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THE EMERGENCE AND EVOLUTION OF COOPERATION ON COMPLEX NETWORKS

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The emergence and evolution of cooperation in complex natural, social and economical systems is an interdisciplinary topic of recent interest. This paper focuses on the cooperation on complex networks using the approach of evolutionary games. In particular, the phenomenon of diversity-optimized cooperation is briefly reviewed and the effect of network clustering on cooperation is treated in detail. For the latter, a general type of public goods games is used with the result that, for fixed average degree and degree distributions in the underlying network, a high clustering coefficient can promote cooperation. Basic quantities such as the cooperator and defector clusters, mean payoffs of cooperators and defectors along their respective boundaries, the fraction of cooperators for different classes as well as the mean payoffs of hubs in scale-free networks are also investigated. Since strong clustering is typical in many social networks, these results provide insights into the emergence of cooperation in such networks.

Keywords: Clustering coefficient; evolutionary game; cooperation; small-world networks; scale-free networks; public goods game.

1. Introduction

Cooperation is ubiquitous in biological, economic and social systems [Colman, 1995]. Understanding the emergence and evolution of cooperation is an outstanding problem in interdisciplinary research. A suitable mathematical framework to address cooperation is evolutionary game theory [Smith, 1982; Gintis, 2000]. In this regard, the types of games that have been studied include those based on pairwise interactions among players such as the Prisoner's Dilemma Game (PDG) and the Snowdrift Game (SG) [Szabó & Vukov, 2004; Perc & Szolnoki, 2008; Zhong *et al.*, 2006; Abramson & Kuperman, 2001; Ebel & Bornholdt, 2002; Wu & Wang, 2007; Wang *et al.*, 2006; Tang *et al.*, 2006; Ren *et al.*, 2007; Wang *et al.*, 2008; Fu *et al.*, 2008; Szolnoki *et al.*, 2008]. A type of games that allow for more general group interactions is Public Goods Games (PGG), models based on which for studying the emergence and the dynamics of cooperation have received a great deal of recent attention [Hauert *et al.*, 2002a; Szabó & Hauert, 2002; Hauert *et al.*, 2002b; Semmann *et al.*, 2003; Brandt *et al.*, 2006; Guan *et al.*, 2007; Huang *et al.*, 2008; Santos *et al.*, 2008]. For example, since agents are connected with each other in a complex manner, they can be regarded as nodes in a network with a complex topology, and the mutual interactions among the nodes are governed by the PGG rules. Similar to PDG and SG, in the PGG defection represents the dominant strategy that can lead to the deterioration of cooperation.

In a typical PGG played by N individuals, each individual can choose to cooperate or defect. Each cooperator contributes an amount c to the PGGs, while defectors do not contribute. The total contribution is multiplied by a factor r, and is then redistributed uniformly among all players. As a result of this redistribution, defectors in a group can usually gain more payoffs than cooperators in the same group. It has been known that in a well-mixed population, for $r < N$, defectors will dominate the whole population [Hauert *et al.*, 2006]. This presents a dilemma as to why cooperation is ubiquitous in real-world systems. There have been efforts to resolve the dilemma. For example, to provide an escape hatch out of an economic stalemate, Hauert *et al.* have introduced the mechanism of voluntary participation and found that it results in a substantial and persistent willingness to cooperate [Hauert *et al.*, 2002a]. Szabó and Hauert have studied voluntary participation in PGGs on a square lattice and found that the existence of loners leads to a cyclic dominance of the strategies and promotes substantial levels of cooperation [Szabó & Hauert, 2002]. The effects of inhomogeneous activity in the PGG have been studied [Guan *et al.*, 2007], where the cooperation level is found to be considerably enhanced. Quite recently, *social diversity* has been introduced by means of heterogeneous graphs [Santos *et al.*, 2008]. It is found that diversity associated with the number and the size of the PGG, where each individual participates and contributes to each game, can promote strong cooperation. This finding is quite surprising as one might expect that cooperation can arise more easily in systems consisting of more homogeneous individuals.

