Nonlinear Dyn (2018) 93:1671–1680
https://doi.org/10.1007/s11071-018-4283-1

ORIGINAL PAPER

Effect of network structural perturbations on spiral wave patterns

Yafeng Wang · Dongmei Song · Xiang Gao · Shi-Xian Qu · Ying-Cheng Lai · Xingang Wang

Received: 8 December 2017 / Accepted: 9 April 2018 / Published online: 20 April 2018
© Springer Science+Business Media B.V., part of Springer Nature 2018

Abstract Dynamical patterns in complex networks of coupled oscillators are of both theoretical and practical interest, yet to fully reveal and understand the interplay between pattern emergence and network structure remains to be an open problem. Among the many outstanding issues, a fundamental one is how the network structure affects the stability of dynamical patterns. To address this issue, we focus on the spiral wave patterns and investigate the effects of systematically added random links on their stability and dynamical evolutions. We find that, as the network structure deviates more from the regular topology and thus becomes increasingly more complex, an originally stable spiral wave pattern can disappear but different types of patterns can emerge. In addition, short-distance links added to a small region containing the spiral tip can have a more significant effect on the wave pattern than long-distance connections. As more random links are introduced into the network, distinct pattern transitions can occur, such as the transition of the spiral wave pattern to a global synchronization state, to a chimera-like state, or to a pinned spiral wave. About the transition points, the network dynamics are highly sensitive to small structural perturbations in that the addition of even a single link can change the pattern from one type to another. These findings provide additional insights into the pattern dynamics in complex networks, a problem that is relevant to many physical, chemical, and biological systems.

Keywords Coupled oscillators · Spiral waves · Complex network · Chimera-like state · Pinned spiral

Y. Wang · D. Song · X. Gao · S.-X. Qu · X. Wang (✉)
School of Physics and Information Technology, Shaanxi Normal University, Xi'an 710062, China
e-mail: wangxg@snnu.edu.cn

Y.-C. Lai
School of Electrical, Computer, and Energy Engineering, Arizona State University, Tempe, AZ 85287, USA

1 Introduction

Pattern formation is ubiquitously observed in spatiotemporal dynamical systems in nature [1,2], ranging from granular materials [3] to ecosystems [4] and plants [5]. Complex dynamical networks such as coupled oscillators are naturally spatiotemporal systems. The past two decades have witnessed a rapid growth of research on various types of dynamical processes in complex networks [6,7], including synchronization [8,9], epidemic spreading [10,11], traffic congestion [12–14], and cascading failures [15,16]. In these studies, a primary issue was to address the interplay between the dynamical processes and the underlying network structure. In exploring the interplay between network structure and dynamics, an interesting topic is the formation of spatiotemporal patterns in complex networks, where one of the central questions to be answered is how the network structure affects the stability and evolution of the dynamical patterns [6–16]. To our knowledge, in spite of the vast literature on dynamics in complex networks, a systematic study of the interplay between pattern formation and network structure is still lacking. The purpose of this paper is to fill this knowledge gap by presenting results on pattern emergence, evolution, and transitions on networks undergoing systematic random structural perturbations.

To probe into the interplay between topological structure and dynamical patterns in a concrete manner, we focus on complex networks of coupled oscillators. An interesting phenomenon in such dynamical networks is that, under certain conditions, the oscillators can be self-organized to form spatial patterns [1]. As the formation of the patterns relies heavily on the symmetry of the coupling structure of the network, an intuitive thinking would suggest that it is unlikely for complex networks to generate dynamical patterns [17,18]. Yet, in natural and man-made systems, there do have situations where well-organized dynamical patterns are formed on systems with complex network structure, e.g., the firing patterns in the human brain [19]. A paradox was then how spatially ordered patterns can emerge from random or disordered coupling structures associated with a complex network. There have been previous efforts devoted to resolving this paradox. For example, pattern formation in complex network of coupled activators and inhibitors was studied, where Turing-like patterns were observed [20]. Complex networks of coupled excitable nodes were also studied [21] with respect to pattern formation in which the technique of dominant phase advanced driving was introduced, leading to the discovery of target-like wave patterns. Desynchronization patterns in complex network of coupled chaotic oscillators were subsequently studied [22], where it was found that reordering network nodes according to the eigenvector of the unstable mode can be effective at identifying the stable synchronous pattern from an asynchronous state. Recently, computational graph algorithms were introduced into the field of network synchronization to study synchronous patterns in large-size complex networks [23,24], where the important role of network symmetry in pattern formation was elucidated. In spite of the existing works, many questions concerning pattern formation and transition in complex networks remain, especially with respect to relatively more sophisticated patterns possessing complex spatial structures, e.g., the spiral waves [25].

The starting point of our study is then spiral waves in coupled oscillator networks, which are patterns observed ubiquitously in physical, chemical and biological systems [1]. Different from other types of patterns such as Turing patterns, the stability of a spiral wave depends crucially on the motion of the spiral tip [26,27], leading to the specially designed methods for analyzing, inducing and controlling spiral waves [28–31]. In most previous works, spiral waves were studied for networks with a regular spatial structure, such as a periodic lattice. Nevertheless, there were works on spiral waves in systems with an irregular spatial structure [32–35]. For example, the formation of spiral waves in a medium possessing random (small-world) connections was studied [33] with the finding that, while the structural irregularity is detrimental to forming and sustaining a spiral wave, a small number of random links can counterintuitively enhance the wave stability. Another work [34] revealed that, random connections added locally to a regular medium can cause the meandering motion of the spiral tip to approach a fixed point. It was later found [35] that random connections introduced globally into a regular medium can lead to rich behaviors in the transition of the system dynamics among global synchronization state, steady state, and multiple spirals. These previous works indicated that the network structure can have nontrivial and intriguing effects on pattern formation and transition, yet a systematic study on the effects of structural perturbations on pattern formation is still lacking. For instance, it remains not clear how to eradicate spiral wave effectively by introducing long-distance connections and, whether the spiral wave can be controlled (switched) to other spatiotemporal patterns by adding or removing just a single link.

