Optimal structure of complex networks for minimizing traffic congestion

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To design complex networks to minimize traffic congestion, it is necessary to understand how traffic flow depends on network structure. We study data packet flow on complex networks, where the packet delivery capacity of each node is not fixed. The optimal configuration of capacities to minimize traffic congestion is derived and the critical packet generating rate is determined, below which the network is at a free flow state but above which congestion occurs. Our analysis reveals a direct relation between network topology and traffic flow. Optimal network structure, free of traffic congestion, should have two features: uniform distribution of load over all nodes and small network diameter. This finding is confirmed by numerical simulations. Our analysis also makes it possible to theoretically compare the congestion conditions for different types of complex networks. In particular, we find that network with low critical generating rate is more susceptible to congestion. The comparison has been made on the following complex-network topologies: random, scale-free, and regular. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2790367]

Complex networks are essential for a modern society. Understanding of dynamics on complex networks with different topologies is not only interesting theoretically, but also can have impacts in real-world applications. Among various dynamics, traffic flow has been studied extensively due to the necessity of designing high efficiency, low cost networks. Examples include communication, computer, traffic, and power distribution networks. Because the amount of information and other physical quantities to be transmitted or transported over the network are ever increasing, congestion detection and optimization have become a topic of recent interest. One of the open questions, for which a relatively complete understanding is still lacking, is how the dynamical property of congestion depends on the network structure. In this work, we study traffic-flow dynamics on complex networks. Our approach is different from previous ones in that we focus on the optimal configuration of capacities to minimize traffic congestion. The critical packet-generating rate is also determined, below which the network is at a free flow state and above which congestion occurs. Our analysis reveals a direct relationship between some controllable parameters of network topology and traffic-flow performance. Moreover, our results permit a theoretical comparison of the congestion conditions among different types of complex networks. Interestingly, we find that the congestion criticality decreases by the following order of network topology: random, scale-free, and regular. This result confirms previously obtained numerical results on phase transition of traffic-flow dynamics on complex networks.

I. INTRODUCTION

In recent years, tremendous interest has been devoted to studying statistical and dynamical properties of large-scale networks with complex structures. This is motivated by the facts that complex networks occur commonly in nature and that they are essential for the infrastructure of a modern society. Examples of such networks include the Internet, world-wide web, telecommunication systems, power grid, social networks, traffic networks, biological networks, such as neural networks, gene regulatory networks, protein-protein interaction networks, etc. The first study on large networks was done by Erdös and Rényi,[1] who analyzed rigorously randomly connected networks. In 1998, Watts and Strogatz[2] discovered that the average shortest paths of a regular network can be drastically reduced and the local structure of the network as measured by clustering coefficient can be maintained by randomly changing only a small portion of links. The resulting networks are called small-world networks, which are representative of real networks such as social and linguistic networks. In 1999, Barabási and Albert[3] discovered that the degree distribution of many complex networks obeys a power law $P(k)\sim k^{-\gamma}$, where k is the number of links (or the degree) of a randomly chosen node and γ is the scale exponent, henceforth the term scale-free networks. This means that the probability of finding a set of nodes with a

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large number of links is not exponentially small, indicating that the degrees are highly heterogeneous, in contrast to random networks whose degree distributions are homogeneous. After those seminal works, extensive research has been carried out, for example, on the following topics: network growth and self-organization, degree and betweenness distribution, complex network resilience and cascading breakdown, epidemiological process, community structure, and network stability and synchronization (see Refs. 4–6 and references therein). Complex networks have become an active field in nonlinear science.

Traffic flow in networks has been extensively studied.[7–20] However, earlier models assumed regular network topology, such as two-dimensional lattices or Cayley trees. Recently, there has been a growing interest in traffic flow on complex networks.[21–27] This is motivated by the fact that the Internet and many other realistic networks are complex to a significant extent, leading to a pressing need to investigate the dynamics of traffic flow on these networks.

