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To cite this article: Run-Ran Liu *et al* 2019 *New J. Phys.* **21** 045002

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OPEN ACCESS

RECEIVED
15 November 2018

REVISED
16 January 2019

ACCEPTED FOR PUBLICATION
8 March 2019

PUBLISHED
12 April 2019

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PAPER

Remote control of cascading dynamics on complex multilayer networks

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Keywords: multilayer network, remote control, cascading failure, percolation

Abstract

To develop effective control strategies to enhance the robustness of multilayer networks against large-scale failures is of significant value. We articulate the idea of 'remote control' whereby adaptive perturbations to one network layer are able to enhance the resilience of not only itself but also other interconnected network layers. We analyze the principle of remote control using percolation dynamics by showing analytically and numerically that, with the adaptive generation of a small number of new links in the control layer, not only is this layer but also other layers become dramatically more resistant to cascading failures. We also find that remote control is more effective for scale-free than for random networks. Remote intervention of multilayer network systems through adaptation has real-world applications, which we illustrate using the rail and coach transportation system in the Great Britain.

1. Introduction

The ability for a complex system to resist/survive random component failures and/or intentional attacks is a problem of continuous interest in modern network science [1–22]. Recent years have witnesses a great deal of attention [23–26] to the robustness of multilayer networks [26–29]. In such a system, a set of nodes belong to many network layers simultaneously and interact with the nodes in different layers, as occurring in diverse contexts such as social [30], technological [31], and biological systems [32]. Because of the common set of nodes belonging to different layers and the links connecting nodes within and across layers [33], nodal failures or influenza viruses [34, 35] in one layer can propagate not only to nodes within the same layer but also to nodes in different interconnected layers. This is essentially a cascading process, the most devastating vulnerability for single layer networks [8], but here the process is more sophisticated because of the multilayer structure [23–26, 36]. To articulate effective control strategies to protect a multilayer system from large scale cascading failures and to enhance the resilience of the system is a problem of significant value and broad interest.

In this paper, we articulate a strategy of *remote control* to enhance the ability of multilayer systems against cascading failures. The general consideration is that, for a multilayer system in the real world, not all layers are externally accessible but failures can still propagate to these layers. The term 'remote control' is thus to be understood with respect to the possibility of protecting the inaccessible network layers through controlling some accessible layers. To formulate the problem in a concrete setting, we exploit percolation dynamics [37–40], a theoretical and computational paradigm to investigate the robustness of networks subject to random failures [24, 41] or malicious attacks [42, 43]. Of particular interest are the occurrence and nature of percolation transitions, which are typically continuous (second order) for single layer networks but can be abrupt (first order) in multilayer systems [24]. The robustness of a network layer in a multilayer system can be characterized by the size of the final giant component after the termination of failure propagation. Given a multilayer system,

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we assume that only one layer is accessible to external control in the sense that its structure can be perturbed with the adaptive implementation of a small number of new links from time to time, whose number can be taken conveniently as the control parameter. Our main result is that adaptive control of the accessible layer can enhance the robustness of the entire multilayer system. A phenomenon is that control is more effective for multilayer systems with a scale-free structure than for those with a random topology.

Our work goes beyond the existing literature in the following three aspects. Firstly, in spite of the tremendous recent interest in linear controllability of complex networks [44–70] and in controlling nonlinear dynamical networks [71–78], the idea of remote control has not appeared in any previous work. Secondly, our idea of control exploits the effects of dynamical adaptation on the robustness of multilayer networks, which have not been studied in the literature. Thirdly, our remote control strategy is physically meaningful and realizable. In particular, traditional study of the robustness of multilayer systems focused on cascading processes in a static setting [23, 24, 36]. In real world systems, the links can be dynamic in response to environmental changes. For example, in a cyberphysical network consisting of a physical and an information transmission layer, the former is inaccessible to control as it is difficult to modify its structure, but realistically the structure of the latter can be readily adjusted [79]. In this specific context, remote control entails manipulating the information layer to achieve desired performance of the physical layer. Another example is air transportation networks composed of airports (nodes) and flight routes (links) operated by multiple companies. Operating flights are constantly adjusted according to external factors such as changes in the passenger flow, business competition, capacity of air traffic control, and accidents [31].