To facilitate computation and make our analysis simple (but not simpler), we adopt the model of coupled map lattices (CML) [36] to investigate the dynamical responses of spiral waves to structural perturbations. Historically, CMLs were used to understand spiral wave patterns in complex media such as granular materials [37–40]. In our work, starting from a two-dimensional regular lattice capable of generating stable spiral waves, we systematically introduce random links into the network and study the transitions in the pattern dynamics as the network structure becomes increasingly random (complex). We uncover dynamical patterns and richer bifurcations that were not reported in previous works [33–35], such as chimera-like states [41–44] where two synchronization clusters coexist with many asynchronous oscillators and the pinned multi-armed spirals [31] in which the arms of the spiral are pinned to a square-shape boundary. An intriguing finding is that, in the region where pattern transitions occur, the dynamics are extremely sensitive to small changes in the network structure. These findings shed new lights on pattern behaviors in complex networks, which may lead to effective methods to control pattern dynamics in spatiotemporal dynamical systems.

In Sect. 2, we introduce our CML model that is capable of generating spiral waves, and describe our strategy to introduce random links. In Sect. 3, we investigate pattern transitions as induced by a systematic change in the network structure, and present evidence of two types of patterns that have not been uncovered previously. In Sect. 4, we demonstrate the sensitivity of the patterns in the transition regions to small structural perturbations, and elucidate the topological properties of the critical links. Discussions will be given in Sect. 5.

2 Model and method

2.1 Network model

We study the following CML network model [36,40]:

$$x_i(n+1) = f(x_i(n))+\varepsilon \sum_{j=1}^{N^2} a_{ij}[f(x_j(n))-f(x_i(n))], \tag{1}$$

with $i, j = 1, \ldots, N$ the nodal indices, $N^2 = N \times N$ the system size, x_i the state of the ith node at time n, and ε the uniform coupling parameter. The isolated dynamics of node i is governed by the nonlinear equation $x_i(n+1) = f[x_i(n)]$. The coupling structure of the system is characterized by the adjacency matrix $\mathbf{A}$ whose elements are given as: $a_{ij} = 1$ if maps i and j are directly connected and $a_{ij} = 0$ otherwise. The degree of node i, the total number of links attached to it, is $k_i = \sum_j a_{ij}$.

Initially, the network takes the structure of two-dimensional regular lattice, where each interior node is coupled to its four nearest neighbors. We assume the free boundary condition and fix the system size to be 300×300 (i.e., $N = 300$). The spatial location of a node in the network is denoted as (l_x, l_y), where $1 \le l_x, l_y \le N$. For the nodal dynamics, we adopt the piecewise linear map [40,46]:

$$f(x) = \begin{cases} ax, & x < x_g, \\ b, & x \ge x_g, \end{cases} \tag{2}$$

where $x \in (0, 1)$, $x_g = 1/a$, a and b the independent parameters. Equation (2) is the discrete version of the differential Chay model used widely in computational neuroscience, and is capable of generating the similar bifurcation scenario of inter-spike interval (ISI) observed from experiments [45]. To be concrete, we fix $(a, b) = (2.5, 0.1)$, for which an isolated node possesses a super-stable period-three orbit [40,46]: $x_1^* = b \equiv A$, $x_2^* = ab \equiv B$, and $x_3^* = a^2b \equiv C$.

2.2 Generation of spiral waves

For a CML system, spiral waves can be stimulated through special initial conditions [40]. For example, we can choose the initial states of the nodes within a narrow strip in the lattice, say $l_x \in [280, 300]$, randomly within the range (0, 1), while nodes outside the strip are set to have the initial value zero. For $\varepsilon = 0.166$, after a transient period of $n = 2.5 \times 10^3$ iterations, a stable three-armed spiral pattern is generated, as shown in Fig. 1a, which is a snapshot of the system state. As the system evolves, the spiral arms rotate in a synchronous fashion and propagate outward from the tip. This feature of wave propagation is similar to that of spiral waves observed in other contexts, e.g., excitable media [1]. A close examination of the motion of the spiral tip reveals a difference: In an excitable medium, the tip trajectory is often regular [47], but in our system of CML it moves randomly inside the central region $100 < l_x, l_y < 200$, as shown in Fig. 1b. In addition, in regions separated by the spiral arms, the nodes are synchronized into three distinct clusters, with nodes in each cluster being synchronized to the trajectory of a periodic point of the period-three orbit. The synchronous clusters have approximately the same size. The spiral arms themselves comprise asynchronous nodes, which constitute the cluster boundaries [46]. In the following studies, we shall adopt the spiral pattern in Fig. 1a as the initial state, and investigate the impacts of the randomly added links on the stability and transition of this pattern.

