Inducing intrinsic localized modes in microelectromechanical cantilever arrays by frequency modulation

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We articulate a control method to induce intrinsic localized modes at programable bielement cell in driven microcantilever arrays. The idea is to excite a preassigned cantilever to an oscillating state with significantly higher amplitude than the average by using feedback signal to modulate the frequency that drives the *whole* array. Our control method is thus global, which is advantageous in microsystems as local pinning actuation may be difficult to implement at small scales. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3216054]

The use of microelectromechanical (MEM) cantilever or clamped beam arrays have become widespread in all kinds of microscale devices. Due to their large output signals, the array systems are suitable for highly sensitive sensors, such as mass sensors[1] and magnetometers.[2] They are also useful in applications such as RF filtering with programmed pass band.[3]

The phenomenon of nonlinear energy localization in MEM cantilever arrays has attracted much recent attention.[4] Such energy states, called *intrinsic localized modes* (ILMs), can occur in a defect-free nonlinear lattice, extending over only a few lattice sites.[5] Advances in microfabrication and optical visualization technologies render feasible detailed experimental studies of ILMs in MEM systems. In a typical experimental setting to excite ILMs,[6] temporal noise is employed. In this case, the actual locations where ILMs would arise are completely random and unpredictable. An interesting question concerns then a suitable driving scheme to excite an ILM at a programmed location. Recently, this goal has been achieved experimentally[7] via a pinning-control method, where impurity mode was induced by laser and used to trap an ILM. We ask in this letter whether it would be possible to use a *global* actuating signal, i.e., a control signal applied identically to all cantilevers, to excite ILMs at any specific location of interest. In practical applications, this question may be meaningful if some particular ILMs correspond to a desirable state of the operation of the device. For example, in experimental research it may be desirable to create certain patterns of ILMs.

A brief description of our control method is as follows. We first set the frequency of the external driving such that the MEM cantilever-array system exhibits spatiotemporal chaos, which is pervasive and can be realized readily by adjusting the frequency of the driving.[8] We then use a measured displacement signal of the target cantilever as input in a feedback scheme to tune the global driving frequency, thereby stabilizing the particular ILM at that cantilever.

The dynamics of a MEM cantilever array is described by[6] $m_i\ddot{x}_i+b_i\dot{x}_i+k_{2i}x_i+k_{4i}x_i^3+k_I(2x_i-x_{i+1}-x_{i-1})=m_i\alpha\cos(\Omega t)$, where x_i ($i=1,\ldots,N$) is the displacement of the end point of the ith cantilever beam of effective mass m_i, b_i is the damping coefficient, k_{2i} and k_{4i} are the on-site harmonic and quadratic spring constants of the ith beam, respectively, and k_I is the harmonic coupling spring constant. Each beam is subject to a common sinusoidal driving of acceleration α and angular frequency Ω. To induce ILM at a preassigned site, say site M, we propose the following frequency-modulating scheme:

$$\begin{cases}\dot{\Omega}_c=\gamma_1\xi\\ \dot{\xi}=-\dfrac{1}{\tau}[\xi+\gamma_2 x_M\sin(\Omega_c t+\phi)],\end{cases}\qquad(1)$$

which can be regarded effectively as a low-pass filter. In Eq. (1), Ω_c is the modulated driving frequency, ξ is a variable related to the phase-angle difference, ϕ is a reference phase, τ is the typical time corresponding to the cutoff frequency of the low-pass filter, and γ_1 and γ_2 are the gains of the modulator. The value $2\pi/\tau$ should be set to be much smaller than Ω_c so that the high-frequency component of $\gamma_2 x_M\sin(\Omega_c t+\phi)$ can be filtered out. Substituting Ω by Ω_c in $x_i(t)=U_i(t)\cos(\Omega t)-V_i(t)\sin(\Omega t)\equiv r_i(t)\cos[\Omega t+\theta_i(t)]$ in Eq. (1), where $r_i=\sqrt{U_i^2+V_i^2}$ and θ_i are the radial and angular coordinates of (U_i,V_i), we can write the $x_M\sin(\Omega_c t+\phi)$ term as $x_M\sin(\Omega_c t+\phi)=r_M(t)\{\sin[2\Omega_c t+\theta_M(t)+\phi]+\sin[\phi-\theta_M(t)]\}/2$, where the high-frequency term $\sin[2\Omega_c t+\theta_M(t)+\phi]$ will be removed by the low-pass filtering process. Equation (1) then becomes $\xi(t)\approx\gamma_2 r_M(t)\sin[\theta_M(t)-\phi]/2$, which depends on the phase-angle difference between the output and the reference and provides perturbations for tuning the driving frequency. The modulated angular frequency is approximately given by

