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Optimal convergence in naming game with geography-based negotiation on small-world networks

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ABSTRACT

We propose a negotiation strategy to address the effect of geography on the dynamics of naming games over small-world networks. Communication and negotiation frequencies between two agents are determined by their geographical distance in terms of a parameter characterizing the correlation between interaction strength and the distance. A finding is that there exists an optimal parameter value leading to fastest convergence to global consensus on naming. Numerical computations and a theoretical analysis are provided to substantiate our findings.

1. Introduction

Naming Game is a model to describe, at a quantitative level, how a multi-agent system can converge toward consensus with respect to the use of a semiotic convention, a fundamental problem in the evolution of human language [1,2]. Models of semiotic dynamics have helped the development of new types of web tools, such as http://del.icio.us and http://www.flickr.com. Users of these webs share information by tagging items such as pictures and web-sites links. Due to theoretical and practical importance of semiotic dynamics, the study of naming game has received increasing attention, particularly in understanding how (linguistic) conventions originate, spread, evolve and compete over time in a population of agents [3–5]. A striking finding from previous research is that global consensus can emerge via local pairwise negotiations without any central coordination [1,2]. Since in naming game an ultimate convergence state can be reached from a multi-opinion state, the dynamics associated with the game is essentially different from that of opinion [6].

A minimal model of Naming Game based on principles of statistical physics was proposed by Baronchelli et al. [7]. This model, while being a simplified version of the original model, can generate various phenomena observed in experiments. Quite recently, the minimal naming-game model was explored from the perspective of complex networks [8–10] with a focus on the effects of topological properties on semiotic dynamics. In network science, it has been known that, compared to regular lattices, small-world and scale-free networks can facilitate the achievement of final consensus [11–14]. In addition, results differing from regular lattices [15] and mean-field framework [7] have been reported. Some modified versions of the minimal naming-game model have been proposed to better characterize the convergent behavior from multi-opinion state and understand the topological effects on consensus dynamics, such as asymmetric negotiation strategy [16] and connectivity-induced weighted words [17]. In other studies, some typical features of human sociality have been incorporated into naming games, such as finite-memory [18], local broadcast [5], and reputation [19].

In this Letter, we investigate geography-induced communication and negotiation in the minimal naming-game model. To our knowledge, geographical effects have not been considered in the studies of consensus from multi-opinion state, although geographical distance plays a significant role in many types of communications and interactions. For instance, physical signals in the transmission process may be weakened by long geographical distance, and communication can thus be restrained by the increase of distance. On the other hand, in some networked systems, long-range connections are important to maintaining systems' functioning, so

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these connections should be strengthened. Motivated by these considerations, we introduce a modified naming-game model to incorporate the effect of geographical distance. In particular, we consider agents on a geographically embedded small-world network, where the geographical distance between any pair of nodes can be meaningfully defined. We then propose a geography-based negotiation strategy to model communication and negotiation in multi-agent systems in a fairly general manner. Interaction strengths among agents are determined by the geographical distances among them in terms of a controlling parameter. Communication frequency can be tuned to be positively or negatively correlated with the distance. Our main finding is the existence of an optimal parameter value that leads to the fastest convergence toward global consensus, implying that a proper correlation between geographical distance and negotiation frequency is key to achieving fast consensus. This is of both theoretical and practical importance because fast convergence not only favors information sharing among agents, but can also help save resources for information storage (as in web servers). Perspectives such as the evolution of different names, dependence of the maximum total names on the parameter, and scaling properties of convergence time, are also explored to explain the emergence of the optimal convergence. A theoretical analysis is provided to support the numerical results.