In many real-world systems designed for information or data exchange, traffic congestion can lead to failures or delays of various system functions. Intuitively, traffic congestion could be largely reduced or completely avoided with a very large average degree of connectivity and/or node capacity for data packet delivery. However, this may not be feasible because of the requirement of potentially high cost. In a network, traffic congestion occurs as soon as the packet-generating rate on each node of the network is greater than a critical value. Network with a smaller critical generating rate is more susceptible to congestion. A recent work[23] has shown that, for two networks with the same average connectivity, node capacity, and total number of nodes, if their topological structures are different, the critical packet-generating rate can be significantly different, indicating that traffic congestion depends sensitively on network structure. It is found that scale-free networks and random networks are more tolerant of traffic congestion than regular networks and Cayley trees. However, between a scale-free and a random network, the former is more prone to congestion. These results imply the existence of some optimal network structure that minimizes traffic congestion.

In order to better understand the problem of optimal network structure for minimal traffic congestion,[18,28] in this paper we study traffic-flow dynamics on complex networks in a more general setting. In particular, our study differs from other works on the same topic[18,23] in that we do not fix the packet-delivery capacity of each node. We shall determine the optimal configuration of capacities to minimize traffic congestion in the network. The critical packet-generating rate is also determined, below which the network is at a free flow state, and above which congestion occurs. Our analysis reveals a direct relationship between some controllable parameters characterizing the network topology and the traffic-flow performance. Specifically we find that, in order to avoid severe traffic congestion, network structure should have two features: uniform distribution of load over all nodes and small network diameter. Our theoretical analysis makes it possible to compare the critical packet-generating rate among different types of complex networks. We find that the critical rate decreases by the following order of the network topology: random, scale-free, and regular. These confirm pertinent previous numerical results concerning traffic dynamics on complex networks.[23]

In Sec. II we present a theoretical analysis to determine the optimal configuration of delivery capacities. In Sec. III we provide computer simulation results to support our analysis. In Sec. IV we give a brief summary and draw conclusions.

II. THEORETICAL ANALYSIS

We consider a general model for traffic flow on complex networks. Each node i generates a data packet with probability λ_i at each time step, which is to be delivered to a randomly selected destination node, say j. The packet is delivered to its destination along the shortest path between nodes i and j. At each time step the data packet can be delivered from one to another if the nodes are connected, such as a pair of nearest-neighboring nodes. The maximum number of packets that node i can deliver at each time step is C_i, the delivery capacity. Although traffic-flow rules defined in this model are simple, they capture the essential characteristics of traffic flow in real networks. Our aim is to determine the relationship between the network structure and the critical packet-generating rate $\lambda_{i,\max}$.[29] To achieve this, we use a recent finding on the critical packet generating rate of node i, i.e., $\lambda_i = C_i/(B_i/N)$,[23] where C_i, B_i, and N are the delivery capacity, the betweenness of node i, and the total number of nodes in the network, respectively. Let $\tilde{\lambda} = (1/N)\Sigma_{i=1}^{N}\lambda_i^2 = N\Sigma_{i=1}^{N} C_i^2/B_i^2$ be the average critical packet-generating rate. In order to determine the optimal values of C_i and $\tilde{\lambda}_{\max}$, we define the following Lagrangian:

$$L = \tilde{\lambda} + \delta\left[\sum_{i=1}^{N} C_i - S\right], \qquad (1)$$

where δ is a Lagrange multiplier, $S = N(1+\beta\langle k\rangle)$ is the total capacity of the network, and $\langle k\rangle$ is the average degree of the network. The total capacity of the network is proportional to the network size and $\langle k\rangle$. This constraint can be justified by the consideration that, in many network designs, network size and the degree of connectivity are usually the main factors determining the cost, and thus they should be limited. The set of C_i maximizes L under the following conditions:

$$\frac{\partial L}{\partial C_i} = 0, \quad i = 1, 2, \ldots, N. \qquad (2)$$

From Eqs. (1) and (2), we get $C_i = -B_i^2\delta/2N$ and $\delta = -2N^2 d\langle k\rangle/\Sigma_{i=1}^{N} B_i^2$, which lead to the optimal delivery capacity: $C_{i,\text{optimal}} = B_i^2 S/\Sigma_{i=1}^{N} B_i^2$. Since λ_i is a positive number, it is maximized when λ_i^2 is maximized. Thus, we have $\lambda_{i,\max} = B_i SN/\Sigma_{i=1}^{N} B_i^2$, where $\lambda_{i,\max}$ is the maximum generating rate of node i.