In a recent work [Yang *et al.*, 2009], we proposed a strategy for achieving maximum cooperation in evolutionary games on complex networks. Each individual is assigned a weight that is proportional to the power of its degree, where the exponent α is an adjustable parameter that controls the level of diversity among individuals in the network. During the evolution, every individual chooses one of its neighbors as a reference with a probability proportional to the weight of the neighbor, and updates its strategy depending on their payoff difference. It was found that there exists an optimal value of α, for which the level of cooperation reaches maximum. The results suggest that, in order to achieve strong cooperation on a complex network, individuals should learn more frequently from neighbors with higher degrees, but only to a certain extent.

In this paper, we address the effect of one generic property of complex networks, namely clustering, on the emergence and evolution of cooperation. A complex network typically exhibits a number of traits, such as the heterogeneous degree distribution, clustering and degree–degree correlation. In this regard, the property of clustering is fundamental to complex networks, as a high degree of clustering is one of the two defining characteristics of small-world networks [Watts & Strogatz, 1998]. The degree of clustering can be characterized by the clustering coefficient, the probability that two neighbors of a given node share also a connection between them [Newman, 2003; Boccaletti *et al.*, 2006]. Topologically, the clustering coefficient is determined by the number of closed triangles in the network. While clustering structures associated with heterogeneous degree distributions can promote cooperation in the PGG [Rong *et al.*, 2010], whether there is a general effect of clustering structures on cooperation is an open issue. To address this question, we implement the PGG on complex networks, vary the clustering coefficient systematically but keep other topological properties of the network fixed. In order to do so, we use the class of homogeneous small-world networks (HOSW) [Santos *et al.*, 2005] and the class of scale-free networks with tunable degree of clustering

[Holme & Kim, 2002]. Our main finding is that strong clustering promotes cooperation in both types of networks. We show that this phenomenon can be explained qualitatively by the dynamical organization of cooperators versus that of defectors, where cooperators tend to form clusters in order to survive. The payoffs of cooperators and defectors along the boundary of clusters with the same type of players are also examined to establish the robustness of the cooperator clusters in resisting to the invasion of defectors. In view of the ubiquity of cooperation in real-world networked systems, our finding points to a possible mechanism for the formation of clustered structures in such systems from the viewpoint of evolutionary games.

In Sec. 2, we describe the PGG model. In Sec. 3, we present results on the dependence of cooperation on the clustering coefficient for homogeneous small-world networks, together with qualitative explanations. In Sec. 4, we demonstrate how clustering coefficient affects cooperation on scale-free networks. The conclusion is presented in Sec. 5.

2. The Model

Our game model is from [Santos *et al.*, 2008]. Initially, cooperators and defectors are randomly distributed among the population with equal probability. During the evolutionary process, each individual x participates in interactions with $k_x + 1$ neighborhoods that center about x and its k_x neighbors, where each neighborhood contains a central node and all nodes that are directly connected to it. Each cooperator contributes a cost $c = 1$ to each neighborhood that it engages. If x is a cooperator, the strategy is $s_x = 1$ and $s_x = 0$ if x is defector. The payoff of an individual x associated with the neighborhood centered at an individual y is given by

$$p_{x,y} = -s_x + \frac{r}{k_y + 1} \sum_{i=0}^{k_y} s_i, \qquad (1)$$

where $i = 0$ stands for y, s_i is the strategy of the neighbor i of y, and k_i is its degree. The total payoff of player x is $P_x = \sum_{y \epsilon \Omega_x} p_{x,y}$, where Ω_x denotes the set of x and x's neighbors. After each time step, all the players update synchronously their strategies according to the following rule. Each individual x chooses at random a neighbor y and compares its payoff P_x with P_y. If $P_y \leq P_x$, no update occurs. Otherwise, x will adopt y's strategy with a probability given by $(P_y - P_x)/M$ for $P_y > P_x$, where the normalization constant M is the maximum possible difference between the payoffs of x and y.