2.3 Effect of adding random links on spiral wave patterns

As the dynamics of the spiral wave is slaved to the tip, applying random structural perturbations to the

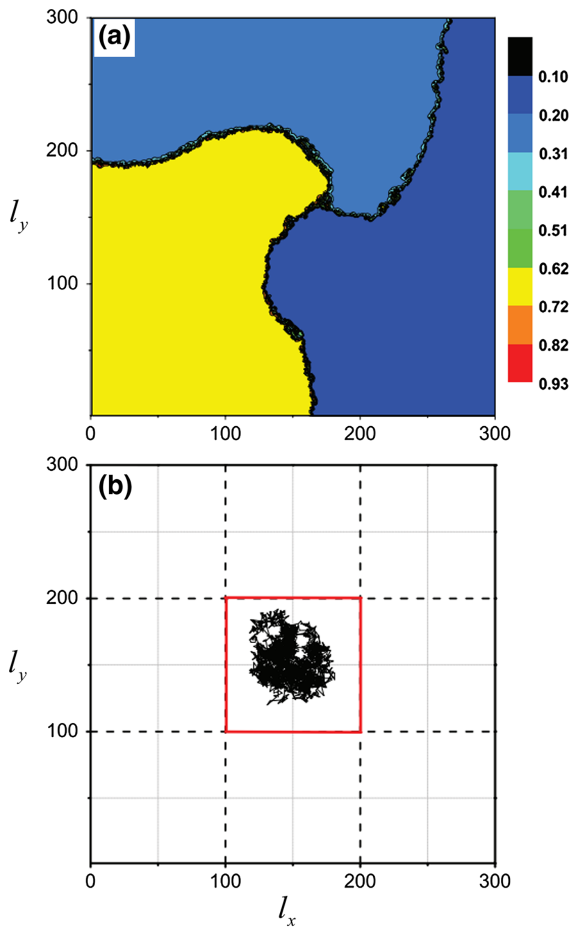

Fig. 1 (Color online) For $\varepsilon = 0.166$, the three-armed spiral wave pattern generated in the regular lattice of 300×300 coupled chaotic maps. **a** A snapshot of the system dynamics taken at the iteration step $n = 2.5 \times 10^3$. **b** The random motion of the spiral tip inside the central area marked by the (red) square

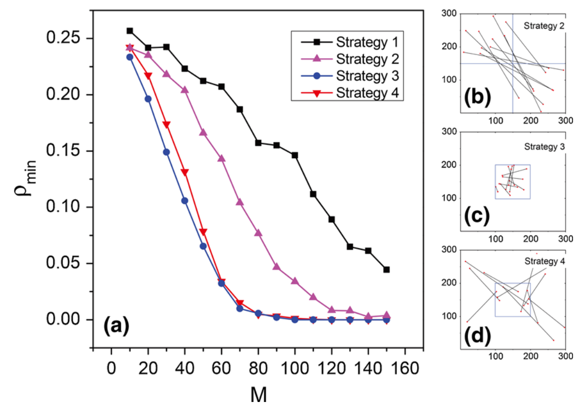

Fig. 2 (Color online) Sustainability of spiral wave pattern subject to different strategies of network structural perturbations. **a** The variation in the normalized size of the smallest synchronous cluster, $\rho_{\min}$, with M, the number of random links newly introduced. Strategy 1: adding random links over the entire network (filled squares); Strategy 2: introducing long-distance random links (filled upper triangles); Strategy 3: supplying random links only in the central area (filled circles); Strategy 4: distributing random links between the central and peripheral regions (filled downward triangles). **b**–**d** Schematic illustrations of strategies 2, 3 and 4. Results in (**a**) are averaged over 200 network realizations

tip region often induce characteristic changes in the wave pattern [26,27]. To gain insights, we first supply links between randomly chosen, unconnected pairs of nodes over the entire network [33–35]. This fashion of introducing new links is named strategy 1 in the present work. The total number of such links is M, with the same coupling function and coupling parameter as the regular links. For a fixed value of M, the network is initialized with the spiral wave pattern in Fig. 1a and the network state is recorded after 5×10^3 iterations. To characterize the deterioration of the perturbed spiral, we introduce the quantities $\rho_{\max} = \max\{N_A, N_B, N_C\}/N^2$ and $\rho_{\min} = \min\{N_A, N_B, N_C\}/N^2$, with N_A (N_B, N_C) the number of nodes with the state A (B,C). $\rho_{\max}$ and $\rho_{\min}$ represent, respectively, the normalized size of the largest and the smallest synchronous clusters associated with the spiral pattern. For the initial spiral [Fig. 1a], as the three clusters have approximately the same size, we have $\rho_{\max} \approx \rho_{\min} \approx 1/3$. If the sizes of the clusters are different, we have $\rho_{\max} > 1/3$ and $0 < \rho_{\min} < 1/3$, indicating a deformed but still sustained spiral. When one cluster is disappeared, we have $\rho_{\min} \approx 0$, but the value of $\rho_{\max}$ may either be close to unity (if the system reaches global synchronization) or 1/2 (if two equal-size clusters coexist). In this case, the original spiral is regarded as destroyed. A simple criterion to determine the destruction of the spiral wave thus is $\rho_{\min} \approx 0$. Figure 2a shows the variation of $\rho_{\min}$ with M (strategy 1). We see that, as M is increased from 10 to 150, $\rho_{\min}$ decreases from the value of about 0.26 to 0.05. That is, by strategy 1, the spiral wave pattern is statistically destroyed when 150 random links are introduced.