2. Model

We consider a class of two-dimensional small-world networks that can be considered as modified networks from the Newman–Watts [20] and the Watts–Strogatz models [21]. Our small-world networks are generated as follows. Initially, a two-dimensional regular lattice of $n \times n$ nodes with periodic boundary conditions is created. Specifically, each node i carries a particular pair of integers (x_i, y_i) to represent its coordinates on the lattice. Small-world property is introduced by adding shortcuts among vertices at random, where duplicated connections are forbidden. This procedure is repeated until m shortcuts have been added. The average degree of connections $\langle k \rangle$ thus is $\langle k \rangle = (4N + 2m)/N$. Based on the coordinates, the geographical distance L_{ij} between two arbitrary nodes i and j is defined as

$$L_{ij} = |x_i - x_j| + |y_i - y_j| < n, \tag{1}$$

where (x_i, y_i) and (x_j, y_j) are the coordinates of i and j, respectively, and $|\cdot|$ represents absolute value [22]. By this definition, the geographical distance between a node and its nearest neighbors is always 1, while a pair of nodes connected by a shortcut has a longer geographical distance. The geographical distance is also called lattice ('Manhattan') distance [23,24].

We now describe our modified minimal model of naming game incorporating a geography-induced negotiation scheme. In the game, N identical agents located on a network observe a single object and try to communicate its name with others. Each agent is assigned an internal inventory or memory to store an unlimited number of different names or opinions. Initially, each agent has an empty memory. The system then evolves as follows:

(i) At each time step, a hearer j is chosen at random and then the hearer chooses one agent from its neighbors (the set of nodes connected to j) as the speaker, the probability that agent i is chosen is proportional to L_{ij}^{β}, where L_{ij} is the geographical distance between nodes i and j and β is a tunable parameter.

(ii) If the speaker i's inventory is empty, it invents a new name and records it. Otherwise, if i already knows one or more names of the object, with equal probability it randomly choose one name from its inventory. The invented or selected word is then transmitted to the hearer.

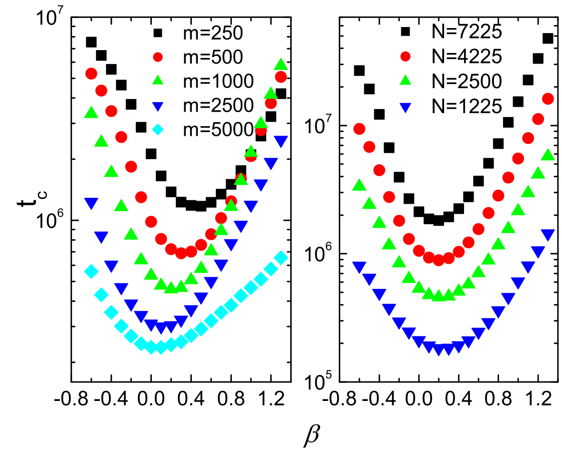

Fig. 1. (Color online.) Convergence time t_c as a function of the geographical parameter β for different values of m with network size 50×50 (left panel) and for different network sizes with fixed average degree of connections $\langle k \rangle = 4.8$ (right panel). Each data point is obtained by averaging over 1000 runs on each of ten different network realizations.

(iii) If the hearer j already has this transmitted name in its inventory, negotiation is regarded as successful, and both agents keep this common name and delete all other names in their memories; otherwise, the negotiation fails, and the new name is included in the memory of the hearer without any deletion, i.e., learns the new word. By repeating the above process, the system evolves.

The geography-induced negotiations refers to the manner of an agent choosing a neighbor as a speaker to communicate according to the geographical distance between them. If $\beta < 0$, the nearest neighbors have more chances to be chosen as a speaker; If $\beta > 0$, the neighbors connected by shortcuts are more likely to be chosen as a speaker. For $\beta = 0$, the model reduces to the previously studied naming-game model [12]. Thus, by tuning the value of β, it is possible to address the roles of shortcuts in the dynamics of language game.

