

Lai and Grebogi Reply: The phenomenon of riddled basin was first reported in Ref. [1] and then in Ref. [2]. The latter gave a rigorous mathematical definition for the phenomenon. Basically, the basin of attraction of a *chaotic attractor* is considered riddled if (1) it has a positive measure and if, (2) in every neighborhood of every point in the basin of the attractor, there is a set of points of positive measure that asymptote to another attractor. The concept of riddling was subsequently *generalized* [3] to focusing on the *dynamical mechanism* that leads to a riddled basin. In particular, the bifurcation leading to riddling, the so-called *riddling bifurcation*, is characterized by the loss of the transverse stability of an unstable periodic orbit embedded in a chaotic set in the invariant subspace. An open set of tongues in the phase space that belong to the basin of the other attractor arises suddenly after the bifurcation, signifying riddling of the chaotic set in the invariant subspace. That occurs because the trajectory is ergodic in the invariant subspace and because the dynamics is nonlinear outside the invariant subspace. In Ref. [3], the chaotic sets considered are chaotic attractors. But the same conditions, and hence riddling bifurcation, occur when there is a chaotic saddle, instead of a chaotic attractor, in the invariant subspace. After the riddling bifurcation, (1) there are transversely stable and transversely unstable periodic orbits embedded in a chaotic set in the invariant subspace and (2) there exists an open set of tongues that belongs to the basin of the attractor not in the invariant subspace. In fact, the conditions for riddling and the behavior after the riddling bifurcation occur for *nonattracting chaotic saddles* as well. Our recent paper [4] addressed this type of riddling in unstable chaotic sets. This fact is clearly stated and conveyed in the title of our paper: “Riddling of Chaotic Sets in Periodic Windows”.

Our paper is not on “riddling of periodic attractors in periodic windows,” as mistakenly stated in the Comment of Terry and Ashwin [5]. In fact, it is an elementary well-known notion that the basin of a periodic attractor must contain open sets, and this was stressed in our paper. The basin of a periodic attractor in the presence of a chaotic saddle, a situation that occurs in every periodic window of a chaotic system, can go through a riddling bifurcation. After the bifurcation, an open set of tongues, which belongs to the basin of the attractor off the invariant sub-

space, is created, and the basin of the periodic attractor in the invariant subspace contains both open volumes and a set of measure zero points that appears riddled. The creation of the open set of tongues is in fact the dynamical mechanism for riddling.

The scaling relations derived analytically and tested numerically in our paper occur only *after* the riddling bifurcation because, before the bifurcation, almost every point in the vicinity of the invariant subspace asymptotes to the periodic attractor. The scaling is, therefore, a property associated with riddling of chaotic saddles in the setting that we considered. In fact, the analytic example that we constructed to derive the scaling relation pointed most emphatically to the fact that we were dealing with riddling of chaotic sets, as the model was resolved by using a random walk type of diffusion equation. The dynamics of the periodic attractor is not directly involved with the scaling relations. The Comment states that any map with a repelling fixed point can exhibit a similar scaling, but we believe it is totally irrelevant to the riddling phenomenon studied in our Letter.

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