3. PGGs on Small-World Networks

We first consider small-world networks, which can be constructed by following the standard rewiring procedure [Watts & Strogatz, 1998]. Starting from an undirected regular graph with fixed connectivity z and size N, a two-step circular procedure is executed: (i) choose two different edges randomly and (ii) swap the ends of the two edges. Here, duplicate connections and disconnected graphs are avoided. The annealed randomness is characterized by the parameter f (the rewiring probability), which denotes the fraction of the swapped edges in the network. HOSW networks have the small-world property and the degree of each individual is kept unchanged. Varying f thus provides a way to systematically vary the clustering coefficient C and the average shortest path L. As shown in Fig. 1, L and C are both decreasing function of f. To simulate PGGs on a network, at $t = 0$ we distribute cooperators and defectors randomly among the population with equal probabilities. Figure 2 shows the cooperator density ρ_c as a function of the multiplication factor r for different values of C and L. It can be seen that ρ_c monotonically increases as r is increased. For $C = 0.6$, $L = 83.8$ and $C = 0.59$, $L = 22.8$, the average network distance L differs

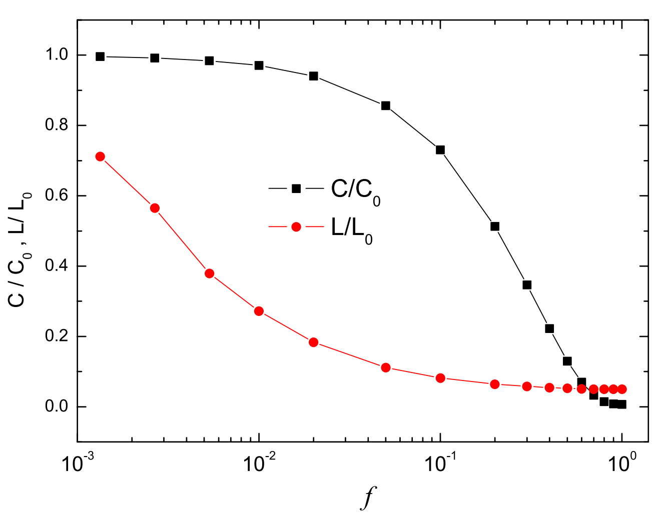


Fig. 1. The cluster coefficient C and the average path length L, divided by their values for regular networks (C_0 and L_0, resp.), as a function of f for the HOSW network.

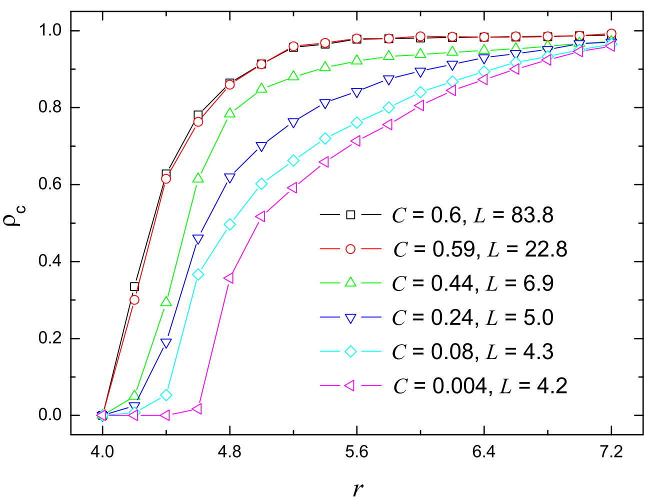


Fig. 2. Cooperator density ρ_c as a function of the multiplication factor r for different values of C and L. The network parameters are $z = 6$ (the number of neighbors in the initial regular network configuration prior to rewiring) and $N = 1000$. Equilibrium cooperator density ρ_c is obtained by averaging over 2×10^3 steps after a transient time of 2×10^5 steps. Each data is obtained by averaging over twenty different network realizations with ten runs for each realization.

obviously, but the clustering coefficient C and the cooperator density ρ_c are approximately the same. For $C = 0.08$, $L = 4.3$ and $C = 0.004$, $L = 4.2$, the average network distance L is approximately the same, but the clustering coefficient C and the cooperator density ρ_c differ obviously. The above results indicate that the degree of cooperation is insensitive to variations in L whereas the clustering coefficient plays a dominant role in affecting the cooperative behavior on the network. In fact, for a fixed value of the game parameter r, ρ_c increases as C is increased, demonstrating that stronger clustering can promote cooperation on small-world networks.

In order to understand the effect of clustering coefficient on, cooperation, in the following, we investigate the cooperator density, the number of clusters, the average size of clusters and the mean payoffs along the boundary as the time evolves. A cooperator (defector) cluster is a connected component (subgraph) fully occupied by cooperators (defectors). Figure 3 shows the time evolution of the cooperator density ρ_c for $f = 0.1$ and $f = 0.5$. One can see that ρ_c decreases rapidly at the early stage while it increases to a steady value for both cases. The steady value of ρ_c is much higher in the case of $f = 0.1$ than that of $f = 0.5$.