3. Simulation results

In the naming-game model, the system can reach an absorbing state in which all agents share one exclusive word after a period of time. The specific time is defined as convergence time t_c, which is a key measure for the convergence efficiency of the system and is of practical importance. Generally, the fast collective agreement on naming objects plays a significant role in the cooperation or communication among individuals for not only intelligent agents but also human beings. We first study the dependence of the convergence time t_c on the geographical parameter β. The left panel of Fig. 1 shows t_c as a function of β for different numbers of added shortcuts m. It can be seen that the convergence time t_c is a non-monotonic function of β. In each case, there is an optimal value of β leading to fastest convergence. This result demonstrates that a proper correlation between geographical distance and negotiation frequency plays the key role in achieving the optimal consensus. Another observed phenomenon is that the optimal value β_{opt} is negatively correlated with the number of shortcuts m. In addition, we also find that, for β quite larger than β_{opt}, e.g., $\beta = 1.2$, t_c associated with larger values of m can be greater than that with lower values of m (see for instance, $m = 250$, $m = 500$, and $m = 1000$). This seems to contradict to intuition that more edges usually better facilitate convergence towards global consensus. We will explain this abnormal phenomenon later. These results indicate that the shortcuts play a significant role for the global convergence, espe-

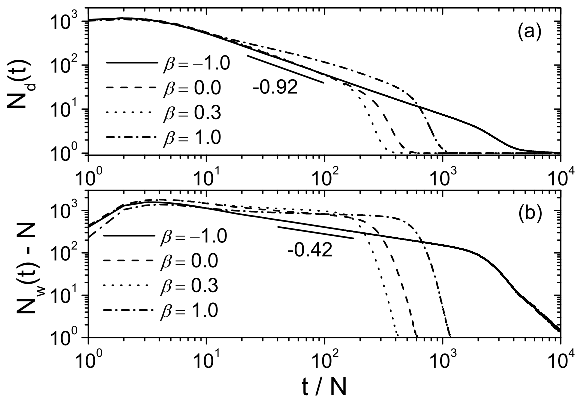

Fig. 2. (a, b) Evolutions of $N_d(t)$, the number of different names, and the total number of names $N_w(t)$, respectively, for different values of β. The line segments indicate the asymptotic power-law decays. The size of all networks is 50×50 and the number of shortcuts is $m = 500$.

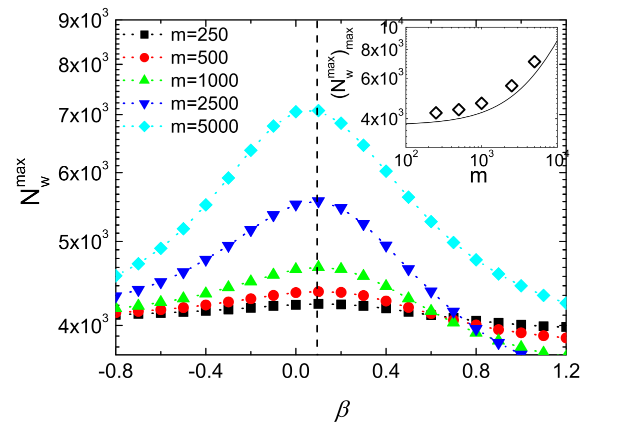

Fig. 3. (Color online.) The maximum number of total names of agents N_w^{max} versus the parameter β for different values of m. We see that N_w^{max} reaches its maximum for $\beta \approx 0.1$. Comparison between the numerically obtained maximum value of N_w^{max} and theoretical predictions is shown in the inset, where 1000 runs for each of ten different network realizations are used to obtain the average value of $(N_w^{max})_{max}$. The size of all networks is 50×50.