The average maximum generating rate is $\tilde{\lambda}_{\max} = (1/N)\Sigma_{i=1}^{N}\lambda_{i,\max}$, which becomes $\tilde{\lambda}_{\max} = SN^2D/\Sigma_{i=1}^{N} B_i^2$, where the betweenness B_i satisfies $\Sigma_{i=1}^{N} B_i = N(N-1)D \approx N^2 D$ for large N (D is the network diameter). To proceed, it is necessary to determine $\Sigma_{i=1}^{N} B_i^2$ with respect to N and D. For a

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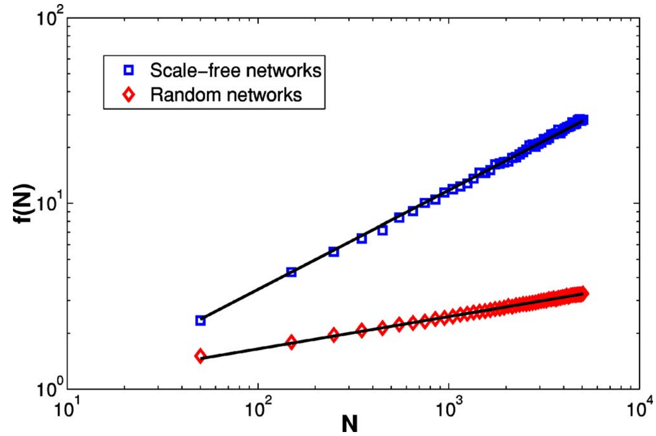

FIG. 1. (Color online) Algebraic scaling of $f(N)$ with network size N for random and scale-free networks. For both types of networks, we fix $\langle k \rangle = 4$ and the simulation results are averaged over 200 realizations. For random networks, we find $r \approx 0.17$ and $c \approx 0.74$. For scale-free networks with degree exponent $\gamma = 3$ (see Refs. 1–5), we find $r \approx 0.54$ and $c \approx 0.28$.

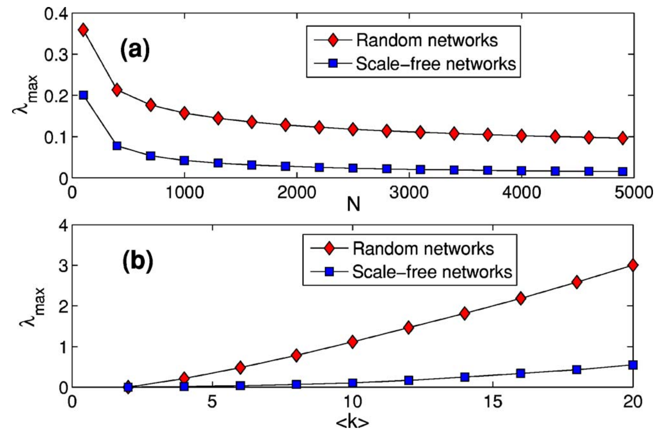

FIG. 2. (Color online) For random and scale-free networks of degree exponent $\gamma = 3$, (a) $\tilde{\lambda}_{max}$ vs network size N for $\langle k \rangle = 4$ and (b) $\tilde{\lambda}_{max}$ vs the average degree $\langle k \rangle$ for $N = 5000$.