In order to understand the role played by the spiral tip in the pattern stability, we design and compare three alternative perturbation strategies [33–35]: introducing long-distance random links (Strategy 2), supplying random links only in the central area (Strategy 3), and distributing random links between the central and peripheral regions (Strategy 4). These three strategies are different from strategy 1, and are schematically illustrated in Fig. 2b–d, respectively. For strategies 2 and 3, the pairs of nodes connected by the new links are randomly chosen from the regions (0, 150) × (150, 300) and (150, 300) × (0, 150) on the lattice, and from the central area (100, 200) × (100, 200), respectively. For strategy 4, one map is randomly chosen from the central area, while another is chosen from the peripheral region. The results of applying perturbation strategies 2–4 are shown in Fig. 2a, where we see that, strategies 3 and 4 are more effective at suppressing the spiral wave pattern than strategies 1 and 2. For example, the value of $\rho_{\min}$ is reduced to 0 at about $M = 140$ for strategy 2; while for strategies 3 and 4, this occurs at about $M = 100$. For the rest of the paper, we will adopt strategy 3 to introduce the new links.

3 Pattern transitions

To uncover and understand the pattern transitions as the network topology deviates from that of a regular lattice and becomes increasingly random, we calculate the variations of $\rho_{\max}$ and $\rho_{\min}$ with M (the total number of randomly added links according to perturbation strategy 3). As shown in Fig. 3, for a few randomly added links, say $M < 10$, we have $\rho_{\max} \approx \rho_{\min} \approx 1/3$. In this region, the network exhibits a stable spiral wave similar to that in Fig. 1a. As M is increased from 10, the value of $\rho_{\max}$ increases but $\rho_{\min}$ decreases. For $M \approx 100$, we have $\rho_{\max} = 1$ and $\rho_{\min} = 0$, signifying that the network has reached a uniform synchronization state without any spatial pattern. Figure 4a shows, for $M = 100$, with time the spiral tip shifts from the central to the peripheral area [Fig. 4a2]. The spiral tip vanishes when it reaches the lattice boundary. Subsequently, one of the synchronous clusters expands while the other two clusters are pushed toward the boundary, leading finally to the state of global synchronization, as shown in Fig. 4a3.

Figure 3 indicates that the global synchronization state is stable for $M \lesssim 550$. As M is increased further, $\rho_{\max}$ decreases gradually but the value of $\rho_{\min}$ remains about 0. For $M \approx 850$, another platform emerges in the variation of $\rho_{\max}$ with M, where $\rho_{\max} \approx 0.9$ for $M \in [850, 1100]$. Since $\rho_{\min} \approx 0$ still holds, there is no spiral wave. In fact, in this region the system contains at most two synchronous clusters. Because the value of $\rho_{\max}$ is close to unity, most nodes in the network are synchronized into a giant cluster. To provide evidence for this scenario, we calculate a series of snapshots of

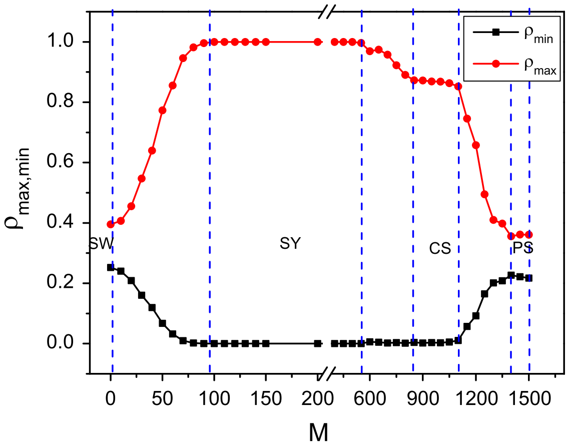

Fig. 3 (Color online) Pattern transitions as the network structure becomes increasingly random. Shown are $\rho_{\max}$ and $\rho_{\min}$ (the normalized sizes of the largest and the smallest synchronous cluster, respectively) versus M, the total number of randomly added links according to perturbation strategy 3. Spiral wave (SW) patterns exist in the parameter interval $M \in [0, 10]$. Global synchronization (SY) occurs for $M \in [100, 550]$. Chimera-like state (CS) arises in $M \in [850, 1100]$. For $M > 1400$, there is pinned spiral (PS). Ensemble average of 200 network realizations is used

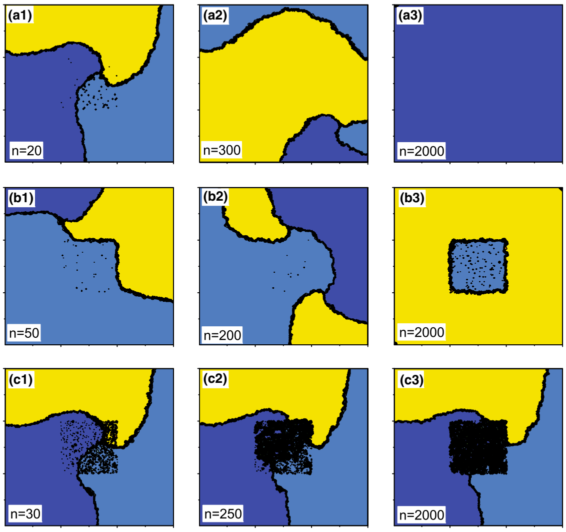

Fig. 4 (Color online) Characteristically distinct dynamical states in the network system that structurally becomes increasingly random for (a1–a3) $M = 100$ (global synchronization) (b1–b3) $M = 1000$ (chimera-like state) and (c1–c3) $M = 1500$ (pinned spiral wave). The left, middle, and right columns are snapshots of the system dynamics taken, respectively, at the initial, middle, and end of the evolution. Black dots in the central area mark the ending nodes of the randomly added links

the system dynamics for $M = 1000$, which are shown in Fig. 4b. We see that, the original spiral wave first breaks into a pair of anti-spiral waves [Fig. 4b2] that move gradually to the system boundary and disappear after reaching it. In the meantime, two synchronous clusters emerge: a small cluster consisting of nodes in the central area (except for the nodes connected by the randomly added links) and a large cluster comprising nodes in the peripheral area. There is a narrow boundary of asynchronous nodes separating the two clusters, as shown in Fig. 4b3. The distinct synchronous clusters represent effectively a chimera-like state observed previously in systems of non-locally coupled oscillators [41–44].