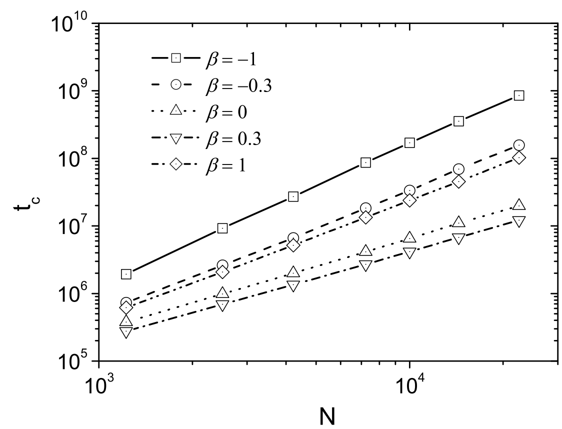

Fig. 4. Convergence time t_c as function of network size N for different values of β. We observe the scaling $t_c \sim N^{\gamma}$. The values of γ are 2.10, 1.85, 1.37, 1.30 and 1.76 for $\beta = -1$, $\beta = -0.3$, $\beta = 0$, $\beta = 0.3$ and $\beta = 1$, respectively. Data points are obtained by averaging over 1000 runs on each of ten different network realizations. The number of shortcuts m is set to be $0.2 \times N$ for each data set.

cially for sparse-shortcut cases. The right panel in Fig. 1 shows the dependence of t_c on β for different network sizes at fixed average degree of connection $\langle k \rangle = 4.8$. We find that larger network sizes result in longer convergence time but the values of β_{opt} are approximately unchanged. That is, β_{opt} is independent of the network size.

In order to understand the role of geography-induced negotiations in achieving consensus, we study the time evolutions of basic quantities such as the number $N_d(t)$ of different words in the system and the number of total words, $N_w(t)$, where $N_d(t)$ reflects the number of word clusters at time t, within which agents share the same word, as shown in Fig. 2. Specifically, each word cluster is contained within boundaries of agents with more than one word (on average, between one and two). When β is negative, few shortcuts are used and consensus is reached through a coarsening process on a two-dimensional lattice [15]. As Ref. [15] points out, the characteristic length of a word cluster's boundary $\xi(t)$ grows as $\sqrt{t}$ on a two-dimensional lattice, and the size of a cluster grows as t, so $N_d(t)$ decays as t^{-1} for negative values of β. For $\beta = 0$, the system exhibits coarsening at first, but the shortcuts induce an abrupt jump toward consensus when the word cluster's average characteristic length of the boundary is comparable to the typical distance between nodes with shortcuts [5,11]. If the negotiation frequency along shortcuts is relatively low, agents can frequently interact with their nearest neighbors. This favors the establishment of local consensus at the early stage, but not global consensus. The links corresponding to long geographical distances usually connect two different word clusters, so enhancing the interaction frequency of agents at the ends of shortcuts can increase the convergence performance. However, if the interaction frequency is too high, agents can hardly interact with their nearest neighbors, which prevents the formation of word clusters. As a result, the decay of $N_d(t)$ slows down at the coarsening stage. Due to the fact that the high interaction frequency along long range links can inhibit the formation of local word cluster, so as to disfavor convergence, the presence of more long range links associated with large values of β can induce larger values of t_c, although more links usually are better for reaching consensus. This can explain the abnormal phenomenon in Fig. 1 that larger number of added links (e.g. $m = 1000$) for $\beta = 1.2$ results in longer convergence time than that for smaller number of added links (e.g., $m = 250$).

Fig. 2(a) demonstrates the decay of $N_d(t)$ for $\beta = -1$. Similar results can be observed for other large negative values of β. Fig. 2(b) shows the total number $N_w(t)$ of names as a function of the rescaled time t/N. At the coarsening stage, the total number of words can be expressed as the number of nodes N plus the number of nodes with more than one word (on average, between one and two) [5], i.e.,

$$N_w(t) - N \sim \frac{N}{\xi(t)^2} \times \xi(t) \sim \frac{N}{\sqrt{t}}. \tag{2}$$

For $\beta = -1$, our simulation results shows that $N_w(t) - N$ decays as $t^{-0.42}$ at the coarsening stage. Fig. 2(b) demonstrates that the parameter β has a significant effect on the maximum total memory of agents, N_w^{max}. It is thus insightful to compute the relation between β and N_w^{max}, as shown in Fig. 3. We find that, when the number of shortcuts is large, the values of β for which N_w^{max} is maximized coincide with the optimal value β_{opt}. An increase in the maximum value of N_w^{max} as m is increased corresponds to a faster convergence toward global consensus. In the next section, we provide theoretical predictions for the value β_{opt} and the maximum values of N_w^{max}.

