

Asymmetry in interdependence makes a multilayer system more robust against cascading failuresRun-Ran Liu^{1,*}, Chun-Xiao Jia,¹ and Ying-Cheng Lai^{2,3}¹*Alibaba Research Center for Complexity Sciences, Hangzhou Normal University, Hangzhou, Zhejiang 311121, China*²*School of Electrical, Computer, and Energy Engineering, Arizona State University, Tempe, Arizona 85287, USA*³*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA*

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Multilayer networked systems are ubiquitous in nature and engineering, and the robustness of these systems against failures is of great interest. A main line of theoretical pursuit has been percolation-induced cascading failures, where interdependence between network layers is conveniently and tacitly assumed to be symmetric. In the real world, interdependent interactions are generally asymmetric. To uncover and quantify the impact of asymmetry in interdependence on network robustness, we focus on percolation dynamics in double-layer systems and implement the following failure mechanism: Once a node in a network layer fails, the damage it can cause depends not only on its position in the layer but also on the position of its counterpart neighbor in the other layer. We find that the characteristics of the percolation transition depend on the degree of asymmetry, where the striking phenomenon of a switch in the nature of the phase transition from first to second order arises. We derive a theory to calculate the percolation transition points in both network layers, as well as the transition switching point, with strong numerical support from synthetic and empirical networks. Not only does our work shed light on the factors that determine the robustness of multilayer networks against cascading failures, but it also provides a scenario by which the system can be designed or controlled to reach a desirable level of resilience.

DOI: [10.1103/PhysRevE.100.052306](https://doi.org/10.1103/PhysRevE.100.052306)**I. INTRODUCTION**

It has been increasingly recognized that, because of the ubiquitous presence of interdependence among different types of systems, a reasonable understanding of a variety of complex phenomena in the real world requires a description based on multilayer [1] or interdependent [2,3] networks. Indeed, the functioning of a complex dynamical system, whether it be physical, biological, or engineered, depends not only on its own components but also on other systems that are coupled or interact with it [4–7]. Examples of this sort abound in the real world, e.g., in social [8], technological [9,10], and biological systems [11]. In most existing models of multilayer networks, the mutual interactions between a pair of network layers are treated as symmetric. This assumption is ideal as the interactions between different types of systems often are asymmetric. There then exists a gap between current theoretical modeling or understanding and real-world situations where asymmetric interdependence is common. The purpose of this paper is to narrow this gap by articulating a prototypical model of dynamics in asymmetrically interacting multilayer networks and investigating its robustness with the finding that interaction asymmetry can surprisingly make the whole system significantly more robust.

To be concrete, we focus on a generic type of dynamical processes on multilayer networked systems: cascading failures that attest most relevantly to the robustness and resilience of the system. There is a large body of literature on cascading

failures in single-layer complex networks [12–29], and there have also been efforts in cascading dynamics in multilayer networks [4,30–34]. The unique feature that distinguishes cascading dynamics in multilayer from those in single-layer systems is that, in multilayer systems, failures can propagate from one network layer to another and trigger large-scale failures in an avalanche manner by the intricate strong node-to-node interaction pattern across the network layers. Because of this, multilayer networks can be vulnerable and collapse in an abrupt manner. While protecting the hub nodes can be an effective strategy to mitigate cascading failures in single-layer networks, in interdependent systems this strategy is less effective [2,3]. Nonetheless, there are alternative methods to generate robust interdependent networks even in the strong dependence regime [31–35], where robustness can be enhanced with second-order phase transitions through mechanisms such as intersimilarity [36], geometric correlations [37,38], correlated community structures [39], and link overlaps [40,41]. In addition, it was found that the vulnerability of interdependent networked systems can be reduced through weakening the interlayer interaction [30]. It was also found that the topological properties of the network layers composing a multilayer system, such as degree correlations [42–45], clustering [46,47], degree distribution [48,49], inner dependency [50,51], and spatial embedding [52,53], can affect the robustness of the whole multilayer system. Another issue of great concern is how to destroy the largest mutually connected component of a given multilayer network deliberately [54,55]. It was found that an effective way to destroy the giant component of a single network, i.e., destruction of the 2-core, does not carry over to multilayer networks. The methods of

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effective multiplex degree [56] and optimal percolation [57] were articulated for multilayer networks to find the minimal damage set that destroys the largest mutually connected component.

A tacit assumption employed in most previous models of cascading failures in multilayer networks [4,30–34] is that the layer interdependence will cause a node to fail completely should any of its neighboring nodes in the other layers become nonfunctional. For convenience, we regard such interdependence as “strong.” As a result of the strong dependence, every pair of interdependent nodes must be connected to the giant component in their respective layer at the same time, motivating the introduction of the notion of mutually connected components to characterizing the robustness of the whole multilayer system [4,29,58]. Analyses based on the percolation theory [59–62] revealed that the mutually connected component generically undergoes a discontinuous phase transition as a function of the initial random damage [3,4]. This result was somewhat surprising because it is characteristically different from the continuous percolation transition typically observed in single-layer networks [63–66]. Moreover, the percolation theory provides a reasonable understanding of the catastrophic cascading dynamics occurred in real interdependent infrastructure systems, such as the interdependent system of power grids and telecommunication networks [67].