The chimera-like state becomes unstable as M is increased through 1100, since $\rho_{\max}$ and $\rho_{\min}$ tend to decrease and increase, respectively. For $M \gtrsim 1400$, the values of $\rho_{\max}$ and $\rho_{\min}$ are stabilized about 1/3 and 1/4, respectively. Figure 4c shows, for $M = 1400$, the typical states emerged during the system evolution. Due to the added random links, the spiral tip first drifts from the central to the peripheral area [Fig. 4c1], but the drift stops at the boundary of the central area after which the tip disappears. During this time interval, the spiral arms are separated from each other. As will be demonstrated below, the three arms are attached to the boundary of the central area and rotate in a synchronous fashion, signifying the phenomenon of pinned spirals [31]. As M is increased further, the pinned spiral state can be maintained (even for $M = 1 \times 10^4$).

Figures 3 and 4 suggest the following transition scenario as more random links are added to the network: spiral wave → global synchronization → chimera-like state → pinned spiral wave, where each state exists in a finite parameter region. Within each region, the values of $\rho_{\max}$ and $\rho_{\min}$ are hardly changed, suggesting that the respective patterns are stable to random structural perturbations. A transition region is associated with dramatic changes in the value of $\rho_{\max}$ or $\rho_{\min}$, in which one type of pattern is destroyed and a new type is born. To better characterize the transition regions, we calculate, by numerical simulations, the probability of certain pattern, p_{state}, with respect to M. The results are shown in Fig. 5. Comparing Figs. 3 and 5, we see that two different patterns coexist in each transition region. Taking $M = 30$ as an example, we see that, over 200 independent network realizations, about 80% of these realizations lead to a spiral wave ($p_{\text{SW}} \approx 0.8$) whereas the remaining cases correspond to the global

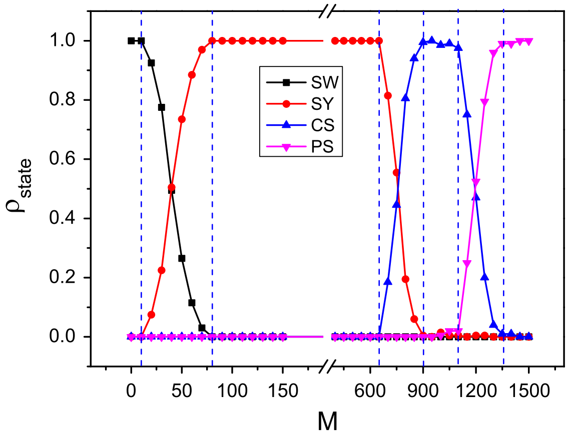

Fig. 5 (Color online) Continuous nature of pattern transition. Shown is p_{state} versus M, which denotes the probability of generating a specific pattern for M randomly added links. The terms SW, SY, CS and PS stand for, respectively, spiral wave, global synchronization, chimera-like state, and pinned spiral wave. Results are averaged over 200 network realizations

synchronization state ($p_{\text{SY}} \approx 0.2$). In the transition region between spiral wave and global synchronization ($10 < M < 100$), p_{SW} decreases from unity to zero, which is accompanied by an increase in p_{SY} in the opposite direction. This feature of gradual and continuous transition appears also in other transition regions, as shown in Fig. 5.

4 Pattern sensitivity in the transition regions

In the transition regions the system dynamics is sensitive to random structural perturbations in the sense that the introduction of a single random link is able to switch the system dynamics from one pattern to another. For example, for a network with $M = 49$ random links, the stable state of the system is a spiral wave pattern. With a new random link being added, the state of global synchronization emerges and becomes stable, as exemplified in Fig. 6a–d, where the time evolutions of the difference between the two pattern states, $\delta\mathbf{X} = \mathbf{X}_{M=49} - \mathbf{X}_{M=50}$, are shown, with $\mathbf{X}_M = \{x_i\}$ denoting the pattern state for the network with M random links. Initially [Fig. 6a], except for the pair of nodes connected by the 50th link (the new link), the two patterns are essentially identical as the two networks (one with 49 and another with 50 random links) start from the same initial condition [the spiral wave in Fig. 1a]. Then, as the two systems evolve, the difference