2. Model

We consider a multilayer networked system of M layers: $A, B, C, \ldots$, each of the same size N. A node in the system is denoted by a pair of labels: (x, X), with $x\,(x = 1, 2, \ldots N)$ and $X\,(X = A, B, C\ldots)$ being the nodal and layer indices, respectively. Nodes within the same layer X follow the degree distribution p_k^X, and nodes with the same Arabic numeral are all replicas of one another. A cascading process is triggered by the initial removal of a fraction $1 - p$ of nodes in all network layers, after which the layers fragment into a set of isolated components. An assumption used in previous work [24] is that nodes in the giant component are functional while others are regarded as having failed. When a node in a layer fails, all its replicas in other layers fail simultaneously. Let network layer A be accessible to external adaptive control, while all other layers are inaccessible. Because of control, all nodes in layer A are potentially immune to failure through the generation of adaptive links. Our control strategy is that, if a node in layer A is isolated from the giant component, m adaptive links will be added to this node for it to be connected to m random nodes in A, provided that the replicas of this node in other network layers are all functional. After reconnection, some isolated nodes in layer A may be reconnected to the giant component, 'rescuing' not only themselves but also their replicas in other layers from failure. If a node in A cannot be reconnected to the giant component, itself and all its replicas are regarded as having failed. Because of the interdependence among the layers, if one node in an uncontrollable layer is isolated from the giant component in this layer, all its replicas will fail simultaneously. Failures in all network layers will induce more fragments, iteratively leading to cascading failures at large scales. When failures have stopped, the entire system reaches a new stable steady state. Schematic illustration of remote control of a double-layer system is shown in figure 1. The size $S^{ABC,\cdots} \equiv s$ of the mutually connected giant component characterizes the robustness of the system [24]. It is worth emphasizing the essence of remote control: only the structure of the control or adaptive layer can be altered with new links, while all other layers cannot be perturbed externally except for the 'automatic' removal of nodes and the associated links as induced by failures.

3. Results

We demonstrate the principle of remote control using a two-layer system of random or scale free networks. Figure 2 shows the size S^{AB} of the mutually connected giant component versus the fraction p of initially reserved nodes. A percolation transition occurs at p_c where S^{AB} becomes zero abruptly and discontinuously. For $p \leqslant p_c$, there are cascading failures. As p deviates from p_c, the number of iterations of the cascading process, denoted as N^I, decreases. The transition point p_c can be precisely determined [80–82] by the peak value of N^I, which occurs at $p = p_c$. In general, a larger value of the control parameter m results in a smaller value of p_c, for which the system is more robust against cascading failures. Figure 3 shows that p_c decreases with m for both random and scale-free networks. Note that, for $m = 0$ (absence of control), the value of p_c is higher for scale-free networks, indicating that multilayer networked systems with a more heterogeneous degree distribution are more vulnerable to cascading failures, as reported in previous work [24]. However, a striking phenomenon arises: as the value of m is increased so that control is intensified, the value of p_c decreases faster for scale-free than for