A deficiency of the assumption of “strong” interdependence is that nodes across different layers in real-world systems typically exhibit weaker types of interdependence. For example, in a transportation system, passengers can travel from city to city through a number of interdependent transportation modes such as coaches, trains, airplanes, and ferries. When one mode becomes unavailable, e.g., when the local airport is shut down, passenger flow into the city may be decreased: Some passengers destined for this city may cancel their travel and the transferring passengers would switch to other cities to reach their final destinations. Thus, although the disabling of the air transportation route can have impacts on the function of the whole interdependent networked system, transportation via other modes is still available, i.e., total destruction will not occur and the system can still maintain a certain level of functioning. For this particular example, the interactions between the air travel network and other transportation network layers are apparently asymmetric. Generally, many real-world infrastructure systems such as electric power, water, or communication networks use backup infrastructures and often have emergency management plans to survive losses of interdependent services. In such a case, the failure of a node in one layer can disable a number of links in other coupled layers, but not necessarily cause the loss of *all* neighboring nodes and links. These considerations motivated recent works on the consequences of “weak” interdependence in multilayer networked systems [7,68]. Another factor of consideration is that the impacts of a failed node on its interdependent partners may depend not only on its position, but also on the positions of the partners. That is, the strength of interdependence of two nodes in different network layers is asymmetric, as in real infrastructure systems. Intuitively, the origin of asymmetry in interdependence can be argued, as follows. In a multilayer networked system, the “important” nodes tend to be highly

connected while the “unimportant” ones are less connected. Probabilistically, the failure of an “unimportant” node thus would not have a great impact on the “important” nodes, but the failure of an “important” node is more likely to have significant effects on the “unimportant” nodes. To investigate the consequence of asymmetrically interdependent interactions may thus lead to a better understanding of robustness and resilience of multilayer networks in the real world.

In this paper, we articulate a class of percolation dynamical models for multilayer networks incorporating asymmetric interdependence and investigate the effects of the asymmetry on the robustness of the whole system. In our model, the strength of the interdependence of nodes with different degrees on its partners in different layers is not identical to the strength in the opposite direction. We introduce a generic parameter θ to characterize the degree or extent of asymmetry of two interdependent network layers. Intuitively, it may occur that asymmetry can make the system more vulnerable to catastrophic failures. For example, disabling some nodes that exert more influence on its partner nodes than the other way around is more likely to lead to failures of these nodes, making cascading failures more probable. However, counterintuitively, we find that increasing the degree of asymmetry can dramatically improve the robustness of the whole multilayer system. In particular, when the highly connected nodes in one layer depend less on the nodes of lower degrees in the other layer than the dependence in the opposite direction, the robustness of the system can be improved significantly as compared with the counterpart system with perfectly symmetric interdependence [4]. Quantitatively, as the degree of asymmetry is systematically increased, the system undergoes a remarkable switch from a first-order percolation transition to a second-order one. We develop an analytic theory to predict the characteristic changes in the nature of the phase transition as induced by asymmetry and the transition points, with strong numerical support based on percolation dynamics in both synthetic and empirical networks. Our results suggest that, in order to enhance network robustness and resilience (as in designing a multilayer infrastructure system), introducing an appropriate level asymmetric interaction among the interdependent layers can be advantageous.

II. MODEL

We consider a percolation process on an interdependent system with two network layers, denoted as A and B , each having the same number N of nodes. The functioning of node a_i ($i = 1, \dots, N$) in network A depends on the functioning of the counterpart node b_i in network B and vice versa. When a_i fails, each link of its dependent partner b_i will be maintained or disabled with probability α_i^b or $1 - \alpha_i^b$, respectively. Similarly, if b_i in B fails, then the links of its dependent partner a_i in network A will be intact or disabled with probability α_i^a or $1 - \alpha_i^a$, respectively. The probabilities α_i^a and α_i^b thus characterize the interdependence strength of node a_i on b_i and vice versa, respectively. When the values of α_i^a or α_i^b approach 1, the interdependence between the two nodes is the weakest, where failures are unable to spread from one network layer to another. The opposite limit where the values of α_i^a and/or α_i^b approach zero corresponds to the case of the strongest

possible interdependence. In general, the values of α_i^a and α_i^b are different and degree dependent. One way to define these parameters is

$$\alpha_i^a = \frac{(k_i^a)^\theta}{(k_i^a)^\theta + (k_i^b)^\theta} \quad \text{and} \quad \alpha_i^b = \frac{(k_i^b)^\theta}{(k_i^a)^\theta + (k_i^b)^\theta}, \quad (1)$$

where k_i^a and k_i^b are the degrees of nodes a_i and b_i , respectively, and θ is a parameter that controls the asymmetry of the interdependent interactions. In particular, for $\theta = 0$, the interdependence between nodes a_i and b_i is symmetric: $\alpha_i^a = \alpha_i^b$, regardless of the nodal degrees. For $\theta > 0$, the interdependence is weak of a high degree node in one network layer on a low degree node in the other network layer and the interdependence of a low degree node in one layer on a high degree node in the other layer is strong. The opposite situation occurs for $\theta < 0$, where the interdependence of a high (low) degree node in one layer on a low (high) degree node in the counterlayer is strong (weak). As described in Introduction, in real multilayer networks, the failure of a less connected node would not have a great impact on the well connected nodes, but the failure of a well connected node is more likely to have significant effects on the less connected nodes. This means the case of negative θ values may seldom appear and positive θ values are more general in realistic scenarios. Tuning the value of θ enables a systematic analysis of the effects of asymmetry in interdependence on the robustness of the whole multilayer system.

We start the percolation process by randomly removing a fraction $(1 - p)$ of the nodes of networks A and B independently. In each network layer, the links connected to the removed nodes are simultaneously removed. This is the case where an initial attack occurs in the two network layers simultaneously. The removal of nodes in one network will cause some nodes to be isolated from the giant component and to fail, and the failure can spread across the whole system through an iterative process. In each iteration, disconnecting certain nodes from the giant component of, e.g., network A , will cause some nodes to be isolated from the giant component of network B through the destruction of some of their links, which in turn will induce more link destruction and nodal failures in A . When the process of failure stops, the whole system reaches a stable steady state. The sizes S^A and S^B of the giant components in the final state of the network layers A or B can be used to measure the robustness of the whole system [4,7].