We have also investigated the scalings of the convergence time and the maximum total memory with respect to the network size. We first fix $\langle k \rangle$ for each network to obtain the dependence of t_c and N_w^{max} on the network size N. Fig. 4 shows t_c as a function of N for different values of β. The algebraic scaling is robust and the value of the scaling exponent depends on the value of β. Fig. 5 displays N_w^{max} as a function of N for different values of β. Similar

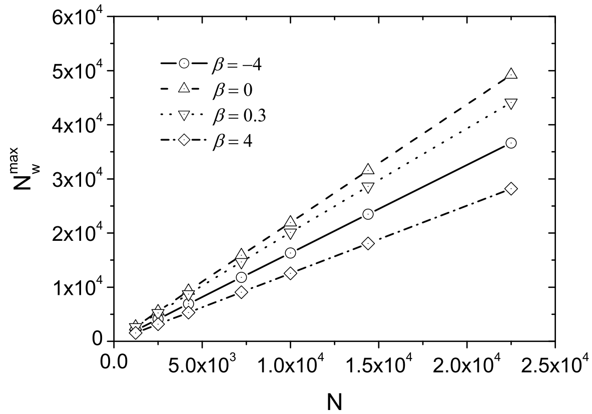

Fig. 5. Maximum total memory N_w^{max} used by agents as function of network size N for different values of β. We see that N_w^{max} scales linearly as N. The number of shortcuts m is set to N for each data set.

to the results in the previous model [7], N_w^{max} scales linearly with the size of the network, but the rate of increase depends on β.

4. Theoretical treatment

Assuming that each agent i interacts with one neighbor j, the probability for repeating such an interaction is

$$P_r = \frac{1}{N}\left(\frac{L_{ij}^{\beta}}{\sum_{l=1}^{N} A_{il} L_{il}^{\beta}} + \frac{L_{ij}^{\beta}}{\sum_{l=1}^{N} A_{jl} L_{jl}^{\beta}}\right), \tag{3}$$

where A_{ij} are elements of the network adjacency matrix. For small-world networks, nodes at the ends of long-range links usually have larger degrees, so the distance between an arbitrary pair of nodes i and j can be estimated by their degrees, i.e., $L_{ij} \sim k_i$ and $L_{ij} \sim k_j$. The repetition probability can thus be approximated as

$$P_r^{ij} = \frac{1}{N}\left(\frac{k_i^{\beta}}{\sum_{l=1}^{N} A_{il} k_l^{\beta}} + \frac{k_j^{\beta}}{\sum_{l=1}^{N} A_{jl} k_l^{\beta}}\right). \tag{4}$$

For a small-world network, degree-to-degree correlation is absent, so the sum in Eq. (4) can be simplified as

$$\sum_{l=1}^{N} A_{il} k_l^{\beta} = k_i \sum_{k'=k_{min}}^{k_{max}} P(k'|k) k'^{\beta} = k_i \sum_{k'=k_{min}}^{k_{max}} \frac{k'^{1+\beta} P(k')}{\langle k\rangle} = \frac{k_i \langle k^{1+\beta}\rangle}{\langle k\rangle}, \tag{5}$$

where the conditional probability $P(k'|k) = k' P(k')/\langle k\rangle$ and the identity

$$\sum_{k'=k_{min}}^{k_{max}} k'^{1+\beta} P(k') = \langle k^{1+\beta}\rangle \tag{6}$$

have been used. We then have

$$P_r^{ij} = \frac{\langle k\rangle}{N\langle k^{1+\beta}\rangle}\left(k_i^{\beta-1} + k_j^{\beta-1}\right). \tag{7}$$

The success rate in the steady range can be obtained by summing over all nodes i and j, as

$$S = \sum_{i,j=1}^{N} P_r^{ij} = \frac{2\langle k\rangle\langle k^{\beta-1}\rangle}{\langle k^{1+\beta}\rangle}. \tag{8}$$