It has been known that cooperators tend to form cluster patterns where cooperators assist each

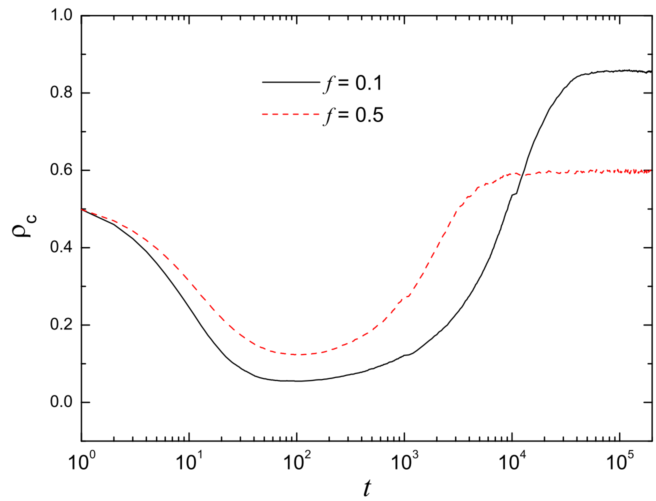
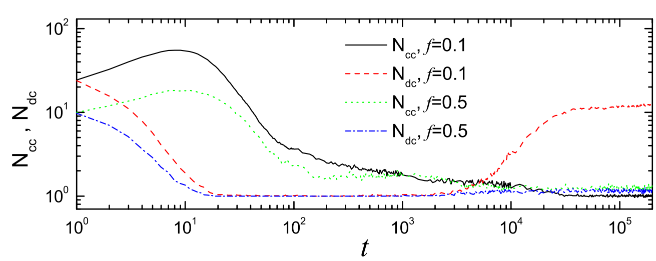
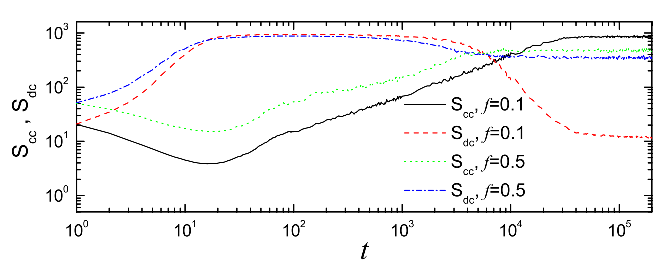


Fig. 3. Time evolution of cooperator density ρ_c for $f = 0.1$ and $f = 0.5$. The multiplication factor $r = 5$ and $N = 1000$.

other to avoid defectors' exploitation in spatial games during the evolutionary process [Guan *et al.*, 2006; Hauert & Doebeli, 2004; Langer *et al.*, 2008]. Figure 4(a) shows that, in both cases of $f = 0.1$ and $f = 0.5$, the number of cooperator clusters N_{cc} increases initially and then gradually decreases to a steady value. For $f = 0.1$, the number of defector clusters N_{dc} first decreases to 1 and then increases



(a)



(b)

Fig. 4. (a) Number of clusters and (b) average size of clusters as a function of time for $f = 0.1$ and $f = 0.5$. The multiplication factor $r = 5$ and $N = 1000$.

to a steady value. For $f = 0.5$, N_{dc} decreases monotonically. Figure 4(b) reports the average cluster size as a function of time. It can be seen that the average size of cooperator clusters S_{cc} decreases followed by an increment, while the average size of defector clusters S_{dc} evolves following a somewhat opposite trend. Figure 4 also shows that for the fixed multiplication factor $r = 5$, large cooperator clusters and many small defector clusters are formed in the equilibrium state for $f = 0.1$. In contrast, for $f = 0.5$, approximately one large cooperator cluster and one large defector cluster coexist in the equilibrium state.