III. THEORY

We develop a theory to understand the asymmetry-induced switch between first- and second-order phase transitions and to predict the transition points. Let p_k^A and p_k^B be the degree distributions of network layers A and B , respectively, where the average degrees are given by $\langle k \rangle^A = \sum_k p_k^A k$ and $\langle k \rangle^B = \sum_k p_k^B k$. The final sizes S^A and S^B of the respective giant components in layers A and B in the steady state can be solved by using a self-consistent probabilistic approach. In particular, define R^A (R^B) to be the probability that a randomly chosen link in network A (B) belongs to its giant component. Suppose we randomly choose a node a_i of degree k_i^a in network A . The

probability of functioning of this node depends the state of its interdependent neighbor b_i in network B . If b_i is disabled, then each of its links can be maintained with the probability α_i^a , so the probability that this link leads to the giant component in A is $\alpha_i^a R^A$. The viable probability of node a_i in A is thus given by $p[1 - (1 - \alpha_i^a R^A)^{k_i^a}]$ if its interdependent neighbor b_i is not viable. If b_i is functional, then the viable probability of node a_i is $p[1 - (1 - R^A)^{k_i^a}]$. With the quantity R^B , we can get the viable probability of node b_i as $p[1 - (1 - R^B)^{k_i^b}]$. Taking into account the probability distributions of k_i^a and k_i^b , the viable probability of a random node in A is

$$\begin{aligned} S^A = & p^2 \left[1 - \sum_{k_i^a} p_{k_i^a}^A (1 - R^A)^{k_i^a} \right] \left[1 - \sum_{k_i^b} p_{k_i^b}^B (1 - R^B)^{k_i^b} \right] \\ & + p \sum_{k_i^a} \sum_{k_i^b} p_{k_i^a}^A p_{k_i^b}^B [1 - (1 - \alpha_i^a R^A)^{k_i^a}] \\ & \times \{1 - p[1 - (1 - R^B)^{k_i^b}]\}, \end{aligned} \quad (2)$$

where the first and second terms denote the cases where b_i is viable and not viable, respectively. Similarly, the viable probability of a node b_i in network B is

$$\begin{aligned} S^B = & p^2 \left[1 - \sum_{k_i^b} p_{k_i^b}^B (1 - R^B)^{k_i^b} \right] \left[1 - \sum_{k_i^a} p_{k_i^a}^A (1 - R^A)^{k_i^a} \right] \\ & + p \sum_{k_i^a} \sum_{k_i^b} p_{k_i^a}^A p_{k_i^b}^B [1 - (1 - \alpha_i^b R^B)^{k_i^b}] \\ & \times \{1 - p[1 - (1 - R^A)^{k_i^a}]\}. \end{aligned} \quad (3)$$

Following a randomly chosen link in network A , we can arrive at a node a_j with degree k_j^a . If its interdependent neighbor b_j of degree k_j^b in network B is viable, then the random link can lead to the giant component with the probability $p[1 - (1 - R^A)^{k_j^a - 1}]$. If b_j is not viable, then each link of node a_j is reserved with the probability α_j^a , and the random link can lead to the giant component with the probability $p\alpha_j^a[1 - (1 - \alpha_j^a R^A)^{k_j^a - 1}]$. These considerations lead to the following self-consistent equation for R^A :

$$\begin{aligned} R^A = & p^2 \left[1 - \sum_{k_j^a} \frac{p_{k_j^a}^A k_j^a}{\langle k \rangle^A} (1 - R^A)^{k_j^a - 1} \right] \\ & \times \left[1 - \sum_{k_j^b} p_{k_j^b}^B (1 - R^B)^{k_j^b} \right] \\ & + p \sum_{k_j^a} \sum_{k_j^b} \frac{p_{k_j^a}^A k_j^a}{\langle k \rangle^A} p_{k_j^b}^B \alpha_j^a [1 - (1 - \alpha_j^a R^A)^{k_j^a - 1}] \\ & \times \{1 - p[1 - (1 - R^B)^{k_j^b}]\}. \end{aligned} \quad (4)$$

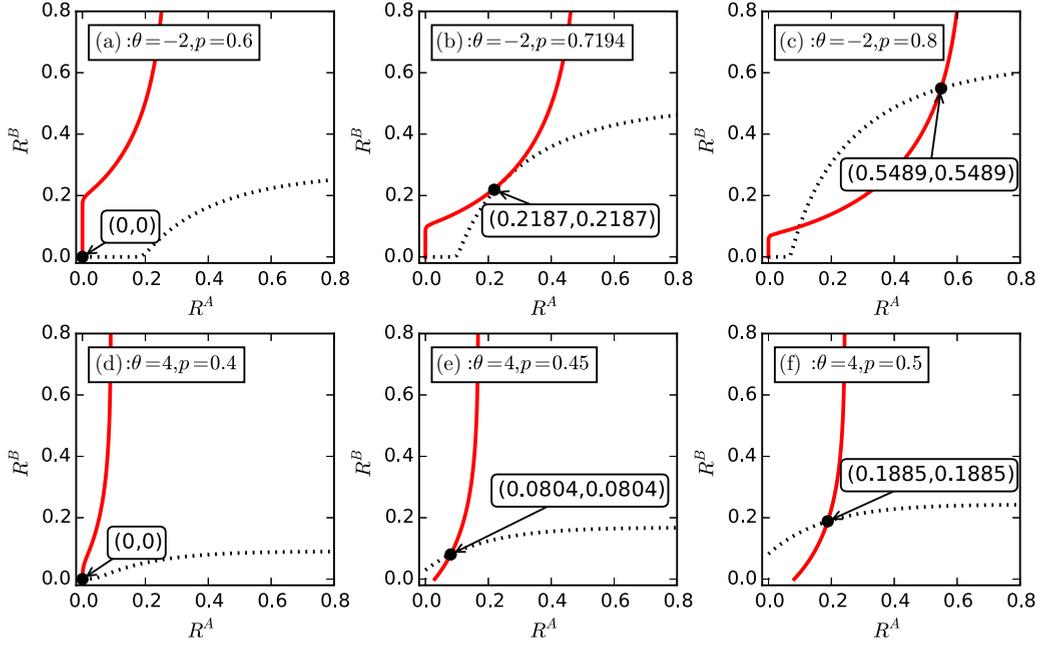


FIG. 1. Solutions of the self-consistent equations for the probabilities that a random node belongs to the giant component in a double-layer system. Shown are the graphical solutions of Eqs. (4) and (5) for different values of θ and p , as marked by the black dots. [(a)–(c)] Results for $p = 0.6$, $p = 0.7194$, and $p = 0.8$, respectively, for $\theta = -2$, and [(d)–(f)] the solutions for $p = 0.4$, $p = 0.45$, and $p = 0.5$, respectively, for $\theta = 4$. The average degree is $\langle k \rangle = 4$.