regular network, all nodes have the same betweenness, $B_{i,\text{reg}} = DN$. Thus, we have $\Sigma_{i=1}^{N} B_{i,\text{reg}}^2 = N^3 D^2$. For a general complex network, we can write $\Sigma_{i=1}^{N} B_i^2 = N^3 D^2 f(N)$, where $f(N)$ is a function that depends on the network structure. For example, for a scale-free network, it is possible to calculate $f(N)$ for different values of γ. To illustrate this, we present our calculations for $\gamma = 3$.[30] First consider the summation $\Sigma_i k_i = 2mN$, where k_i and $2m$ are the degree of node i and the average degree of the network, respectively. The summation can be replaced by an integration, i.e., $\Sigma_i k_i \approx a \int_{k_{\min}}^{k_{\max}} P(k) dk \sim N$, where we have used the degree distribution function $P(k) = a k^{-\gamma}$ with constant a, and have assumed that the network diameter satisfies $D \ll N$ for large N. Since $k_{\max} \sim N^{1/(\gamma-1)}$ (Ref. 31) and $k_{\min}$ is a small constant for scale-free networks, we have $a \sim N$ for large N. Next, we focus on the summation $\Sigma_i B_i \approx N^2 D$. It can also be approximated by integration: $\Sigma_i B_i \approx \int_{k_{\min}}^{k_{\max}} B(k) P(k) dk \sim N^2$. Using the scaling relation $B(k) = b k^{\eta}$ with $\eta \approx 1.5$ for $\gamma = 3$,[32,33] we find $b \sim N$ for large N. The sum $\Sigma_i B_i^2$ can then be calculated, as follows:

$$\sum_i B_i^2 \approx \int_{k_{\min}}^{k_{\max}} B^2(k) P(k) dk \sim N^3 N^{(2\eta-\gamma+1)/(\gamma-1)}. \qquad (3)$$

We then get $f(N) \sim N^{0.5}$. This result has been verified by numerical simulations, as shown in Fig. 1. As is apparent from the analysis, the function $f(N)$ for different γ can be determined in a similar way. Numerically, we find the algebraic law $f(N) = cN^r$ holds even for random networks, as shown in Fig. 1.

The average maximum rate over the whole network generally can be written as

$$\tilde{\lambda}_{\max} = \frac{1 + \beta \langle k \rangle}{D f(N)}, \qquad (4)$$

where the critical rate $\tilde{\lambda}_{\max}$ is inversely proportional to the network diameter D and the scaling function $f(N)$. This means that, if two structurally distinct networks have the same average degree, the same node capacity, and the same system size, the one with smaller value of $Df(N)$ is more robust against traffic congestion. As we will explain, $f(N)$ has larger values when total load of the network is concentrated on a small number of nodes. Therefore, networks with small diameter and uniform load distribution are more robust against traffic congestion.

III. NUMERICAL SUPPORT

In order to characterize the transition from free traffic flow to congestion, we use the order parameter $\eta = \lim_{t \to \infty} \langle \Delta \Theta \rangle / \tilde{\lambda} \Delta t$,[15] where $\Delta \Theta = \Theta(t + \Delta t) - \Theta(t)$, $\Theta(t)$ is the total number of packets in the network at time t, and $\langle \cdots \rangle$ indicates average over a time window of length Δt. For $\tilde{\lambda} < \tilde{\lambda}_{\max}$, the network is in a free flow state. In this case, we have $\Delta \Theta \approx 0$ and $\eta \approx 0$. For $\tilde{\lambda} > \tilde{\lambda}_{\max}$, traffic congestion occurs so that $\Delta \Theta$ increases with Δt. In our simulations, scale-free and random networks are generated by using the general network model proposed in Ref. 34. Betweenness is calculated by using the algorithm introduced in Ref. 35. For all cases considered, η is approximately zero when $\tilde{\lambda}$ is small, but it suddenly increases when $\tilde{\lambda}$ is larger than a critical value $\tilde{\lambda}_{\max}$. Figure 2(a) shows $\tilde{\lambda}_{\max}$ versus network size N for random and scale-free networks, respectively. We see that $\tilde{\lambda}_{\max}$ decreases with N, which means that larger networks are more susceptible to congestion than smaller networks. This is because packets take longer time to be delivered in a larger network, which in turn leads to higher chances for the network to be congested. In the extreme case, where network size is infinite, situations can arise where packets will never reach their respective destinations in finite time. Hence, the critical packet generating rate decreases with system size. This behavior is predicted by our theoretical result Eq. (4), which says that $\tilde{\lambda}_{\max}$ is inversely proportional to $Df(N)$ and therefore decreases with N. Figure 2(b) shows that for both scale-free and random networks, $\tilde{\lambda}_{\max}$ increases with the average degree. This is because an increase in the average degree usually reduces the lengths of shortest paths. Equation