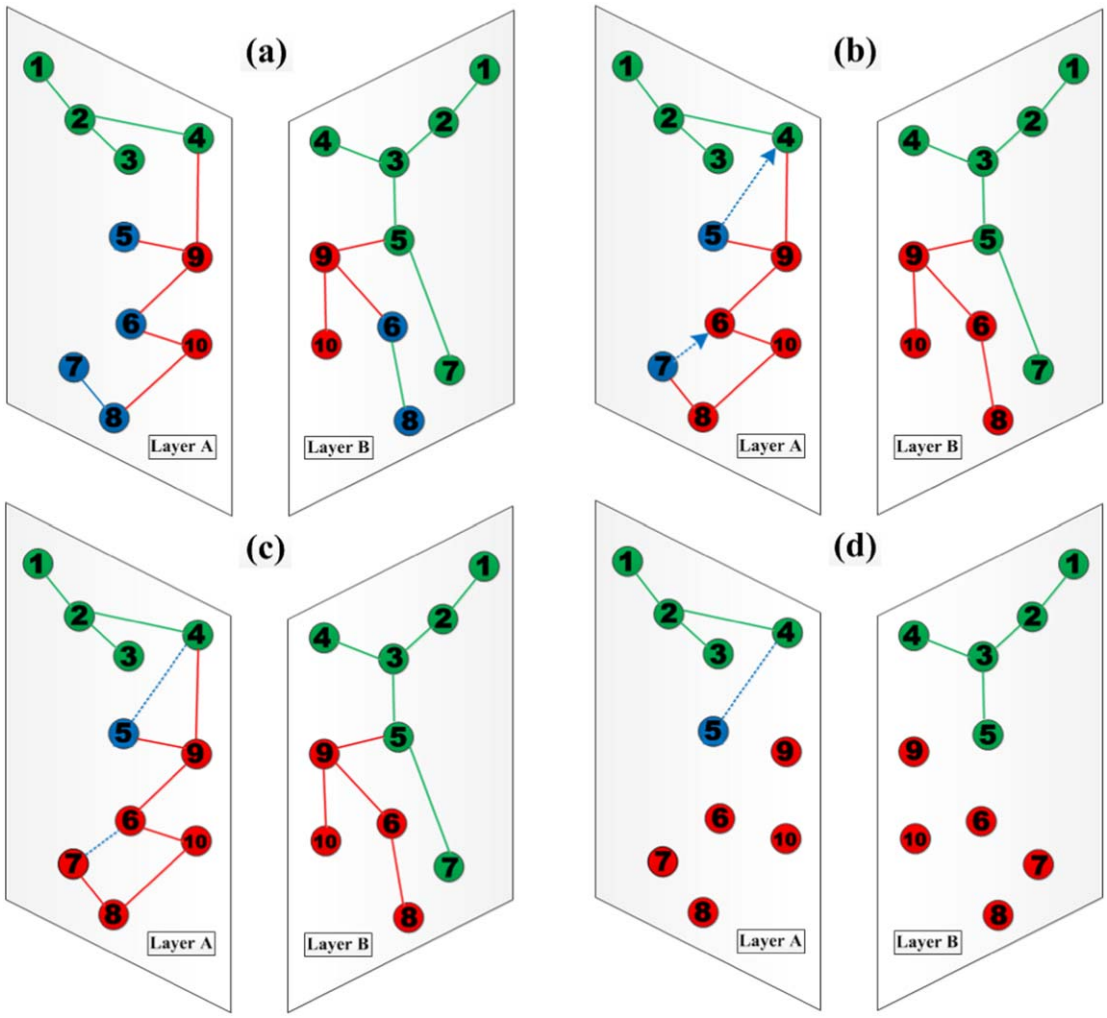

Figure 1. *Schematic illustration of remote control of a double-layer system.* The system has two layers of networks, where layer A is accessible to external control action (the control layer) and layer B is the remote layer that is not externally accessible. The goal of remote control is to improve the robustness of layer B against large scale failures through control perturbations to layer A. (a) Red circles denote the initially removed nodes from layer A and its replicas in layer B. The blue and green circles represent the isolated nodes from the giant component and the functional nodes in the giant component, respectively. (b) Each node isolated from the giant component in layer A generates an adaptive link ($m = 1$) and is reconnected to a random node if its replica is still in the giant component of layer B. Otherwise, the node and its replica are regarded as having failed immediately. (c) The isolated nodes are not able to link to the giant component of layer A through adaptive links and hence fail. (d) The replicas of the failed nodes in layer A fail and the system reaches a new stable steady state.

random networks. This means that remote control is more effective for scale-free networks to maintain its robustness than for random networks. Note that, theoretical results (to be presented below) are also included in figure 2, which agree with the numerical results quite well.

We develop a general theory to support the principle of remote control of multilayer networked systems. Differing from the conventional percolation theory for multilayer systems where all layers are on the equal footing, in our theory the final structure of the control layer A comprises heterogeneous links that contain the original and the adaptive ones generated during control. In particular, a node in A may be temporally isolated in the cascading process. When this occurs, it can acquire one of the two types of adaptive links: one generated by itself (active) and the other from other nodes to which it is connected (passive). It is necessary to treat the two types of links separately as they have different probabilities to lead to the giant component. For the control layer, there are then three types of links altogether: the original links, the actively and passively adaptive links as a result of control.

For simplicity, we assume that all network layers follow the same degree distribution p_k and the number of layers is $M \equiv n$. Let R be the probability that a randomly chosen, original link in the control layer A belongs to the giant component, and R' be the probability of the same nature in an inaccessible layer X ($X \in B,\ C,\ D,\ldots$). Let T^a (T^p) be the probability that an active (passive) link is connected to the giant component in A. To determine the probabilities R, R', T^a and T^p, we generalize the standard approach of generating functions