We also study the evolution of the mean payoffs of cooperators and defectors along the boundary ($\overline{P}_{\text{c\_bound}}$ and $\overline{P}_{\text{d\_bound}}$). We define a cooperator (defector) staying at the boundary if it has at least a defector (cooperator) neighbor and vice versa. In Fig. 5, we see that, both $\overline{P}_{\text{c\_bound}}$ and $\overline{P}_{\text{d\_bound}}$ decreases initially and then increases to a steady value, exhibiting the same trend as the evolution of the cooperator density ρ_c. During the decreasing period, $\overline{P}_{\text{c\_bound}}$ is less than $\overline{P}_{\text{d\_bound}}$; while $\overline{P}_{\text{c\_bound}}$ exceeds $\overline{P}_{\text{d\_bound}}$ gradually after they begin to increase. In particular, from time $t = 10^2$ to $t = 10^4$, the difference between $\overline{P}_{\text{c\_bound}}$ and $\overline{P}_{\text{d\_bound}}$ for $f = 0.1$ is much higher than that for $f = 0.5$, e.g. ρ_c are the same for $f = 0.1$ and $f = 0.5$ at $t = 10^4$ (see Fig. 3), but $\overline{P}_{\text{c\_bound}} - \overline{P}_{\text{d\_bound}} \approx 5.5$ for $f = 0.1$, which is larger than $\overline{P}_{\text{c\_bound}} - \overline{P}_{\text{d\_bound}} \approx 2.8$ for $f = 0.5$.

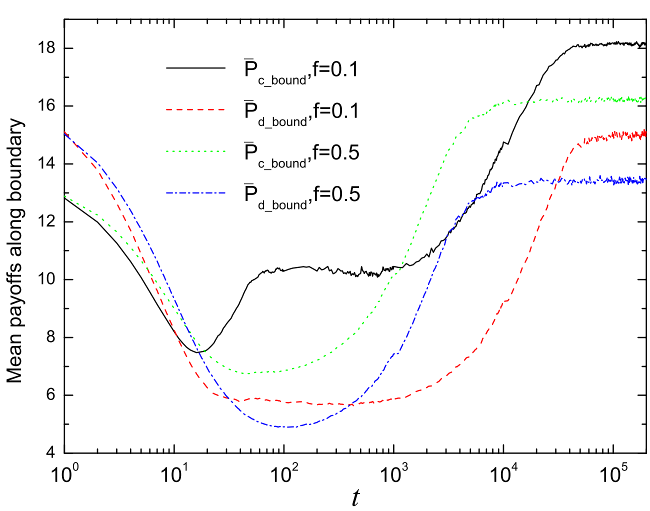


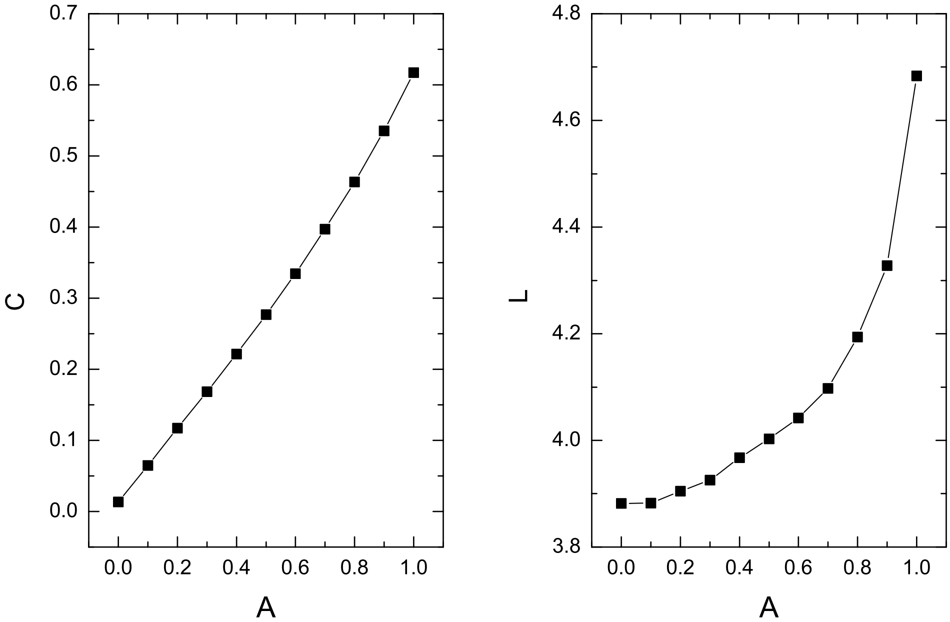
Fig. 5. The mean payoffs of cooperators and defectors along the boundary as a function of time for $f = 0.1$ and $f = 0.5$. The multiplication factor $r = 5$ and $N = 1000$.