During the evolution, there are two factors contributing to the change in $N_w(t)$: one is success of a negotiation through an interaction between two agents, which will result in the deletion of names in both agents; the other is failure, which will result in one name being included in the hearer's memory. Since the small-world networks are homogeneous with respect to node degrees, we can assume that the memory of agents are approximately identical, i.e., N_w/N. We have examined that the assumption is valid for relatively large m given network size N, e.g., $m \geqslant 250$ for $N = 2500$. The evolution of N_w can thus be expressed as

$$\frac{dN_w(t)}{dt} = -S(t)\cdot 2\left[\frac{N_w(t)}{N} - 1\right] + \left[1 - S(t)\right], \tag{9}$$

where the first term on the right-hand side represents the contribution of the success to the change in N_w, while the second term is due to failure. Since we found that the maximum value N_w^{max} of $N_w(t)$ arises when the success rate stays in steady state, the quantity N_w^{max} can be obtained by inserting the steady value of S into Eq. (9) and setting

$$\frac{dN_w(t)}{dt} = 0, \tag{10}$$

which yields

$$N_w^{max} = \frac{N}{2}\left(1 + \frac{\langle k^{\beta+1}\rangle}{2\langle k\rangle\langle k^{\beta-1}\rangle}\right). \tag{11}$$

As $\beta \to \infty$ or $\beta \to -\infty$, we have $\langle k^{\beta+1}\rangle \approx \langle k^{\beta-1}\rangle$ and, hence,

$$N_w^{max}(\beta \to \infty) \approx N_w^{max}(\beta \to -\infty) = \frac{N}{2}\left(1 + \frac{1}{2\langle k\rangle}\right). \tag{12}$$

The maximum value of N_w^{max} can be expected in the vicinity of $\beta = 0$, giving rise to the largest ratio of $\langle k^{\beta+1}\rangle$ to $\langle k^{\beta-1}\rangle$. The maximum value of N_w^{max} can then be approximated by considering $\langle k^{\beta+1}\rangle \approx \langle k\rangle^{\beta+1}$ and $\langle k^{\beta-1}\rangle \approx \langle k\rangle^{\beta-1}$. We obtain

$$\left(N_w^{max}\right)_{max} \approx \frac{N}{2}\left(1 + \frac{\langle k\rangle}{2}\right). \tag{13}$$

Since for small-world networks, $\langle k\rangle = (4N + 2m)/N$, so the maximum value of N_w^{max} as a function of m can be expressed as

$$\left(N_w^{max}\right)_{max} \approx \frac{3N + m}{2}. \tag{14}$$

Theoretical predictions, as represented by Eqs. (13) and (14), agree well with numerical simulations, as shown in Fig. 3. In particular, the maximum value of N_w^{max} occurs for $\beta \approx 0$. For large value of m, the value of β for the fastest convergence can thus be predicted by Fig. 3, since when the maximum value of N_w^{max} is attained, the convergence to global consensus is the fastest.

5. Conclusion

In summary, we have proposed and studied a naming-game model on small-world networks embedded in Euclidean geographical space. Negotiation frequency between two agents is determined by their geographical distances in terms of a controlling parameter, β. We find that there exist optimal values of β that can lead to the fastest convergence toward global consensus. This finding unveils that a proper correlation between geographical distance and communication frequency can optimize the achievement of final consensus; while biased communication in local or global connections can hamper or even prevent convergence in the agreement dynamics. We have qualitatively explained the dependence of the convergence time on β, in terms of the evolutions of the number of different names and of the total maximum memory used by agents. In the intermediate range of β, we found a peak of the total maximum memory, where the fastest convergence occurs for relatively large numbers of shortcuts. Numerical results of the total maximum memory can be predicted theoretically, providing insight into the mechanism for the fastest convergence. We have also found robust scaling behaviors for the convergence time and the maximum total memory. Geographical effects are common in a variety of real communication networks, and we expect our work to provide new insights into agreement dynamics and other related dynamical processes on such networks.

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