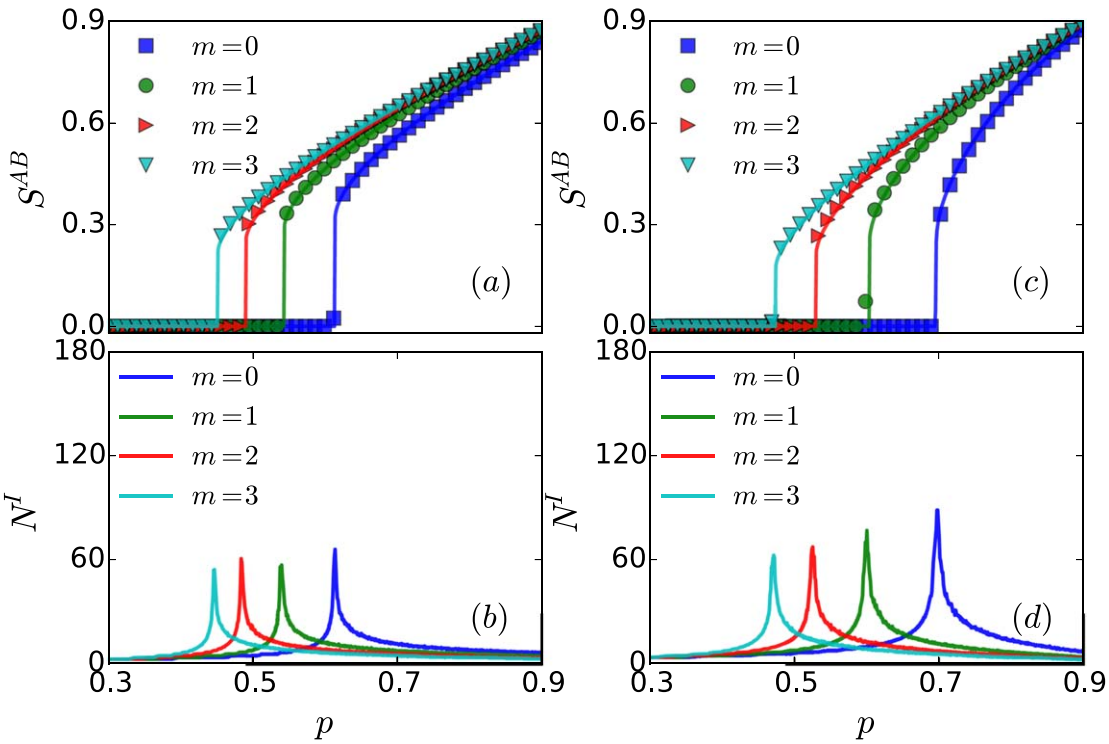

Figure 2. Percolation transition in two-layer systems. (a), (b) For random networks, the size of mutually connected component S^{AB} and the number of iterations N^I versus the fraction p of initially reserved nodes, respectively, for different values of the control parameter m. As the value of m is increased, the transition point shifts towards the left, indicating an increased level of robustness. The symbols and solid lines denote the numerical and theoretical results, respectively. (c), (d) Similar results but for scale-free networks. For both random and scale-free interdependent networks, the average degree is set as 4 (the minimum degree of scale-free network is 2 and power exponent of degree distribution is -2.6), and network size is $N = 5 \times 10^5$. Each data point is the result of 40 independent statistical averages.

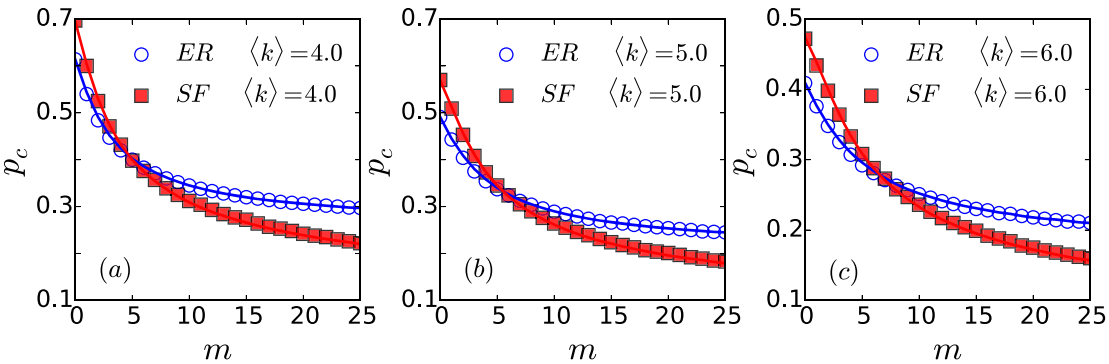

Figure 3. Percolation transition point p_c of scale-free and random networks versus the control parameter for (a) $\langle k\rangle = 4$, (b) $\langle k\rangle = 5$, and (c) $\langle k\rangle = 6$ (the minimum degree of scale-free network is two and the power exponents of degree distribution is -2.6, -2.3 and -2.1 for $= 4$, 5 and 6, respectively). The solid lines and symbols denote the theoretical predictions and simulation results determined by the peak of N^I, respectively. These results reveal a remarkable phenomenon: remote control is more effective for scale free than for random networks.

