak and R. M. May, Nature (London) **359**, 826 (1992); G. Szabó and C. Hauert, Phys. Rev. Lett. **89**, 118101 (2002); C. Hauert and M. Doebeli, Nature (London) **428**, 643 (2004).

[9] E. Fehr and S. Gächter, Nature (London) **415**, 137 (2002); R. Boyd, H. Gintis, S. Bowles, and P. J. Richerson, Proc. Natl. Acad. Sci. USA **100**, 3531 (2003); M. A. Nowak and K. Sigmund, Nature (London) **437**, 1291 (2005); A. Traulsen and M. A. Nowak, Proc. Natl. Acad. Sci. USA **103**, 10952 (2006); C. Hauert, A. Traulsen, H. Brandt, M. A. Nowak, and K. Sigmund, Science **316**, 1905 (2007); H. Ohtsuki, Y. Iwasa, and M. A. Nowak, Nature (London) **457**, 79 (2009).

[10] E. Sicardi, H. Fort, M. Vainstein, and J. Arenzon, J. Theor. Biol. **256**, 240 (2009).

[11] D. Helbing and W. Yu, Proc. Natl. Acad. Sci. USA **106**, 3680 (2009); S. Meloni, A. Buscarino, L. Fortuna, M. Frasca, J. Gómez-Gardeñes, V. Latora, and Y. Moreno, Phys. Rev. E **79**, 067101 (2009); L.-L. Jiang, W.-X. Wang, Y.-C. Lai, and B.-H. Wang, *ibid.* **81**, 036108 (2010); H.-X. Yang, Z.-X. Wu, and B.-H. Wang, *ibid.* **81**, 065101(R) (2010).

[12] W.-X. Wang, J. Ren, G. Chen, and B.-H. Wang, Phys. Rev. E **74**, 056113 (2006); S.-M. Qin, Y. Chen, X.-Y. Zhao, and J. Shi, *ibid.* **78**, 041129 (2008).

[13] G. Szabó, J. Vukov, and A. Szolnoki, Phys. Rev. E **72**, 047107 (2005); J. Vukov, G. Szabó, and A. Szolnoki, *ibid.* **73**, 067103 (2006); J. Ren, W.-X. Wang, and F. Qi, *ibid.* **75**, 045101 (2007).

[14] F. C. Santos, J. M. Pacheco, and T. Lenaerts, Proc. Natl. Acad. Sci. USA **103**, 3490 (2006); F. C. Santos, M. D. Santos, and J. M. Pacheco, Nature (London) **454**, 213 (2008); H.-X. Yang, W.-X. Wang, Z.-X. Wu, Y.-C. Lai, and B.-H. Wang, Phys. Rev. E **79**, 056107 (2009); M. Perc and A. Szolnoki, *ibid.* **77**, 011904 (2008).

[15] Y.-Z. Chen, Z.-G. Huang, S.-J. Wang, Y. Zhang, and Y.-H. Wang, Phys. Rev. E **79**, 055101(R) (2009).

[16] W.-B. Du, X.-B. Cao, M.-B. Hu, and W.-X. Wang, Europhys. Lett. **87**, 60004 (2009).

[17] A. Szolnoki and G. Szabó, Europhys. Lett. **77**, 30004 (2007); A. Szolnoki, M. Perc, and G. Szabó, Eur. Phys. J. B **61**, 505 (2008); A. Szolnoki and M. Perc, New J. Phys. **10**, 043036

(2008); G. Szabó and A. Szolnoki, Phys. Rev. E **79**, 016106 (2009); A. Szolnoki, M. Perc, G. Szabó, and H. U. Stark, *ibid.* **80**, 021901 (2009).

[18] There are relevant previous works on well mixed populations. See, for example, C. Hauert, M. Holmes, and M. Doebeli, Proc. R. Soc. London Ser. B **273**, 2565 (2006); J. Alonso, A. Fernández, and H. Fort, J. Stat. Mech. (2006) P06013; A. Arenas, J. Camacho, J. Cuesta, and R. Requejo, J. Theo. Biol. **279**, 113 (2011).

[19] W.-X. Wang, R. Yang, and Y.-C. Lai, Phys. Rev. E **81**, 035102(R) (2010); W.-X. Wang, Y.-C. Lai, and D. Armbruster, Chaos **21**, 033112 (2011).

[20] Z.-G. Huang, Z.-X. Wu, A.-C. Wu, L. Yang, and Y.-H. Wang, Europhys. Lett. **84**, 50008 (2008).

[21] In real-life situations the punishment can be different from the sucker's payoff, and this sometimes can have a dramatic effect on evolutionary-game dynamics. However, our computations indicate that our problem of optimizing cooperation through the use of time delays does not depend on the choices of parameters such as the temptation, temperature, and payoff-matrix parameters.

[22] The random rewiring mechanism can affect the network dynamics, especially the occurrence of cascading dynamics. Numerically, we find that random rewiring can make the system more susceptible to large-scale cascading failures. This is so because without random rewiring, agent death tends to reduce the network connectivity, which can inhibit the spreading of the defective strategy and suppress any possible cascading process. In contrast, random rewiring serves to retain the network connectivity so that the defective strategy can "invade" the cluster of cooperative agents effectively. In a defective environment, an agent's partners can make it more vulnerable to bankruptcy by maintaining some high-degree value without improving the agent's payoff. As a result, cascading failures are more likely. In the presence of random rewiring, the death of agents can transform the original power-law distribution into a Poisson distribution. During the cascading process, the center of the Poisson distribution moves toward some moderate-degree value for $\beta < 0$ or toward some high-degree value for $\beta \geqslant 0$.

[23] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).

[24] F. C. Santos and J. M. Pacheco, Phys. Rev. Lett. **95**, 098104 (2005); F. C. Santos, J. F. Rodrigues, and J. M. Pacheco, Proc. R. Soc. London Ser. B **273**, 51 (2006); F. C. Santos, J. M. Pacheco, and T. Lenaerts, Proc. Natl. Acad. Sci. USA **103**, 3490 (2006).