PHYSICS

Special Topic: Network Science

Controlling complex, non-linear dynamical networks

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INTRODUCTION

An outstanding problem in the field of complex dynamical systems is to control non-linear dynamics on complex networks. Indeed, the physical world in which we live is non-linear, and complex networks are ubiquitous in a variety of natural, social, economical, and man-made systems. Dynamical processes on complex networks are thus expected to be generically non-linear. While the ultimate goal to study complex systems is to control them, the coupling between non-linear dynamics and complex network structures presents tremendous challenges to our ability to formulate effective control methodologies. In spite of the rapid development of network science and engineering toward understanding, analyzing and predicting the dynamics of large complex network systems in the past 15 years, the problem of controlling non-linear dynamical networks remains to be outstanding.

Control of non-linear dynamical systems

Controlling chaos in non-linear dynamical systems has been studied for more than two decades since the seminal work of Ott, Grebogi, and Yorke [1]. The basic idea was that chaos, while signifying random or irregular behavior, should not be viewed as a nuisance in applications of non-linear dynamical systems. In fact, given a chaotic system, that there are an infinite number of unstable periodic orbits embedded in the underlying chaotic invariant set means that there are an equally infinite number of choices for the operational state of the system depending on need, provided that any such state can be stabilized. Then, the intrinsically sensitive dependence on initial conditions, the hallmark of any chaotic system, implies that it is possible to apply small perturbations to stabilize the system about any desirable state. Controlling chaos has since been studied extensively, and examples of successful experimental implementation are abound in physical, chemical, biological, and engineering systems [2]. The vast literature on controlling chaos, however, has been limited to low-dimensional systems, systems that possess one or a very few unstable directions (i.e. one or a very few positive Lyapunov exponents). Complex networks with non-linear dynamics are generally high dimensional, rendering inapplicable existing methodologies of chaos control.

Linear controllability of complex networks

In the past several years, a framework for determining network controllability based on control and graph theories emerged [3–5], leading to quantitative understanding of the effect of network structure on its controllability. For example, a structural controllability framework was proposed [4], revealing that the ability to steer a complex network toward any desired states, as measured by the minimum number of driver nodes, is determined by the set of maximum matching, which is the maximum set of links that do not share starting or ending nodes. A main result was that the number of driver nodes required for full control is determined by the network's degree distribution [4]. The framework was established for weighted and directed networks. More recently, an alternative framework, the exact controllability framework, has been formulated [5], which is based on the principle of maximum multiplicity to identify the minimum set of driver nodes required to achieve full control of networks with arbitrary structures and link-weight distributions. The deficiency of such rigorous mathematical frameworks of controllability is that the nodal dynamical processes must be assumed to be linear. Even for linear dynamics, when a complex network is controllable according to the mathematical controllability theories, often the actual control would require unreasonably large amount of energy, raising the serious issue of physical controllability of complex networks. For non-linear nodal dynamics, the mathematical framework on which the controllability theories are based, namely the classic Kalman's controllability rank condition [6, 7], is not applicable. While controllability for non-linear control can be formulated based on Lie brackets [8], it may be difficult to implement the abstract framework experimentally for complex networks. To control non-linear dynamics on complex networks is at the present an outstanding and challenging problem.

PERSPECTIVE OF CONTROLLING COMPLEX, NON-LINEAR DYNAMICAL NETWORKS

That chaos control can be done for low-dimensional systems and controllability theory of complex networks is limited to linear dynamics calls for drastically different approaches to controlling non-linear dynamics on complex networks. While there have been previous works on specific control methods such as pinning control [9–11] and brute-force control that relies on altering the state variables of the underlying system that in realistic situations can be difficult to implement, a general framework for actual control of complex networks with non-linear dynamics through realistic, physical means has not been achieved. The main difficulty in this field lies in the extremely diverse non-linear dynamical behaviors that a network can generate, making it practically impossible to define general mathematical framework for control. Our idea is that, in the formulation of control of non-linear networked systems, a physically meaningful approach may not require detailed control of all state variables. Potentially, this would enable us to develop a general framework of controllability of non-linear dynamical networks based on physical/experimental considerations.