[3, 83]. Let $G_0(x) = \sum_k p_k x^k$ be the function that gives rise to the degree distribution associated with the original links for random nodes in each network layer, and $G_1(x) = \sum_k p_k k x^{k-1}/\langle k\rangle$ be the corresponding generating function from the branching process, which generates the distribution for the number of outgoing original links, where $\langle k\rangle$ is the original average degree. For the control layer A, two sets of generating functions are needed: one for the active and another for the passive links. Specifically, $G_0^a(x) = x^m$ and $G_1^a(x) = x^{m-1}$ are the generating functions for active links, and $G_0^p(x) = \sum_k p_k^* x^k$ and $G_1^p(x) = \sum_k p_k^* k x^{k-1}/\langle k\rangle^p$ are the generating functions

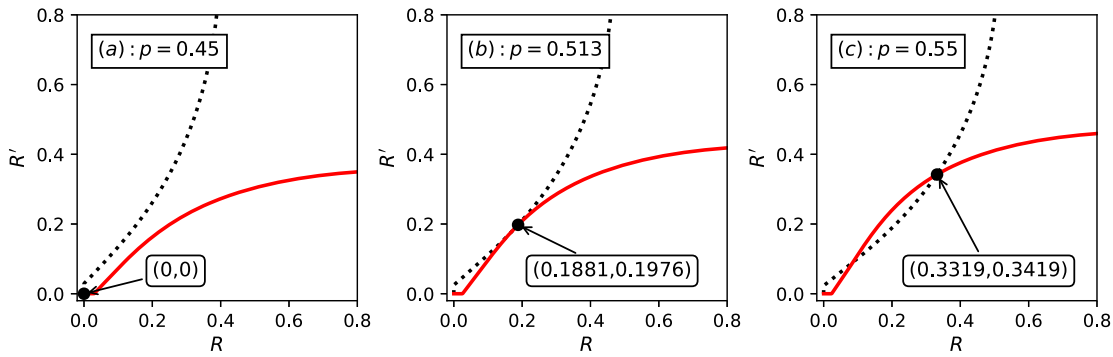

Figure 4. Determination of the theoretically predicted phase transition point. Shown are graphical solutions (the large black dots) of equations (1) and (2) for a two-layer scale-free network with $m = 1$. The minimum degree of the network is two and the algebraic exponent of degree distribution is −2.3. (a)–(c) Results for $p = 0.5 < p_c$, $p = 0.513 \approx p_c$, and $p = 0.55 > p_c$, respectively.

for passive links, where p_k^* and $\langle k\rangle^p$ are the degree distribution of nodes and average degree for passive links, respectively.

Using the various generating functions and taking advantage of the locally treelike property of the networks, we can obtain the respective self-consistent equations for the probabilities R, R', T^p and T^p. In particular, following a randomly chosen link in layer A, we arrive at a node (x, A), which can be linked to the giant component by the outgoing links, active or passive. The probabilities that (x, A) is not linked to the giant component by the three types of links are $G_1(1 - R)$, $G_0^a(1 - T^a)$, and $G_0^p(1 - T^p)$, respectively. The total probability that a randomly chosen link leads to the giant component is thus $1 - G_1(1 - R)G_0^p(1 - T^p)G_0^a(1 - T^a)$. Since the probability that the replicas of (x, A) are all functional is $p[1 - G_0(1 - R')]^{n-1}$, we obtain the self-consistent equation for R as

$$R = p[1 - G_0(1 - R')]^{n-1}[1 - G_1(1 - R)G_0^p(1 - T^p)G_0^a(1 - T^a)]. \tag{1}$$

Similarly, the equation for R' is

$$R' = p[1 - G_1(1 - R')][1 - G_0(1 - R')]^{n-2}[1 - G_0(1 - R)G_0^p(1 - T^p)G_0^a(1 - T^a)]. \tag{2}$$

Since an active link connects to a node randomly, the probability that the link can connect the giant component is the size of the giant component: $T^a = s$. Using T^a and the branching process on the control layer A, we obtain the self-consistent equation for T^p. Following a passive link of a random node (x, A), we arrive at another node (y, A) that has generated this link and the remaining $m - 1$ active links. The probability that (y, A) is not connected to the giant component by its remaining active links is $G_1^a(1 - T^a)$. Since the probability that the node (y, A) is not connected to the giant component by passive links is $G_0^p(1 - T^p)$, the equation for T^p is

$$T^p = 1 - G_1^a(1 - T^a)G_0^p(1 - T^p). \tag{3}$$

The degree distribution of passive links p_k^* hinges on the fraction of nodes that are not connected to the giant component through the original links in the component, which is $p[1 - G_0(1 - R')]^{n-1}G_0(1 - R)$, so p_k^* follows the Poisson distribution with the average degree $\langle k\rangle^p = mp[1 - G_0(1 - R')]^{n-1}G_0(1 - R)$. In the final steady state, the equation for the size of the giant component is thus

$$s = p[1 - G_0(1 - R')]^{n-1}[1 - G_0(1 - R)G_0^p(1 - T^p)G_0^a(1 - T^a)]. \tag{4}$$