Higher values of $\overline{P}_{\text{c\_bound}} - \overline{P}_{\text{d\_bound}}$ suggest that defectors along the boundary are more likely to be replaced by cooperators, promoting the diffusion of cooperators.

4. PGGs on Scale-Free Networks

We next consider scale-free networks. Starting from an initial core of m_0 fully connected nodes, at each time step, a new node i is added to the network with links to m of the previously existent nodes. The first link follows a preferential attachment rule [Barabási & Albert, 1999]. The remaining $m - 1$ links are attached in two different ways: (i) with probability A, node i is connected to a randomly chosen neighbor of node j and (ii) with probability $1 - A$, the preferential-attachment rule is applied again, and node i is connected to another one of the previously existent nodes. The resulting scale-free network has the average connectivity $z = 2m$ and degree distribution $P(k) \sim k^{-3}$. The clustering coefficient can be systematically varied by changing the value of A. In particular, for $A = 0$, we obtain the standard scale-free network [Barabási & Albert, 1999] for which the clustering coefficient C tends to be zero as the network size N becomes infinite. As A is increased, C monotonically increases, so does the average shortest path of the network, as shown in Fig. 6.

In our simulations, we set $m_0 = m = 3$ and $N = 3000$. Figure 7 shows ρ_c as a function of the multiplication factor r for different values of C. We observe that, for networks with higher values of C, cooperation is enhanced. It has been known that on scale-free networks, hubs (high-degree individuals) play a prominent role in maintaining the cooperation [Santos & Pacheco, 2005; Santos *et al.*, 2006; Santos *et al.*, 2008]. To show how hubs influence the evolution of cooperation on scale-free networks in the presence of dense clustering structures, we divide individuals into three classes according to their degrees k_i on a scale-free network: (i) low-degree class: $k_i < z$; (ii) medium-degree class: $z \leq k_i < k_{\max}/3$ and (iii) high-degree class: $k_{\max}/3 \leq k_i < k_{\max}$. Here $k_{\max}$ is the maximum degree of a scale-free network. Figure 8 shows the fraction of cooperators for each of the three classes as a function of time. For $A = 0$, low-degree and medium-degree classes rapidly become defectors, while high-degree individuals insist on their initial strategies for a short period but then turn to



(a) (b)

Fig. 6. For a scale-free network, (a) the cluster coefficient C and (b) average path length L as a function of A (see text for details).

defectors [see Fig. 8(a)]. For $A = 1$, the fractions of cooperators for low-degree and medium-degree classes decrease initially but finally increase to 1. In contrast, individuals of high-degree hold their initial strategies for a short period and then turn to cooperators. The occupation of cooperators on the hubs promotes the diffusion of cooperators in the whole population [see Fig. 8(b)]. Figure 9 shows the mean payoffs of hubs with time. We see that, for $A = 1$, initially the mean payoff of the cooperator hubs (C-hubs) is lower than that of the defector hubs (D-hubs), but after a short period, the payoff of C-hubs becomes much higher than that of D-hubs.