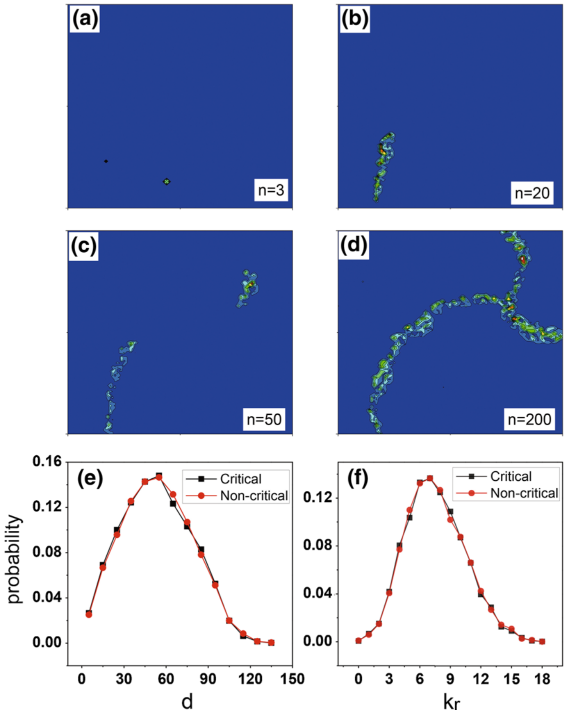

Fig. 6 (Color online) Pattern sensitivity in the transition regions. **a**–**d** Time evolution of the difference $\delta\mathbf{X}$ between the spiral wave ($M = 49$) and the global synchronization state ($M = 50$). **e** Normalized distance distribution of the critical (filled squares) and non-critical links (filled circles). **f** Normalized distribution of the local connectivity of the critical (filled squares) and non-critical links (filled circles). Results in **e** and **f** are averaged over 500 realizations of the 50th link

at one of the ending nodes of the 50th link disappears, whereas the difference at the other ending node sustains and gradually propagates to its neighboring region, as shown in Fig. 6b. At a later time the difference at this node vanishes but appears on a remote ending node of a random link in the lattice, as shown in Fig. 6c. This propagation and switching process occurs repeatedly in the central area, resulting in the formation of a three-armed spiral gradually, as shown in Fig. 6d. Finally, a spiral wave similar to that in Fig. 1 is generated. Similar behaviors arise in other transitional regions, e.g., $M \in (550, 850)$ and $M \in (1100, 1400)$.

Do the critical connections observed in the transition regions possess any special topological property? To address this question, we focus on still the transition from spiral wave to global synchronization [Fig. 6a–d], but adding the 50th link in the central area in a random fashion. If the system evolves finally into the state of global synchronization, we mark this link as critical and record the locations of the ending nodes, denoted by (l_x, l_y) and (l'_x, l'_y). For comparison, we also record the ending nodes of non-critical links that do not lead to the destruction of the spiral wave. We first examine the Euclidean distances of the critical links, defined as $d = [(l_x - l'_x)^2 + (l_y - l'_y)^2]^{1/2}$. Previous studies [48,49] revealed that long-distance links have a more significant effect on the network dynamics than short range links. A higher probability for a critical link to be long-ranged can then be intuitively expected. Figure 6e shows the normalized distribution of the distances of the critical links. Surprisingly, the distribution is unimodal with the maximum probability occurring at about $d = 50$. For comparison, the distance distribution of the non-critical is also shown, where we see that the two distributions are nearly identical. As the averaged distance between nodes in the central area is also about 50, the analysis thus indicates that the critical links are uncorrelated with the distance.

The local connectivity of the ending nodes associated with the critical links represents another topological feature. To examine it, for each ending node, we count the number n_r of nodes of degree larger than 4 within the distance $d_r = 10$ from it, for the reason that there is at least one random link attached to such a node. If the critical links were attached to the existing nodes following the preferential attachment rule, such links would be more likely to lie in regions containing large degree nodes. In this case, the distribution of n_r should exhibit a heavy tail. A representative distribution of k_r, the sum of the critical links of the ending nodes, is shown in Fig. 6f, which exhibits a unimodal feature too. For comparison, the k_r distribution of the non-critical links is also shown, which is indistinguishable from that associated with critical links. We find that the distance d has no effect on the unimodal feature, i.e., the local connectivity is uncorrelated with the critical links.

Examination of additional topological properties [6] such as the degree assortativity, network modularity, and average network diameter revealed no clear difference between the critical and non-critical links. Topological analyses have also been carried out for other transition regions, with the results essentially the same. Our conclusion is that, so far as the topological properties are concerned, there is no clear difference between the critical and non-critical links.

5 Discussion and conclusion

In a regular lattice of coupled nonlinear oscillators capable of generating spiral wave patterns, a sufficient number of random links will destroy the patterns, but how? This paper addresses this question through a detailed computational study of the effect of random links on spiral wave pattern as their number is systematically increased. From the point of view of network structure, adding random links to a regular network makes it complex, thus our work effectively addresses the general problem of pattern formation and transition in complex networks. We find, as the number of randomly added links is increased, the underlying networked system can exhibit distinct types of dynamical patterns and rich transitions among them. Our study has revealed two types of patterns in complex networks, which to our knowledge have not been reported previously: a chimera-like pattern and the pinned multi-armed spiral waves. We also find that a transition between two distinct types of patterns can be triggered through only a single, critical random link. The topological properties of the set of critical links are found to be of no difference from those of the non-critical links. With respect to spiral waves, our study reveals that random links added into the region of the spiral tip can have a devastating effect on the pattern, a result that is consistent with those from previous works [33–35].

A key difference from previous studies of networked oscillators is that, in our CML model, the tip of the spiral is moving randomly in the central area, which could be the underlying reason for the sensitivity of the system pattern to structural perturbations in the transition regions. This has consequences. In particular, in previous studies of synchronization transitions in complex networks [9], a general finding is that as the network becomes increasingly more complex, the order parameter characterizing the degree of network synchronization will be increased progressively. As demonstrated in the present work, for the spiral wave patterns, the order parameter, which is characterized by the normalized size of the largest synchronous cluster ($\rho_{\max}$), exhibits a non-monotonous behavior. As shown in Fig. 3, with the increase of M (the number of random links), the order parameter first increases (in the transition from spiral wave to global synchronization), then reaches 1 (in the global synchronization state), and finally decreases (in transitions from global synchronization to the chimera-like state and pinned multi-armed spiral). This peculiar phenomenon of non-monotonous behavior of the order parameter is disappeared when the links are introduced randomly over the entire lattice (strategy 1). When random links are added to the network on a global scale (as with the Newman–Watts small-world network model [50]), preliminary simulations show that as the network structure becomes more complex, the order parameter of the system is monotonically increased (not shown).