A self-consistent equation for R^B can be obtained in a similar way. We get

$$\begin{aligned}
 R^B &= p^2 \left[1 - \sum_{k_j^b} \frac{p_{k_j^b}^B k_j^b}{\langle k \rangle^B} (1 - R^B)^{k_j^b - 1} \right] \\
 &\times \left[1 - \sum_{k_j^a} p_{k_j^a}^A (1 - R^A)^{k_j^a} \right] \\
 &+ p \sum_{k_j^a} \sum_{k_j^b} p_{k_j^a}^A \frac{p_{k_j^b}^B k_j^b}{\langle k \rangle^B} \alpha_j^b [1 - (1 - \alpha_j^b R^B)^{k_j^b - 1}] \\
 &\times \{1 - p[1 - (1 - R^A)^{k_j^a}]\}. \quad (5)
 \end{aligned}$$

Figure 1 shows, for random networks A and B with a Poisson degree distribution [69,70] $p_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$, graphical solutions of R^A and R^B for different values of θ and p . For simplicity, we consider the case where A and B have the identical degree distribution: $p_k^A = p_k^B \equiv p_k$. For $\theta = -2$, there is a trivial solution at the point $(R^A = 0, R^B = 0)$ for $p = 0.6$, indicating that both networks A and B are completely fragmented. For $p = 0.7194$, the solutions are given by the tangent point $(0.2187, 0.2187)$, giving rise to a discontinuous change in both R^A and R^B that is characteristic of a first-order percolation transition. For $\theta = 4$, the crossing point for R^A and R^B changes continuously from $(0,0)$ to some nontrivial values, indicating a continuous (second order) percolation transition.

The critical point for both first- and second-order types of percolation transition can be obtained, as follows. For $p_k^A = p_k^B \equiv p_k$, we have $\langle k \rangle^A = \langle k \rangle^B \equiv \langle k \rangle$, $R^A = R^B \equiv R$. Equation

(4) or (5) can then be reduced to

$$\begin{aligned}
 R &= p^2 \left[1 - \sum_{k_j^a} \frac{p_{k_j^a}^A k_j^a}{\langle k \rangle} (1 - R)^{k_j^a - 1} \right] \left[1 - \sum_{k_j^b} p_{k_j^b} (1 - R)^{k_j^b} \right] \\
 &+ p \sum_{k_j^a} \sum_{k_j^b} \frac{p_{k_j^a}^A k_j^a}{\langle k \rangle} p_{k_j^b} \alpha_j^a [1 - (1 - \alpha_j^a R)^{k_j^a - 1}] \\
 &\times \{1 - p[1 - (1 - R)^{k_j^b}]\} \equiv h(R). \quad (6)
 \end{aligned}$$

For the first-order transition, the straight line $y = x$ and the curve $R = h(R)$ from Eq. (6) become tangent to each other at the point (R_c, R_c) , at which the derivatives of both sides of Eq. (6) with respect to R are equal:

$$\left. \frac{dh(R)}{dR} \right|_{R=R_c, p=p_c^I} = 1. \quad (7)$$

Equations (6) and (7) can be solved numerically to yield the first-order percolation transition point p_c^I .

In the regime of second-order percolation transition, the probability R tends to zero as p approaches the percolation point p_c^II . We can use the Taylor expansion of Eq. (6) for $R \equiv \epsilon \ll 1$:

$$h(\epsilon) = h'(0)\epsilon + \frac{1}{2}h''(0)\epsilon^2 + O(\epsilon^3) = \epsilon. \quad (8)$$

Since $\epsilon \in (0, 1)$, we obtain $h'(0) + \frac{1}{2}h''(0)\epsilon + O(\epsilon^2) = 1$ by dividing both sides of Eq. (8) by ϵ . Neglecting high-order terms of ϵ , we have that the second-order percolation point p_c^II is determined by the solutions of

$$h'(0) = p_c^II \sum_{k_j^a} \sum_{k_j^b} \frac{p_{k_j^a}^A k_j^a}{\langle k \rangle} p_{k_j^b} (\alpha_j^a)^2 (k_j^a - 1) = 1. \quad (9)$$

If, for any node, we have $\alpha_j^a \rightarrow 1$, then there will be no interdependence across the network and Eq. (9) can be reduced to the case of a single-layer network. In this case, the percolation transition point becomes $p_c^{\text{II}} = \langle k \rangle / \langle k(k-1) \rangle$, which is the same result for single-layer generalized random networks and can be validated to be consistent with the previous ones [64,65]. Since the interdependence strength of a pair of nodes across two network layers is determined by their degrees, the situation $\alpha_j^a \rightarrow 0.5$ arises if all pairs of interdependent nodes have exactly the same degree, which provides a special symmetrical case for any given value of θ .