$$\dot{\Omega}_c(t)=\gamma_1\xi(t)\approx\frac{\gamma_1\gamma_2}{2}r_M(t)\sin[\theta_M(t)-\phi].\qquad(2)$$

The output phase angle can thus be stabilized about the desired reference value that corresponds to a high-energy oscil-

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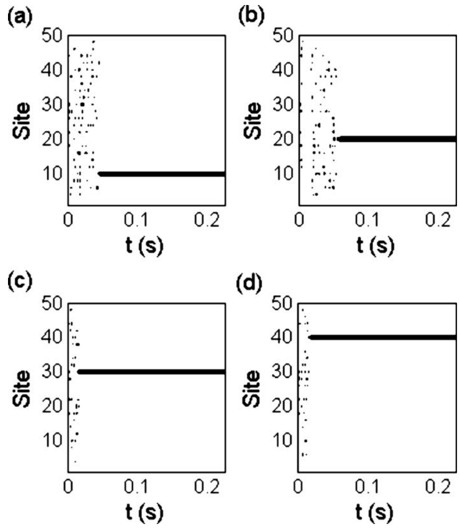

FIG. 1. For a MEM cantilever system of $N=50$ beams of alternating length, space-time plots of four examples of inducing ILMs at sites $M=10$ (a), 20 (b), 30 (c), and 40 (d). The initial value of Ω_c is $\Omega_c(t=0)=9.24\times 10^5$ rad/s. Dark lines indicate higher amplitudes. See text for various parameters.

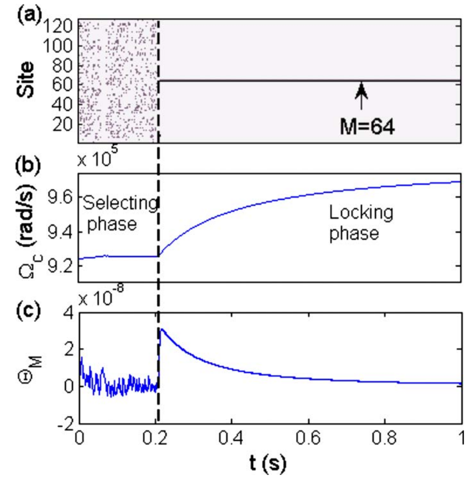

FIG. 2. (Color online) For a MEM cantilever system of $N=128$ beams, an example of stabilizing an ILM at site $M=64$: (a) space-time plot of beam amplitude, (b) time series of the modulating frequency $\Omega_c(t)$, and (c) time series of the phase difference $\Theta_M(t)$ for $M=64$. The initial value of Ω_c is set to be $\Omega_c(t=0)=9.24\times 10^5$ rad/s. The vertical dashed line at $t\approx 0.21$ s denotes the boundary between the selecting and the locking phases.

lation state at site M. The reference phase angle should be set as the one associated with the ILM.