We would like to note that the three-armed spiral, while is sensitive to the long-distance connections, is hardly affected by the parameter or state perturbations. Numerical results show that given the maps are of period-three orbits ($a \in [2.16, 3.16]$), the three-armed spiral can always be generated by the described initial conditions, even under small noise perturbations. In the meantime, we have to mention that the phenomena are observed for only the case of three-armed spiral so far, and it remains a challenge to us how to generate more complicated spirals, e.g., four-armed spiral, by a proper setting of the initial conditions. Further studies on the generation of more complicated spirals and their responses to network structural perturbations might lead to intriguing and fascinate new findings.

To summarize, by the model of coupled map lattice, we have studied the dynamical responses of spiral wave pattern to network structural perturbations. It is found that as more random connections are introduced into the network, the system dynamics undergoes rich bifurcations. In particular, two new patterns, namely the chimera-like state and the pinned multi-armed spiral wave, are reported for the first time. Furthermore, in the transition regions of the patterns, it is found that the system dynamics could be changed from one type of pattern to another one by just adding (removing) one critical connection. Our study sheds new lights on the pattern dynamics in complex networks and the findings provide insights into the issue of pattern control on networks.

Acknowledgements We thank Profs. H. Zhang and J. Ma for helpful discussions. This work was supported by the National Natural Science Foundation of China under the Grant No. 11375109 and by the Fundamental Research Funds for the Central Universities under the Grant No. GK201601001. YFW was supported by the Fundamental Research Funds for the Central Universities under the Grant No. 2017TS005. YCL would like to acknowledge support from the Vannevar Bush Faculty Fellowship program sponsored by the Basic Research Office of the Assistant Secretary of Defense for Research and Engineering and funded by the Office of Naval Research through Grant No. N00014-16-1-2828.

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interests regarding the publication of the paper.