When the conditions for the first- and second-order transitions are satisfied simultaneously, i.e., $p_c^{\text{I}} = p_c^{\text{II}}$, the percolation transition switches from first to second order (or vice versa). Substituting p_c^{II} from Eq. (9) into Eq. (8), we have

$$\frac{1}{2}h''(0)\epsilon^2 + O(\epsilon^3) = 0. \quad (10)$$

For the first-order percolation transition, ϵ_c is always nontrivial and Eq. (10) is not applicable any more. Apparently, if the system undergoes a second-order percolation transition, then the value of ϵ is at the transition point $\epsilon_c = 0$ and Eq. (10) is naturally satisfied. On the boundary between the first- and second-order percolation transitions, the value of ϵ_c is negligibly small. We have

$$\begin{aligned} h''(0) = p_c^{\text{II}} & \left[2p_c^{\text{II}} \sum_{k_j^a} p_{k_j^a} k_j^a (k_j^a - 1) \right. \\ & + p_c^{\text{II}} \sum_{k_j^a} \sum_{k_j^b} \frac{p_{k_j^a} k_j^a}{\langle k \rangle} p_{k_j^b} (\alpha_j^a)^2 \\ & - \sum_{k_j^a} \sum_{k_j^b} \frac{p_{k_j^a} k_j^a}{\langle k \rangle} p_{k_j^b} (k_j^a - 1)(k_j^a - 2)(\alpha_j^a)^3 \\ & \left. - p_c^{\text{II}} \sum_{k_j^a} \sum_{k_j^b} \frac{p_{k_j^a} k_j^a}{\langle k \rangle} p_{k_j^b} (k_j^a - 1)(\alpha_j^a)^2 \right] = 0. \quad (11) \end{aligned}$$

For a given degree distribution p_k , we can obtain the crossover point θ_c of the first- and second-order transition points by solving Eq. (11) numerically.

IV. RESULTS

Figures 2(a) and 2(b) show the sizes of the giant components in network layers A and B, denoted by S^A and S^B , versus the fraction p of initially preserved nodes for interdependent random and scale-free networks, respectively. For a negative value of the asymmetry parameter θ (e.g., $\theta = -2$), S^A and S^B percolate discontinuously at a threshold p_c^{I} . For a positive value of θ (e.g., $\theta = 4$), the networks A and B percolate continuously with a reduced value of the transition point p_c^{II} , leading to a crossover in the percolation transition and a higher degree of system robustness. Further increase in the value of θ leads to little change in the transition point p_c^{II} , indicating that the ability to enhance the system robustness by increasing the value of θ is saturated, for both interdependent random and

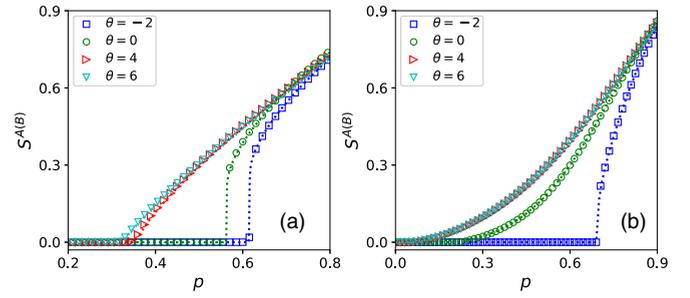


FIG. 2. Simulation results for first- and second-order percolation transitions on interdependent random (a) and scale-free (b) networks. Shown are the fractions S^A and S^B of nodes in the respective giant component at the end of a cascading process as a function of p for $\theta = -2, 0, 4, 6$. The results are obtained by averaging over 40 independent realizations, where the network size is $N = 5 \times 10^5$ with the average degree $\langle k \rangle = 5$ for both random and scale-free networks. For scale-free interdependent networks, the minimum degree is 2 and the power-law exponent of degree distribution is -2.3 . The dotted curves underlying the symbols represent the theoretical predictions obtained from Eq. (6), all agreeing well with the numerical results.

scale-free networks. Theoretical predictions are also included in Fig. 2, which agree with the numerical results quite well.

Figures 3(a) and 3(b) show the percolation transition points p_c^{I} (p_c^{II}) versus θ for a random and a scale-free interdependent networked system, respectively. In Fig. 3(a), for each average-degree value tested, the phase diagram is divided into two distinct regions by a critical point: For $\theta < \theta_c$, the transition is discontinuous (first order) while it is continuous (second order) for $\theta > \theta_c$ with relatively smaller values of the percolation threshold p_c^{II} . Similar behaviors occur for scale-free interdependent networks, as shown in Fig. 3(b). For both types of interdependent networks, as θ is increased, the percolation transition point p_c^{I} (p_c^{II}) moves toward lower values, indicating that more nodes can be removed before a phase transition occurs and, consequently, the whole system becomes more robust. Nonetheless, as θ is further increased, the transition point p_c^{I} (p_c^{II}) becomes saturated. A distinct feature in the changes of p_c^{I} and p_c^{II} versus θ is that, near the crossover point θ_c , the transition point for the scale-free networked system is more sensitive to asymmetry in the interdependent interactions than the random networked system. This result suggests that, near θ_c , the robustness of the scale-free interdependent network can be compromised by the asymmetry. Figures 3(c) and 3(d) show the the critical size $S_c^{A(B)}$ of giant component at the percolation transition point as functions of the asymmetrical parameter θ . Above the switch point θ_c , $S_c^{A(B)}$ is finite characterizing a discontinuous phase transition, whereas below the switch point θ_c , $S_c^{A(B)}$ is zero and the system percolates as a continuous phase transition.

For lower values of the asymmetrical parameter θ , large-degree nodes in one network depend on the small-degree nodes in the other network with a large coupling strength. In this case, the failure of a low-degree node in one network can destroy a high-degree node in the other network. The small-degree nodes are sensitive to nodal or link removal and have a high risk to fail in a cascading process. Although the large-degree nodes are “stubborn,” they are more destructive than