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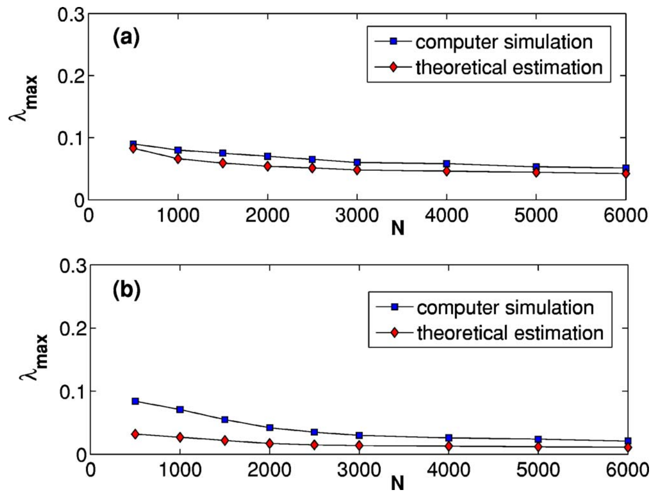

FIG. 3. (Color online) Comparison of critical generating rate $\tilde{\lambda}_{max}$ from theoretical prediction (diamonds) and from direct numerical simulation (squares) for (a) random networks and (b) scale-free networks with degree exponent $\gamma=3$. For all networks considered, $\langle k\rangle=4$.

(4) predicts that the critical generating rate is proportional to the average degree. Again the simulation results are consistent with our theoretical analysis.

To compare Eq. (4) with numerical results in a more quantitative way, in Fig. 3 we show values of $\tilde{\lambda}_{max}$ from direct numerical simulations and from Eq. (4) for random [Fig. 3(a)] and scale-free networks [Fig. 3(b)]. In both cases, a reasonably good agreement is observed, especially when N is large.

Previous numerical simulations have shown that scale-free networks are more vulnerable to traffic congestion than random networks with the same number of nodes and average degree.[33] Our result confirms this fact and provides a theoretical explanation. In particular, say we consider the ratio $r=\tilde{\lambda}^{R}_{max}/\tilde{\lambda}^{SF}_{max}$, where $r>1$ means that scale-free (SF) network is more easily congested than random (R) network. Since $D_{SF}\sim \ln N/\ln\ln N$, $f_{SF}(N)\sim N^{0.54}$ (for scale-free network with degree exponent $\gamma=3$; see Refs. 1–5) and $D_R \sim \ln N$, $f_R(N)\sim N^{0.17}$ for networks with size N and $\langle k\rangle=4$, the ratio r can be written as

$$r \sim (N^{0.54}\ln N/\ln\ln N)/(N^{0.17}\ln N) = N^{0.37}/\ln\ln N. \quad (5)$$

We see that r is always greater than 1 for large N. The same results hold for $\langle k\rangle>4$. This is because $f(N)$ is larger in a scale-free network than in a random network. Although the network diameter D shows the opposite behavior, the difference of D between two types of networks is insignificant to affect the value of $f(N)$.

Figure 4 shows the order parameter η versus the packet-generating rate $\tilde{\lambda}$ for networks of different diameters, as the networks are changed from regular to small-world and to random by a continuous rewiring process with rewiring probability p.[2] When p is small, the network is small-world. For $p=1$, all links are randomly changed and the network becomes completely random. As p is increased, N and $\langle k\rangle$ are fixed, but D is reduced. We thus expect $\tilde{\lambda}_{max}$ to decrease with p, as confirmed by Fig. 4. We see that random networks are