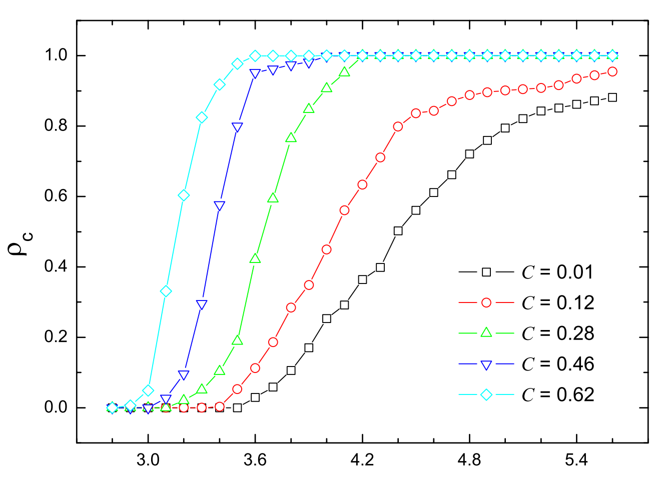
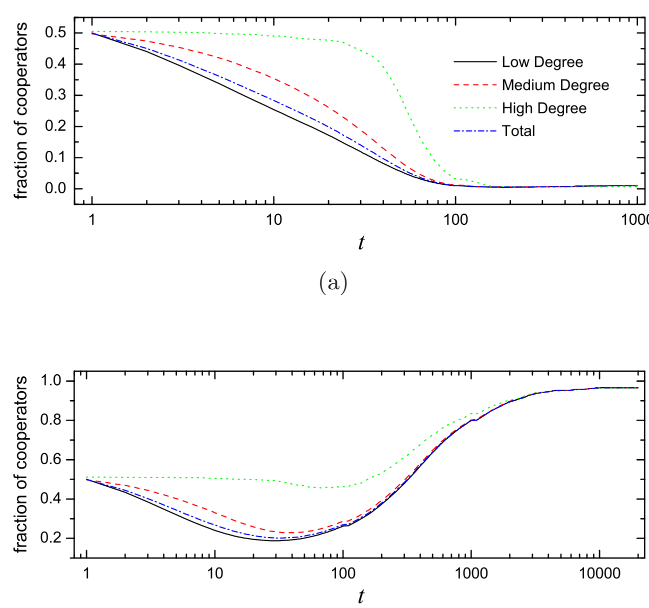


Fig. 7. Cooperator density ρ_c as a function of the multiplication factor r for different values of C.



(b)

Fig. 8. Time evolution of the fraction of cooperators for each of three classes. (a) $A = 0$ and (b) $A = 1$. The multiplication factor is $r = 3.5$.

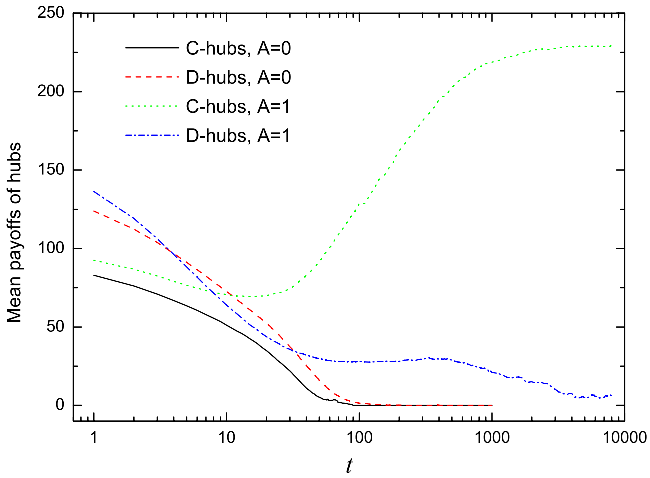


Fig. 9. Time evolution of the mean payoffs of C-hubs and D-hubs for $A = 0$ and $A = 1$. The multiplication factor $r = 3.5$.

While for $A = 0$, the mean payoff of C-hubs is always lower than that of D-hubs, leading to the extinction of cooperation.

5. Conclusion

We have studied the effects of clustering coefficient on cooperation in spatial PGGs on two common types of complex networks: small-world and scale-free. We have found that high clustering coefficient can promote the emergence of cooperation for both types of networks. While this phenomenon seems intuitive, we note that, in the prisoner's dilemma game, high clustering coefficient does not always tend to enhance cooperation [Santos *et al.*, 2005; Assenza *et al.*, 2008]. This is because in this case, individuals play games exclusively with their nearest neighbors, but in the PGG, each individual participates in the neighborhoods that center about itself and its neighbors, meaning that individuals' payoffs are not only related with their nearest neighbors, but also with their next-nearest neighbors. Due to the presence of group interactions in the PGG, highly clustered topologies can promote cooperation despite the fact that the defection strategy can usually lead to more payoffs. This can be a motivation for individuals to build more connections to form clusters in order to resist the invasion of defectors. Our finding may help explain the widely observed clustering structure in the real-world networks from the viewpoint of evolutionary games.

Acknowledgments

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