The general philosophy underlying our idea is the fact that the traditional control theoretical tools for linear dynamical systems aim to control the detailed states of all of the variables, which is in fact an overkill for most systems. A common feature of non-linear dynamical systems is the emergence of a large number of distinct, coexisting attractors [12]. Often the performance and functions of the system are determined by the particular attractor that the system has settled into, to which the detailed states of the dynamical variables are not relevant. The key is thus to develop control principles whereby we nudge a complex, non-linear system from attractor to attractor through small perturbations to a set of physically or experimentally feasible parameters. Here, we wish to convey the message that a controllability framework can be developed for non-linear dynamical networks based on the control of attractors.

To describe generally how control can be articulated and implemented, we recall that the reason for control is that the current system is likely to evolve into an undesirable state (attractor) or the system is already in such a state, and one wishes to apply perturbations to bring the system out of the undesirable state and steer it into a desirable state. The first step is then to identify a final state or attractor of the system that leads to the desirable performance. The next step is to choose a set of experimentally adjustable parameters and determine whether small perturbations to these parameter can bring the system to the desirable attractor. That is, under physically realizable perturbations there should be a control path between the undesirable and the desirable attractors. The path can be directly from the former to the latter, or there can be intermediate attractors on the path. For example, due to the physical constraint on the control parameters and the ranges in which they can be changed, one can drive the system into some intermediate attractor by perturbing one set of parameters, and then from the intermediate attractor to the final attractor by using a different set of parameter control. For a complex, non-linear dynamical network, the number of coexisting attractors can be large. Given a set of system performance indicators, one can classify all the available attractors into three categories: the undesirable, desirable, and the intermediate attractors. We say a non-linear network is controllable if there is a control path from any undesirable attractor to the desirable attractor under finite parameter perturbations. Regarding each attractor as a node, and the control paths as directed links or edges, we can construct an 'attractor network', whose properties determine the controllability of the original networked dynamical system. For example, the average path length from an undesirable to a desirable attractor and the 'control energy' (or the amount of necessary parameter perturbations) can serve as quantitative measures to characterize the controllability of the original network.

CONSTRUCTION OF ATTRACTOR NETWORKS

Given a non-linear dynamical network, the attractor network can be constructed, as follows. We first identify all possible asymptotic states, or attractors, of the system. We then choose a set of system parameters that can be perturbed. Setting the system in a specific attractor i, for a reasonable combination of adjustable parameters, we can determine the set of attractors into which the system can evolve from the original attractor i. Effectively, under the given set of parameters, there is a link from attractor i to any of the new attractors after the parameter perturbations. Repeating this procedure for all attractors in the system, we build up an attractor network that provides a blueprint for driving the whole networked system from any attractor to any other attractor in the system, assuming at the time the latter attractor would lead to desirable function of the system as a whole. All these can be done using small perturbations. We have obtained preliminary evidence, using a class of gene regulatory networks (GRNs) of increasing size, that our idea of attractor network is effective and represents an innovative, experimentally implementable way to control non-linear dynamical networks.

The idea of attractor network provides a realistic framework to control non-linear dynamics on complex networks. Strikingly, it leads to another counterintuitive idea that control can be benefited from noise. In particular, more than three decades of intense research in non-linear dynamical systems has led to great knowledge about the role of noise, in terms of phenomena such as stochastic resonance, coherence resonance, and noise-induced chaos [12], etc. Common to all these phenomena is that a proper amount of noise can in fact be beneficial, for example, to optimize the signal-to-noise ratio, to enhance the signal regularity or temporal coherence, or to facilitate the transitions among the attractors. Since the foundation of our method is control paths in the attractor network that characterizes how 'easy' or 'difficult' for the system to transition from one attractor to another, we speculate that, under a given set of physical control parameters and given range of parameter variations, noise can facilitate the transition process, leading to the emergence of control paths that were previously not possible or to more favorable control paths with less energy requirement. The inevitable presence of noise in any realistic system and the possibility that noise has the potential to greatly facilitate the control of non-linear dynamical networks open up a new avenue to harness complex, non-linear networked systems in realistic applications.