For the special case $m = 0$ (no control), we have $R' = R$, and the equations reduce to those in the standard percolation model for interdependent networks [27, 28]: $R = p[1 - G_1(1 - R)][1 - G_0(1 - R)]^{n-1}$ and $s = p[1 - G_0(1 - R)]^n$. It is not feasible to get closed-form expressions for R, R', T^a, T^p and s. It is necessary to resort to numerical solutions for a given degree distribution. In particular, the percolation transition point p_c can be graphically represented on the (R, R') plane, as shown in figure 4. For $p < p_c$, the solution is given by the crossing point of the curves at the origin. However, for $p \approx p_c$, the solution is given by the tangent point, leading to a discontinuous change in both R and R'. Exploiting this behavior of the solutions for R and R', we can determine the value of p_c. The analytic results are included in figure 2, which agree with the results from direct numerical simulations quite well.

4. Heterogeneous adaptivity of nodes and control efficiency

Adding links in a real system can be costly. An effective way to achieve control efficiency under this realistic constraint is to allocate the adaptivity of nodes to improve the robustness of the system for *a given quota* of

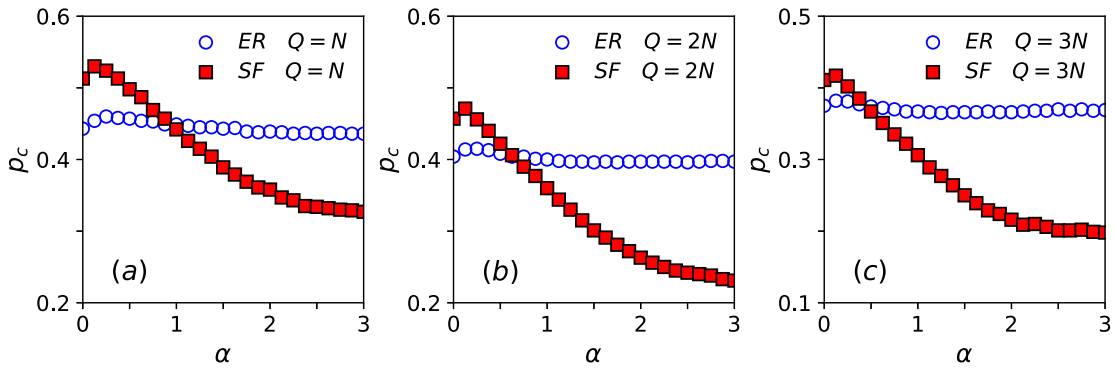

Figure 5. Effectiveness of degree based adaptivity distribution scheme in protecting multiplex network systems under *a given quota* of adaptive links. Shown are simulation results for the percolation transition point p_c for scale-free and random networks versus the parameter α: (a) $Q = N$, (b) $Q = 2N$, and (c) $Q = 3N$. The average degree of the networks is five. For the scale-free networks, the minimum degree is two and algebraic degree distribution exponent is -2.3.

adaptive links. In a two-layer system, a pair of interdependent nodes in different network layers have different degrees, e.g. a high-degree node in one layer may depend on a low-degree node in the other layer. Since the nodes with small degrees are fragile and sensitive to node or link removal in the percolation process, the failures of some low-degree nodes in one network layer can bring down high-degree nodes in the other network layer [84], magnifying the damage associated with the cascading process. Protecting the small-degree nodes in the accessible network layer with high-degree counterparts in the other layer thus represents an efficient way to mitigate cascading dynamics.

More specifically, the control strategy can be described, as follows. For a pair of nodes (i, A) and (i, B), we assign a number m_i of adaptive links to the node (i, A) in the adaptive layer according to

$$m_i = \frac{(k_i^B/k_i^A)^\alpha}{\sum_x^N (k_x^B/k_x^A)^\alpha} Q, \tag{5}$$

where the ratio k_i^B/k_i^A characterizes the degree difference between nodes (i, B) and (i, A), Q denotes the given quota of the adaptive links, and the parameter α controls the nodal heterogeneity in adaptive control. In particular, for $\alpha = 0$, the quota is evenly distributed among all nodes in the adaptive layer. For $\alpha > 0$, the quota is distributed selectively according to the nodal degree. Figure 5 shows the simulation results for the percolation transition point p_c versus α in a two-layer system for different quota Q of adaptive links. It can be seen that the percolation transition point p_c decreases faster for scale-free than for random networks. Because of the heterogeneity of the degree distributions for scale-free networks, the degree difference between two interdependent nodes is significant, making the adaptivity allocation scheme particularly effective in improving the robustness of the whole system. On the contrary, the degree distribution of a random network is homogeneous, making the degree based adaptivity distribution scheme less effective.