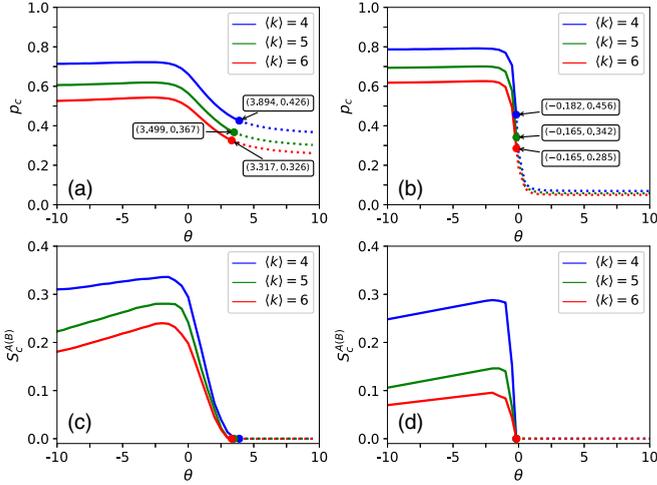


FIG. 3. Dependence of the percolation transition point and the critical size of giant component at the percolation transition point on the asymmetry parameter θ . [(a) and (b)] The transition point p_c versus θ for a system of interdependent random and scale-free networks, respectively. [(c) and (d)] The critical size $S_c^{A(B)}$ of the giant component at the percolation transition point versus θ for a system of interdependent random and scale-free networks, respectively. For both random and scale-free networks, the average degree $\langle k \rangle$ is 4, 5, and 6 from high to low (For scale-free networks, the minimum degree is 2 and the corresponding power-law exponent of degree distribution is -2.6 , -2.3 , or -2.1). For each $\langle k \rangle$ value, there exists a critical point θ_c , marked by the large solid dots, that divides the θ interval into two subregions with distinct types of phase transitions: first-order (solid curves) and second-order (dotted curves), respectively. At first-order phase transition points, the critical size $S_c^{A(B)}$ of the giant component is nonzero and the transition is abrupt. At the second-order phase transition points, the critical size $S_c^{A(B)}$ of the giant component is zero and the transition is continuous. Increasing the value of the asymmetry parameter θ from a negative to some positive value has two advantages: (i) a decreased value of the critical transition point p_c regardless of the nature of the transition (i.e., first or second order), indicating that more nodes can be removed before the occurrence of a phase transition, and (ii) a switch in the transition from first to second order, where the former is often catastrophic while the latter can be benign.

the low-degree nodes in case of failures. That is, a low value of θ can reinforce the dependence of the destructive nodes on high-risk nodes, amplifying the systematic risk for the whole interdependent system. As the value of θ is increased to become positive, the dependence strength of the high-degree nodes on the low-degree nodes is reduced, making the system relatively more robust.

Since the interdependence of a pair of nodes is controlled by the degree difference of them in terms of the asymmetrical parameter θ , we introduce a parameter ω to control the fraction of overlapping links and the degree difference of interdependent nodes in a double-layer network. An overlapping link is defined in terms of a pair of links that connect two pairs of interdependent nodes in different network layers, respectively. In particular, say there exist a link that connects two nodes a_i and a_j in layer A . The link connecting nodes b_i and b_j in layer B is an overlapping link. For $\omega \rightarrow 0$, there

is no overlapping link and the system reduces to the one studied above. For $\omega \rightarrow 1$, all the links are overlapping links and the degrees of nodes in network A are the same as the degrees of their respective interdependent nodes. In this case, the interdependence is symmetric and $\alpha = 0.5$ for any given value of θ .

Figure 4 shows the simulation results for percolation transitions in random networks with overlapping links. We find that, for a fixed value of ω , the value of the percolation transition point p_c decreases with the increase in the asymmetrical parameter θ . This means that the system is robust when large-degree nodes in one network layer depend strongly on large-degree nodes in the other layer, but the system becomes vulnerable when there is strong interdependence between large-degree nodes in one layer and small-degree nodes in the other layer. We also find that the curves of the percolation transition point p_c versus θ for different values of ω intersect at the point $\theta_c \approx 2$, as shown in Fig. 4(d). For $\theta < \theta_c$, the percolation point p_c decreases with the increase in the value of ω , indicating that an increase in the fraction of overlapping links makes the system more robust. However, for $\theta > \theta_c$, the percolation point p_c increases and the system becomes less robust as the value of ω is increased. As the networks become fully overlapped ($\omega \rightarrow 1$), the differences in the degrees of the interdependent nodes across the network layers decrease and the value of the interdependence strength approaches 0.5 irrespective of the value of θ . In this case, increasing θ will not lead to any appreciable change in the asymmetry of the interdependence strength among the network layers. This is the reason why the overlap does not contribute to enhancing the system robustness in the region of large θ values, as exemplified in Fig. 4. These results suggest that the role of overlapping links in the robustness of a system depends on the value of the asymmetrical parameter θ .

What about percolation transitions in multilayer systems with more than two layers? To address this question, we study three-layer systems with asymmetrical interdependencies. To be concrete, we consider the following configuration of interdependence among the three layers (A , B , and C): layer A depends on layer B , layer B depends on layer C , but layers A and C have no direct dependence on each other. Depending on the extent of asymmetrical interdependencies, multiple percolation transitions can occur. Figure 5 shows that the transition point p_c decreases and the system becomes more robust as the value of the asymmetrical parameter θ is increased. We also find that multiple percolation transitions occur for relatively large values of θ , and layers A and C percolate first, followed by layer B , after which another phase transition occurs in layers A and C . However, for small values of θ , the phenomenon of multiple percolation transitions disappears because the three layers tend to percolate at the same point. These results are consistent with those in Ref. [7], demonstrating that both asymmetry in the interdependence and layer position can be important for the functioning of the multilayer interdependent systems.