To demonstrate these ideas, we have carried out detailed studies using small non-linear networks at Arizona State University. The models include a two-node GRN with four coexisting attractors and a three-node GRN with eight attractors, both being derived from experiments. We identify the coupling strengths as control parameters, because they can be experimentally adjusted by applying targeted drugs. Under the constraint that the control parameters can be perturbed in given ranges, attractor networks are constructed, providing complete information about the feasibility of driving the system from one attractor to another and consequently a quantitative understanding of the controllability of the underlying network. There is also preliminary evidence that noise can enhance the controllability significantly.

CONCLUSION

The ability to control complex networks is of uttermost importance to many critical problems in science and engineering, and has the potential to generate great technological breakthroughs. We argue that it is possible to develop a controllability framework for complex, non-linear dynamical networks based on the idea of attractor networks. For the field of complex dynamical systems, the framework will lead to landscape changes, revolutionizing our ability to control the systems. Another field that will benefit tremendously from this is systems and synthetic biology, where a basic problem is to control GRNs. To be able to control complex, non-linear networks will also have significant impacts on fields such as computer networks, wireless networks, cybersecurity, biological networks, and even social and economical networks.

Our framework of attractor networks has the appealing features that (a) it is applicable to non-linear dynamical networks in general, (b) the attractor network can possess quite simple structure even for large, complex networks, and (c) noise can enhance the controllability. The benefit of noise, while counterintuitive, has its origin in well-known phenomena in non-linear dynamical systems such as stochastic resonance and coherence resonance. There are, however, difficulties with the attractor network framework. For example, for large networks the construction of an attractor network may be quite challenging—the scalability issue. In addition, the structure of the attractor network in general depends on the system parameters. While we emphasize the need to focus on physically meaningful and experimentally accessible parameter perturbations, there can still be a large number of attractor networks depending on the parameters, making it difficult to formulate a rigorous mathematical framework. We believe that these issues can and will be satisfactorily addressed in the near future, finally realizing the grand goal of controlling non-linear dynamical networks.

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REFERENCES

1. Ott, E, Grebogi, C and Yorke, JA. *Phys Rev Lett* 1990; **64**: 1196–9.
2. Boccaletti, S, Grebogi, C, Lai, YC *et al. Phys Rep* 2000; **329**: 103–97.
3. Lombardi, A and Hörnquist, M. *Phys Rev E* 2007; **75**: 056110.
4. Liu, YY, Slotine, JJ and Barabási, AL. *Nature* 2011; **473**: 167–73.
5. Yuan, ZZ, Zhao, C, Di, ZR *et al. Nat Commun* 2013; **4**: 2447.
6. Kalman, RE. *SIAM J Appl Math* 1963; **1**: 152–92.
7. Lin, CT. *IEEE Trans Autom Control* 1974; **19**: 201–8.
8. Slotine, JJ and Li, W. *Applied Nonlinear Control*. New Jersey: Prentice-Hall, 1991
9. Wang, XF and Chen, G. *Physica A* 2002; **310**: 521–31.
10. Li, X, Wang, XF and Chen, G. *IEEE Trans. Circuits Syst. I* 2004; **51**: 2074–87.
11. Sorrentino, F, di Bernardo, M, Garofalo, F *et al. Phys Rev E* 2007; **75**: 046103.
12. Lai, YC and Tél, T. *Transient Chaos—Complex Dynamics on Finite-Time Scales*. New York: Springer, 2011.

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MULTIDISCIPLINARY

Special Topic: Network Science

Theoretical progress and practical challenges in controlling complex networks

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The issue of control has gained significant prominence in engineering, useful for aircraft control, manufacturing processes, communication systems and so on. Yet, the control of complex networks was barely studied, as we lack powerful theories to address it in a quantitative fashion.

Fair recently, considerable efforts were made to address the controllability issue of complex networks [1]. As a key notion in control theory, controllability concerns our ability to drive a dynamic system from any initial state to any final state in finite time [2], which agrees well with our intuitive notion of control. To model complex networks as dynamic systems, we normally adopt the so-called *nodal dynamics*, i.e. we associate each node in a network with a state variable, whose time evolution crucially depends on the state variables of the node itself and/or its neighbors.