5. Remote control of a real-world networked system

We apply the remote control principle to a real world system: the national transportation network in the Great Britain [85]. A statistical data analysis indicates that the inter-urban connections consist mainly of rail and coach (about 98.1% of all the traffic in October 2011), while ferry and air transportation account for the remaining 1.9%. We thus focus on the former to construct a two-layer network where, because of the necessity to have a giant component shared by both rail and coach carriers, they act as two interdependent network layers. In this system, nodal removal can occur as a result of natural disasters, accidents or curfew in response to terrorist attacks in a city, etc, possibly leading to a total breakdown of the transportation system in utility for passengers and profitability for carrier. For example, if a coach station is closed, the passenger flow into this city will be reduced, especially for transit passengers, further decreasing the flow through the rail station. As a result, the operating cost per passenger for either a coach or a rail station will increase, causing the company to decrease their vehicle trips through the city for cost control. For passengers, having a connected component in the transportation network shared by both coach and train is important for convenience and affordable fares.

In general, there is a synergistic effect between the coach and the railway transportation networks, as passengers often switch from one network to another to reach their finial destinations. Consider a typical travel itinerary that contains several coach and some train trips. If a railway station fails, the passenger will have no access to the coach stations located in the same city. As a result, he/she has to cancel or reroute the itinerary. This

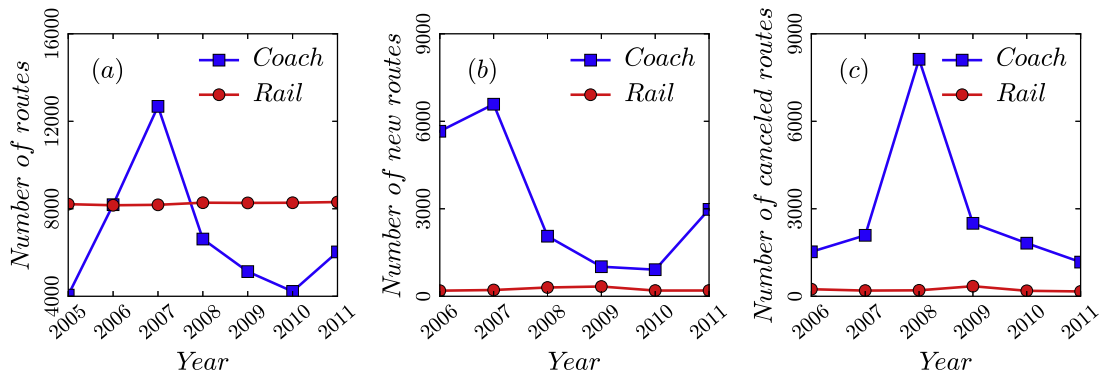

Figure 6. Fluctuations in the numbers of various routes in a real coach-rail transportation system in the Great Britain. (a)–(c) For both coach and rail layers, the numbers of routes, canceled routes, and new routes, respectively, versus time (in units of year) with respect to the corresponding numbers in the previous year.

scenario is typical of ones that give rise to the interdependence between the coach and railway stations. However, the failure of a coach station may not lead to the failure of the rail station, although the failure can have an effect on the convenience for passengers and on the traffic flow of the rail station on which its profit depends. When we construct a multilayer network model for such a coach-railway system, cascading failures manifest themselves as the propagation of service unavailability for passengers or profit loss for the carriers.

To realize control, we analyze the time tables of coach and rail stations for a full week in October between 2005 and 2011, and calculate the yearly fluctuations of two quantities characterizing the effect of control: the numbers of canceled routes and of new routes as compared with those in the previous year. The results are shown in figure 6, where the number of rail routes is essentially stable with few route changes, but the number of coach routes varies frequently, which validates the basic assumption employed in formulating our control strategy: certain network layer has the ability to adjust its connections in response to external perturbations. For the coach/rail double-layer system, it is the coach network that can adjust the routes to maintain business stability in response to random failures or attacks, providing a concrete justification for our designating the number of adaptive links as a physically meaningful control parameter. The control layer A is thus the coach network, while the remote layer B is the rail network. A larger value of the control parameter m means that the control layer is more adaptive and the corresponding carrier has a stronger ability to handle emergency situations, thereby reducing the chance of large scale cascading failures. As an example, the multilayer networked system in a week of October 2011 has $N = 4090$ nodes (cities with at least one coach station or rail station), where there are 2490 nodes in the coach layer, 1874 nodes in the rail layer, and 274 overlapping nodes in both layers, and the sizes of the giant components in the two layers are different. To implement our principle of remote control, we regard nodes with replicas in the coach layer as the adaptive ones. Figure 7 shows that controlling network A through increasing the value of m enhances the robustness of the entire system. In particular, adaptive control of network A not only enhances its own robustness, but also improves the robustness of network B, providing concrete support for the principle of remote control in that the robustness of some inaccessible layers in a real multilayer system can be improved through adaptive control applied to an externally accessible layer. For the coach-rail transportation system *per se*, our results not only have implications for managing the transportation system in response to emergencies, but also provide a potential strategy for carriers to optimize business performance.