A practical implication is that the asymmetrical parameter can be exploited for modulating or controlling the characteristics of the percolation transition [7]. In particular, for relatively low degree of asymmetry (e.g., $\theta = -2$), the percolation transitions are abrupt and discontinuous. In this case,

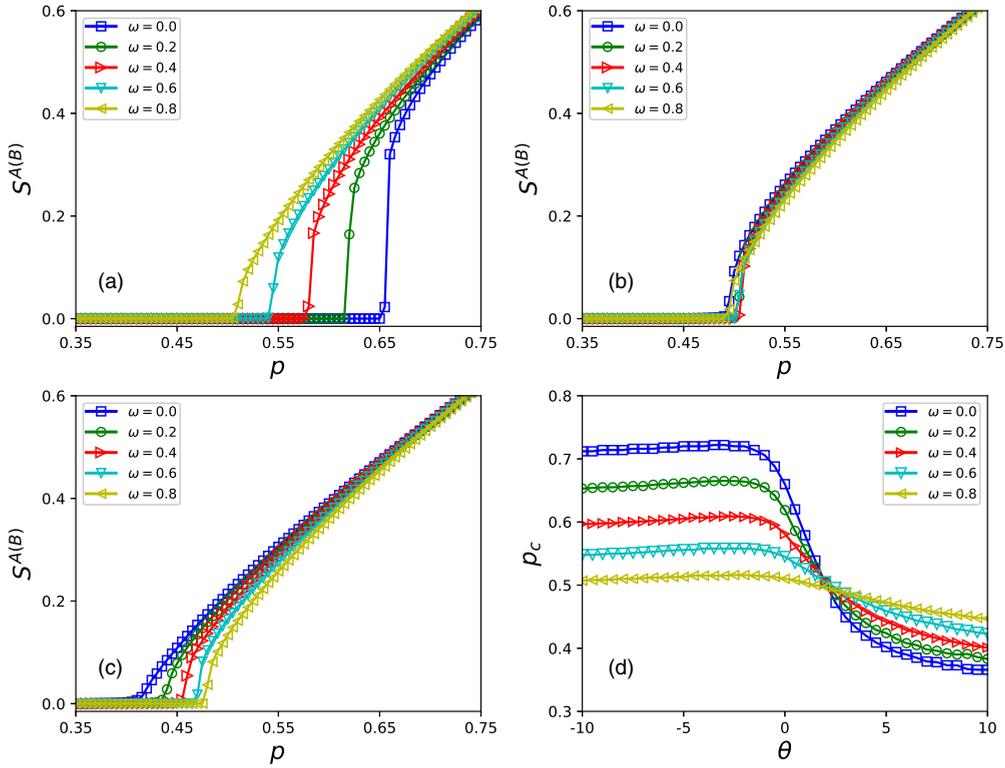


FIG. 4. Simulation results of percolation transitions in random networks with overlapping links. [(a)–(c)] The fractions S^A and S^B of nodes in the respective giant component at the end of a cascading process as a function of p for different fractions of overlapping links for $\theta = 0$, $\theta = 2$, and $\theta = 4$, respectively. (d) The percolation transition point p_c as a function of θ for different values of ω . The results are obtained by averaging over 40 statistical realizations. The network size is $N = 5 \times 10^5$ and the average degree is $\langle k \rangle = 4$.

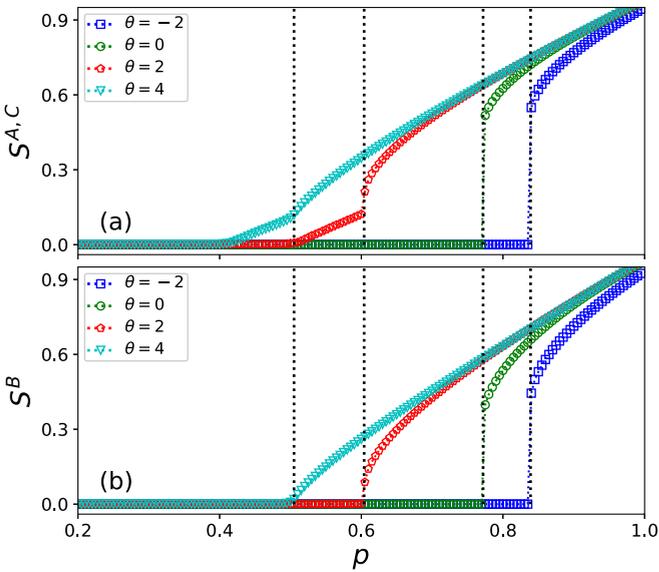


FIG. 5. Percolation transitions in three-layer random networks. The fractions $S^{A(C)}$ (a) and S^B (b) of nodes, respectively, in the corresponding giant component at the end of a cascading process as a function of p for different values of the asymmetrical parameter θ . The straight vertical dotted lines denote the positions of percolation transition of B . The results are obtained by averaging over 40 statistical realizations. The network size is $N = 5 \times 10^5$ and the average degree is $\langle k \rangle = 4$.

the interdependent system is not resilient and is likely to collapse suddenly as random nodal failures or intentional attacks intensify. To improve the resilience of the system, a larger value of θ can be chosen (e.g., $\theta = 4$) to make the system collapse, if at all inevitable, to occur in a continuous fashion. While the whole system still collapses eventually, the manner by which the collapse occurs is benign and the value of the critical point p_c^{II} is smaller as compared with that associated with first-order phase transitions.

We demonstrate the role of asymmetric interdependence in enhancing the robustness of multilayer networked systems in a controllable manner by studying a real-world networked system with asymmetrical interdependence: an autonomous systems of the Internet and the power grid of the western states of the U.S. [71]. The autonomous systems of the Internet consist of 6474 nodes [72] and the power grid has 4941 nodes [73] with each being a generator, a transformer, or a substation. We randomly choose a number of nodes from the power grid as the dependent partners of the nodes in the autonomous level Internet, and define an interdependence link between a power grid node and an Internet node until all the selected power grid nodes and the Internet nodes are connected. The dependency strengths of the power grid and the Internet nodes are assigned according to Eq. (1). That is, if a node in the power grid fails, then the Internet node that depends on it will suffer a loss of some links because of the interdependence and reserve some links because of the existing buffering effect and vice versa. Figure 6 shows the sizes of the giant components of the Internet and the power