References

1. Cross, M., Greenside, H.: Pattern Formation and Dynamics in Nonequilibrium Systems, 1st edn. Cambridge University Press, Cambridge (2010)
2. Hoyle, R.: Pattern Formation—An Introduction to Methods, 1st edn. Cambridge University Press, Cambridge (2006)
3. Aranson, I.S., Tsimring, L.S.: Patterns and collective behavior in granular media: theoretical concepts. Rev. Mod. Phys. **78**, 641–692 (2006)
4. Rietkerk, M., van de Koppel, J.: Regular pattern formation in real ecosystems. Trends Ecol. Evol. **23**(3), 169–175 (2008)
5. Willemsen, V., Scheres, B.: Mechanisms of pattern formation in plant embryogenesis. Annu. Rev. Genet. **38**, 587–614 (2004)
6. Newman, M.E.J.: Networks: An Introduction. Oxford University Press, Oxford (2010)
7. Barrat, A., Barthélemy, M., Vespignani, A.: Dynamical Processes on Complex Networks. Cambridge University Press, Cambridge (2008)
8. Barahona, M., Pecora, L.M.: Synchronization in small-world systems. Phys. Rev. Lett. **89**, 054101 (2002)
9. Arenas, A., Díaz-Guilera, A., Kurths, J., Moreno, Y., Zhou, C.S.: Synchronization in complex networks. Phys. Rep. **469**, 93–153 (2008)
10. Pastor-Satorras, R., Vespignani, A.: Epidemic spreading in scale-free networks. Phys. Rev. Lett. **86**(14), 3200–3203 (2001)
11. Balcan, D., Vespignani, A.: Phase transitions in contagion processes mediated by recurrent mobility patterns. Nat. Phys. **7**, 581 (2011)
12. Arenas, A., Díaz-Guilera, A., Guimerà, R.: Communication in networks with hierarchical branching. Phys. Rev. Lett. **86**, 3196–3199 (2001)
13. Zhao, L., Lai, Y.-C., Park, K., Ye, N.: Onset of traffic congestion in complex networks. Phys. Rev. E **71**, 026125 (2005)
14. Morris, R.G., Barthelemy, M.: Transport on coupled spatial networks. Phys. Rev. Lett. **109**, 128703 (2012)
15. Motter, A.E., Lai, Y.-C.: Cascade-based attacks on complex networks. Phys. Rev. E **66**, 065102(R) (2002)
16. Buldyrev, S.V., Parshani, R., Paul, G., Stanley, H.E., Havlin, S.: Catastrophic cascade of failures in interdependent networks. Nature (London) **464**(7291), 1025–1028 (2010)
17. Park, K., Huang, L., Lai, Y.-C.: Desynchronization waves in small-world networks. Phys. Rev. E **75**, 026211 (2007)
18. Wang, X.G., Guan, S.-G., Lai, Y.-C., Li, B.-W., Lai, C.-H.: Desynchronization and on–off intermittency in complex networks. Europhys. Lett. **88**, 28001 (2009)
19. Basar, E.: Brain Function and Oscillation. Springer, Berlin (1998)
20. Nakao, H., Mikhailov, A.S.: Turing patterns in network-organized activator–inhibitor systems. Nat. Phys. **6**, 544–550 (2010)
21. Qian, Y., Huang, X., Hu, G., Liao, X.: Structure and control of self-sustained target waves in excitable small-world networks. Phys. Rev. E **81**, 036101 (2010)
22. Fu, C., Zhang, H., Zhan, M., Wang, X.G.: Synchronous patterns in complex systems. Phys. Rev. E **85**, 066208 (2012)
23. Pecora, L.M., Sorrentino, F., Hagerstrom, A.M., Murphy, T.E., Roy, R.: Cluster synchronization and isolated desynchronization in complex networks with symmetries. Nat. Commun. **5**, 4079 (2014)
24. Lin, W., Fan, H., Wang, Y., Ying, H.P., Wang, X.G.: Controlling synchronous patterns in complex networks. Phys. Rev. E **93**, 042209 (2016)
25. Huett, M.-T., Kaiser, M., Hilgetag, C.C.: Perspective: network-guided pattern formation of neural dynamics. Philos. Trans. R. Soc. B **369**, 20130522 (2014)
26. Barkley, D.: Euclidean symmetry and the dynamics of rotating spiral waves. Phys. Rev. Lett. **72**, 164–167 (1994)
27. Guo, H., Liao, H., Ouyang, Q.: Relation between the wave front and the tip movement of spirals. Phys. Rev. E **66**, 026104 (2002)
28. Wu, N.J., Gao, H.J., Ma, J., Ying, H.P.: Adjustment of spiral drift by a traveling wave perturbation. Nonlinear Dyn. **67**, 159–164 (2012)
29. Wang, C.N., Ma, J., Liu, Y., Huang, L.: Chaos control, spiral wave formation, and the emergence of spatiotemporal chaos in networked Chua circuits. Nonlinear Dyn. **67**, 139–146 (2012)
30. Ma, J., Hu, B.L., Wang, C.N., Jin, W.Y.: Simulating the formation of spiral wave in the neuronal system. Nonlinear Dyn. **73**, 73–83 (2013)
31. Pan, D., Gao, X., Feng, X., Pan, J., Zhang, H.: Removal of pinned scroll waves in cardiac tissues by electric fields in a generic model of three-dimensional excitable media. Sci. Rep. **6**, 21876 (2016)
32. Panfilov, A.V.: Spiral breakup in an array of coupled cells: the role of the intercellular conductance. Phys. Rev. Lett. **88**, 118101 (2002)
33. He, D., Hu, G., Zhan, M., Ren, W., Gao, Z.: Pattern formation of spiral waves in an inhomogeneous medium with small-world connections. Phys. Rev. E **65**, 055204(R) (2002)
34. Wang, X., Lu, Y., Jiang, M., Ouyang, Q.: Attraction of spiral waves by localized inhomogeneities with small-world connections in excitable media. Phys. Rev. E **69**, 056223 (2004)
35. Sinha, S., Saramäki, J., Kaski, K.: Emergence of self-sustained patterns in small-world excitable media. Phys. Rev. E **76**, 015101(R) (2007)
36. Kaneko, K.: Theory and Application of Coupled Map Lattice. Wiley, Chichester (1993)
37. Venkataramani, S.C., Ott, E.: Spatiotemporal bifurcation phenomena with temporal period doubling: patterns in vibrated sand. Phys. Rev. Lett. **80**, 3495–3498 (1998)
38. Venkataramani, S.C., Ott, E.: Pattern selection in extended periodically forced systems: a continuum coupled map approach. Phys. Rev. E **63**, 046202 (2001)
39. Harrison, M.A.F., Lai, Y.-C.: Continuum coupled-map approach to pattern formation in oscillating granular layers: robustness and limitation. Int. J. Bifurc. Chaos **18**, 1627–1643 (2008)

40. Kapral, R., Livi, R., Oppo, G.-L., Politi, A.: Dynamics of complex interfaces. Phys. Rev. E **49**, 2009–2022 (1994)
41. Kuramoto, Y., Battogtokh, D.: Coexistence of coherence and incoherence in nonlocally, coupled phase oscillators. Nonlinear Phenom. Complex Syst. **5**, 380 (2002)
42. Abrams, D.M., Strogatz, S.H.: Chimera states for coupled oscillators. Phys. Rev. Lett. **93**, 174102 (2004)
43. Zhu, Y., Zheng, Z.G., Yang, J.Z.: Chimera states on complex networks. Phys. Rev. E **89**, 022914 (2014)
44. Yao, N., Huang, Z.-G., Grebogi, C., Lai, Y.-C.: Emergence of multicluster Chimera states. Sci. Rep. **5**, 12988 (2015)
45. Mo, J., Li, Y., Wei, C., Yang, M., Gu, H., Ren, W., Qu, S.: Interpreting a period-adding bifurcation scenario in neural bursting patterns using border-collision bifurcation in a discontinuous map of a slow control variable. Chin. Phys. B **19**(8), 080513 (2010)
46. Yang, K., Wang, X.G., Qu, S.-X.: Cyclic synchronous patterns in coupled discontinuous maps. Phys. Rev. E **92**, 022905 (2015)
47. Li, G., Ouyang, Q., Petrov, V., Swinney, H.L.: Transition from simple rotating chemical spirals to meandering and traveling spirals. Phys. Rev. Lett. **77**, 2105 (1996)
48. Qian, Y.: The influence of long-range links on spiral waves and its application for control. Chin. Phys. B **21**(8), 088201 (2012)
49. Xu, K., Huang, W., Li, B., Dhamala, M., Liu, Z.: Controlling self-sustained spiking activity by adding or removing one network link. Europhys. Lett. **102**, 50002 (2013)
50. Newman, M.E.J., Watts, D.J.: Renormalization group analysis of the small-world network model. Phys. Lett. A **263**, 341–346 (1999)