Numerical results illustrating our method for inducing ILMs are shown in Figs. 1(a)–1(d) in which the fourth-order Runge–Kutta method with time step 6.4×10^{-5} s is employed to integrate the dynamical equations. The coupled MEM cantilever system consists of two groups of beams of different length, arranged alternatively in space. The structural parameters of the system are chosen according to their respective experimental values,[6] $(m_i, b_i, k_{2i}, k_{4i}) = (5.46\times 10^{-13}$ kg, 6.24×10^{-11} kg/s, 0.303 N/m, 5×10^8 N/m^3) 5.46×10^{-13} kg, 6.24×10^{-11} kg/s, 0.303 N/m, 5×10^8 N/m^3 for odd i, the long beams, and $(m_i, b_i, k_{2i}, k_{4i}) = (4.96\times 10^{-13}$ kg, 5.67×10^{-11} kg/s, 0.353 N/m, 5×10^8 N/m^3) for even i, the short beams. The parameters of the driving are chosen to be $(\alpha, \gamma_1, \gamma_2, \tau) = (1.56\times 10^4$ m/s^2, 5×10^{12}, 0.1361 s). The total number of cantilevers is $N=50$ and the phase in Eq. (1) is set to be $\phi=-1.4641$ (arbitrarily). The initial value of the driving frequency is set to be $\Omega_c(t=0)=9.24\times 10^5$ rad/s so that the system exhibits spatiotemporal chaos. The initial displacements and velocities of all beams are set to be randomly with standard deviation of 10^{-7} m and 10^{-7} m/s, respectively. The four panels in Figs. 1(a)–1(d) correspond to the cases where an ILM has been induced at site $M=10$, 20, 30, and 40, respectively. Apparently, our method is capable of inducing robust ILMs at any desirable cell in the system. Note that the device employed in experiment[6] consists of multiple bielement cells and each cell is a dual-coupled beam with different beam lengths. For this class of systems, ILMs can be induced at the shorter beam with even sequence number in any bielement cell.

The above simulation results are for ideal cantilever arrays. In real experimental devices fabrication errors are inevitable. It is thus necessary to address the effect of fabrication errors on the effectiveness of control. We have carried out a number of simulations and found that our control method is robust to such errors. For example, when there is 10% systematic mismatch and 1% random mismatch among both the lengths and thicknesses of the beams, ILMs can be induced at any desirable cell, which is essentially the same result as in Fig. 1. We remark that the main requirement for our control scheme is measurement of the position of the target cantilever, which can be realized by optical microprobes.[9] Other quantities necessary for the control are driving frequency and phase angle of the stable ILM, which can be obtained by theoretical analysis and computations.[10]

To understand the working of our control method, we consider a MEM array system of $N=128$ beams (64 cells). Say the desirable site for ILM is $M=64$. Figures 2(a)–2(c) show a representative space-time plot, a typical time series of the modulating frequency $\Omega_c(t)$, and a signal of the filtered phase-angle difference

$$\Theta_M(t) = r_M(t)\sin[\theta_M(t) - \phi], \tag{3}$$

respectively. The control process consists of two distinct stages, as shown in panels (b) and (c). The first stage, a selecting phase, is for $t\leq 0.21$ s (indicated by the vertical dashed line), where the quantity $\Theta_M(t)$ fluctuates about zero but the modulating frequency $\Omega_c(t)$ remains approximately at its initial constant value. In this stage, the system exhibits spatiotemporal chaos, as indicated by the space-time plot in Fig. 2(a). The second stage, a locking phase, corresponds to the desirable state where an ILM has occurred at the target site. At the beginning of this phase, $\Theta_M(t)$ is excited to a relatively high value. It then decays slowly to zero. The modulating frequency $\Omega_c(t)$, however, increases with time, approaching asymptotically a constant value as the ILM state becomes stable.

From Eq. (2), one can see the value of Θ_M determines the frequency modulation rate. It has been found that multistable dynamical states can coexist in different frequency ranges in MEM cantilever array systems.[8] As one tunes the frequency from low to high value, the system will transit from spatiotemporal chaotic state (denoted SC regime) to a state where all beams oscillate with low energy (denoted as LES regime). When the frequency is increased further, the system enters a multistable regime where stable ILMs and LES coexist. The phase angles of the LES and SC state are