6. Discussion

To summarize, we have proposed a remote control scheme with theoretical justification and numerical support to enhance the ability of complex multilayer networks to resist large scale cascading failures. The basic principle is to apply adaptive control to one network layer that is externally accessible, not only to make itself more resilient to failures but, more importantly, to induce the positive effect of improving the robustness of other interdependently connected layers that are inaccessible to control. A finding is that remote control is more effective for systems with a scale free than a random topology. A real world coach-rail transportation system has been used to demonstrate the control principle. In spite of the existing literature on controlling complex networks, the idea of remote control is novel and general, and we expect it to find broad applications in addressing the challenging problem of controlling nonlinear and collective dynamics on in complex dynamical systems.

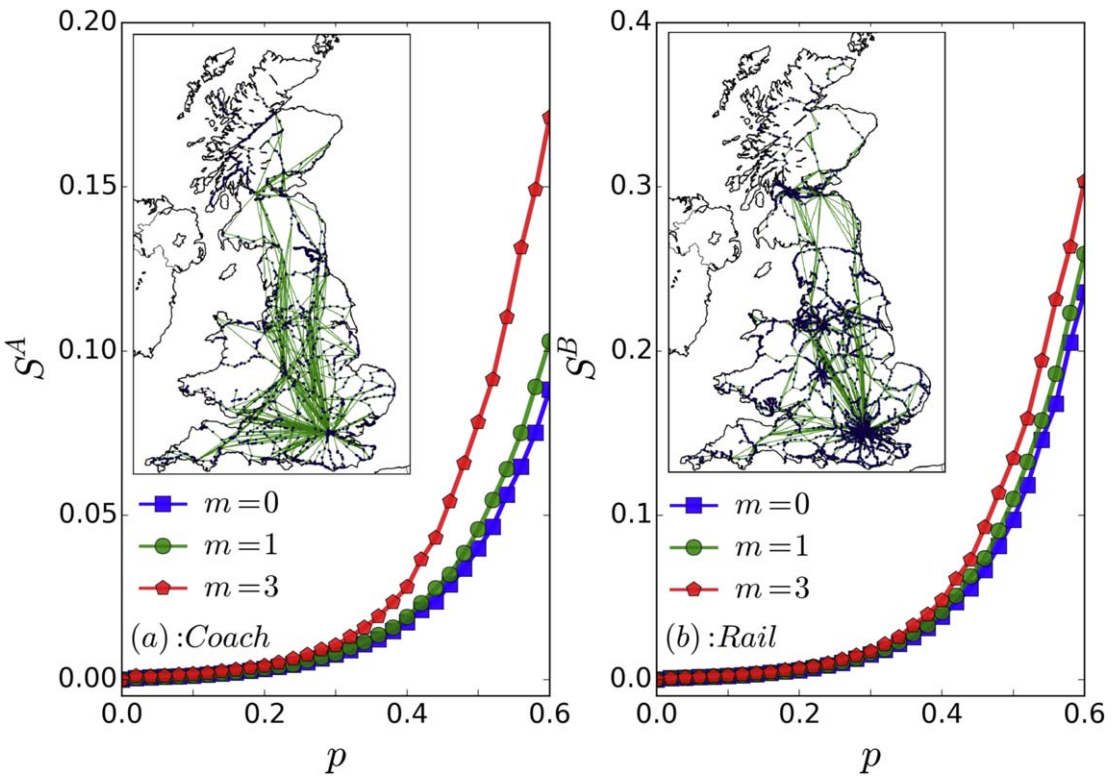

Figure 7. Application of remote adaptive control to enhance the robustness of the entire multilayer system. Coach network is chosen as the control layer *A*, while the rail network is the inaccessible layer *B*. (a), (b) The sizes of the giant components of *A* and *B* versus *p* for different values of the control parameter *m*, respectively. Control can enhance the robustness of both layers. The data points are result of averaging over 1000 statistical realizations. In each panel, the upper left inset shows the actual route map for the corresponding layer.

Acknowledgments

RRL was supported by the National Natural Science Foundation of China under Grant Nos. 61773148 and 61673150. YCL would like to acknowledge support from the Vannevar Bush Faculty Fellowship program sponsored by the Basic Research Office of the Assistant Secretary of Defense for Research and Engineering and funded by the Office of Naval Research through Grant No. N00014-16-1-2828.

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