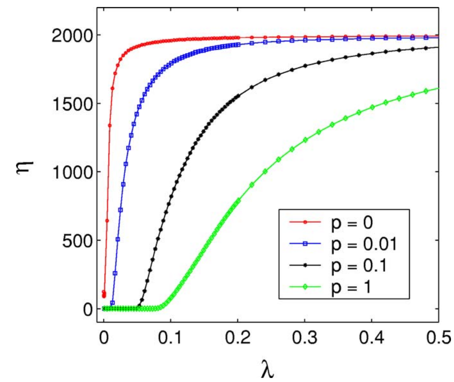

FIG. 4. (Color online) Order parameter η vs the packet-generating rate λ for networks of different diameters. Circles, squares, asterisks, and diamonds correspond to results for $p=0$ (regular network), $p=0.01$ (small-world network), $p=0.1$ (partially random network), and $p=1$ (random network), respectively. For each data point, the network size is $N=2000$ and 50 realizations are averaged.

apparently more robust against congestion than regular networks. This can also be seen by considering the ratio $r=\tilde{\lambda}^{p=1}_{max}/\tilde{\lambda}^{p=0}_{max}$. From Eq. (4) and using $D=N/2$ for $p=0$, we see that the ratio becomes $r\sim(N/2)/(N^{0.17}\ln N) \sim N^{0.83}/\ln N>1$ for large N.

As shown in Fig. 1, $f(N)$ has relatively larger values when the total load of the network is concentrated on a small number of nodes, as for a scale-free network. In order to avoid severe traffic congestion, network structure should have the following two features: (1) total load in the network should be distributed uniformly over all nodes as much as possible and (2) network diameter should be small. One intuitive but efficient method to generate such a network from a network model with $p=1$ in Fig. 4 is as follows. Instead of detaching a link from each node and connecting it to a randomly selected node without taking into consideration the degree of the newly connected node, we detach all links connected to next nearest neighbors and connect them to a randomly selected node which has less than k links. As a

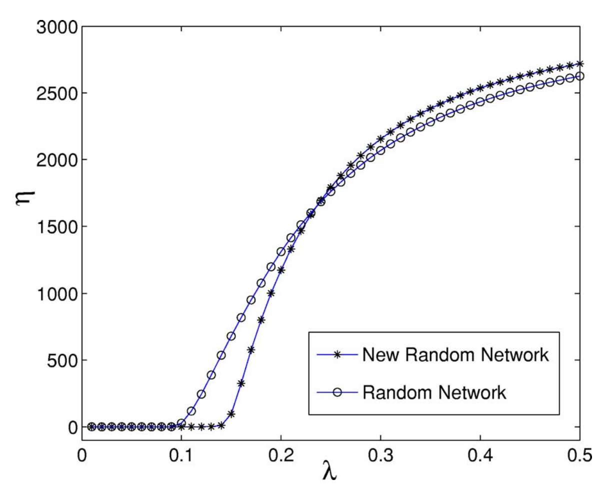

FIG. 5. (Color online) Order parameter η vs the packet-generating rate λ for classical random networks (circles), and for the new random networks proposed in this paper (asterisks). Network parameters are $p=1$, $N=3000$, and $\langle k\rangle=4$. For each data point, 30 realizations are averaged.

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result, each node in the network will have exactly k links, and thus the load of the network is distributed uniformly. We expect this network to be less congested than the random network with $p=1$ in Fig. 4. This has indeed been observed, as shown in Fig. 5, where the onset of traffic congestion occurs for larger value of λ for our uniform random network.

IV. DISCUSSION

In conclusion, we have presented a theoretical analysis and simulation results for traffic flow processes on complex networks. Our motivation comes from the desire to understand the influence of network structure on the traffic dynamics. We have obtained the optimal configuration of node capacity for data delivery in complex networks and the maximum packet-generating rate above which traffic congestion occurs. Our analysis reveals that traffic congestion depends on factors such as network size, network diameter, average degree, and load distribution. Our finding suggests that, in order to mitigate or avoid traffic congestion, load should be uniformly distributed in the network and the network diameter should be small. Our results have practical implications for designing computer networks and other communication networks where minimizing traffic congestion is a central goal.

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