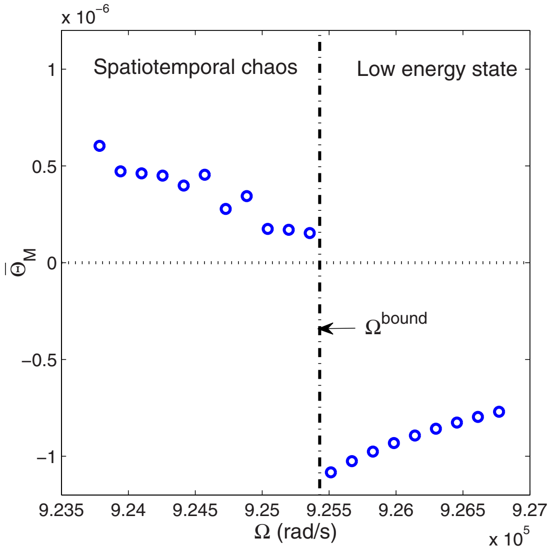

FIG. 3. (Color online) Average value of the phase difference $\bar{\Theta}_M$ vs the driving angular frequency Ω, where Ω^{bound} is the boundary value of Ω between the SC and the LES regimes. We see that $\bar{\Theta}_M > 0$ when the system is in spatiotemporal chaos ($\Omega < \Omega^{\text{bound}}$) and $\bar{\Theta}_M < 0$ when the system is in some low-energy state ($\Omega > \Omega^{\text{bound}}$).

qualitatively different. The one for LES is less than ϕ, the phase angle of the targeted ILM, resulting in $\Theta_M < 0$ [Eq. (3)]. The phase angle for SC state is larger than ϕ, leading to $\Theta_M > 0$. Figure 3 shows the average value of Θ_M for $\Omega \in (9.235, 9.27) \times 10^5$ rad/s, where $\Omega^{\text{bound}} \approx 9.254 \times 10^5$ rad/s is boundary value of Ω between the SC and the LES regimes obtained numerically by a standard averaging method. One can see that the phase-angle difference is different on the different sides of Ω^{bound}. In particular, Θ_M is positive for $\Omega < \Omega^{\text{bound}}$, where the system is in a chaotic state. However, Θ_M becomes negative for $\Omega > \Omega^{\text{bound}}$, where the system is in a low-energy state. From Eq. (2), we then have $\dot{\Omega}_c = \gamma_1 \gamma_2 \bar{\Theta}_M / 2 > 0$ in SC regime and $\dot{\Omega}_c < 0$ in LES regime.

The above results suggest a general strategy for inducing ILMs in a MEM cantilever-array system. When the system is in the SC regime, $\dot{\Omega}_c > 0$ so that the frequency will increase and exceed Ω^{bound}. If the system cannot get into the basin of a high-energy mode (ILM), it will go to the basin of a low-energy state. If this happens, $\dot{\Omega}_c$ will become negative, reducing the driving frequency. As a result, if ILM does not occur, the modulated driving frequency Ω_c will switch back and forth between some values in the chaotic and in the low-energy regimes. During this process the value of $\bar{\Theta}_M$ is changing constantly, leading to a nonzero probability for the system to get into an ILM state through frequency chirping.[6] When this occurs, $\bar{\Theta}_M$ will maintain a positive value but it will decrease slowly as the system evolves into a stable ILM state defined by $\theta_M = \phi$, as shown in Fig. 2(c). In this selecting process, the selecting time is random due to the randomness associated with spatiotemporal chaotic state. Using Monte Carlo simulations, we find that the average selection time is about 50 ms (with the same parameters as in Fig. 1). This time is about one order of magnitude's larger than the one with the pinning control method (2.5 ms).[4] Once a particular cantilever has been locked in an ILM state, the random fluctuations in $\bar{\Theta}_M$ are reduced significantly [Fig. 2(c)], making it practically improbable for cantilevers at other sites to enter some ILM states. This is the mechanism responsible for the locking phase.

In conclusion, a method based on global frequency modulation is articulated for exciting ILMs at a programable spatial location in MEM cantilever-array systems. The idea is to take advantage of the spatial heterogeneity and temporal irregularity offered by the inherent nonlinear dynamics of MEM cantilever array, and to exploit a typical frequency modulation scheme to drive the system into an ILM state occurred at preassigned location.

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