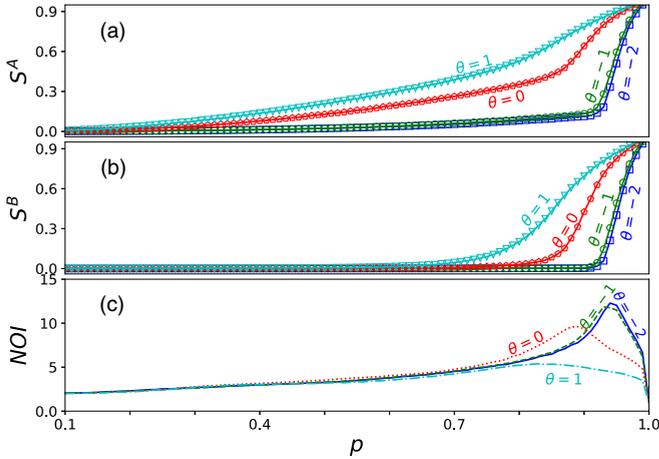


FIG. 6. Role of asymmetric interdependence in enhancing the robustness of an Internet-power system in a controllable manner. [(a) and (b)] The sizes of the giant components of the autonomous system of the Internet and the power grid versus p for different values of θ , respectively. (c) The number of iterations (NOI) as a functions of p for different values of θ for the Internet and power grid networks. The data points are the result of averaging over 1000 statistical realizations.

grid versus p for different values of θ . For negative values of θ (e.g., $\theta = -2$), the sizes of the giant components reduce drastically as p is decreased from 1, as indicated by a relative large number of iterations in the cascading process. While for a relatively large positive value of θ , the changes in the sizes of the giant components versus p are smooth, signified by fewer iterations in the cascading process.

Another example is the rail and coach transportation system in Great Britain [74], which includes rail, coach, ferry, and air transportation layers. Here we use the data in October 2011 to set up the multilayer network and conduct our numerical experiments. An analysis of the data shows that the rail and coach occupy 98.1% of all the interurban connections, and ferry and air transportation account for the remaining 1.9%. We thus focus on the former to construct a two-layer network. Due to the interdependence of passenger flows between different traffic layers, the dependence strengths of the coach station and the rail station are assigned according to Eq. (1). Figure 7 shows the sizes of the giant components of the rail and coach layers versus p for different values of θ . For negative values of θ (e.g., $\theta = -2$), the sizes of the giant components are always lower than that of the case when θ is positive (e.g., $\theta = 0$ or $\theta = 2$).

These results demonstrate that the proposed principle of asymmetrical interdependence can be effective for enhancing the robustness of the functioning of the interdependent system. Especially, an appropriate amount of asymmetry in the interdependence can be quite beneficial to preventing a first-order transition from happening which leads to sudden, system wide failures.

V. DISCUSSION

Interdependent multilayer networked systems in the real world are generally asymmetric in layer-to-layer interactions, and the asymmetry will inevitably have an impact on the robustness of the whole system. In most existing works on dynamical processes in multilayer networks, the mutual interactions between a pair of network layers are treated as symmetric, giving rise to a knowledge gap in our general understanding of the dynamics on multilayer networked systems and their robustness against failures and/or attacks. The present work aims to narrow the gap by investigating a

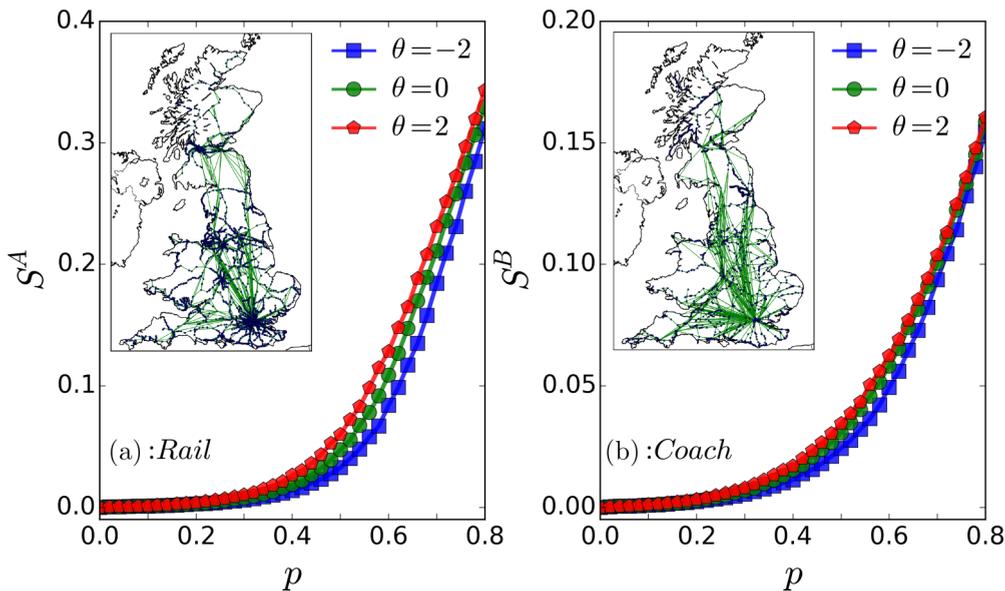


FIG. 7. Role of asymmetric interdependence in enhancing the robustness of a transportation system in a controllable manner in rail and coach transportation system in Great Britain. Shown are the sizes of the giant components of the rail layer (a) and the coach layer (b) versus p for different values of θ , respectively. The data points are the result of averaging over 1000 statistical realizations. In each panel, the upper left inset shows the actual route map for the corresponding layer.

generic type of dynamics on multilayer networks: cascading failures.

For simplicity and to facilitate analysis but without sacrificing generality, we have studied double-layer networked systems and focus on the percolation dynamics, where a cascading process can be triggered by random removal of nodes and links, which can cause a dramatic reduction in the sizes of the giant components in both network layers and possibly lead to total fragmentation of the system. There are two characteristically distinct failure scenarios: as the fraction of removed nodes is increased, the sizes of the giant components will inevitably reduce to near zero values, either in a discontinuous or in a continuous manner, corresponding to a first- or a second-order phase transition, respectively. From the standpoint of network robustness and resilience, a first-order transition is undesired as the system can become fragmented abruptly. Even if a total system breakdown is inevitable, it is desired that the process occurs gradually and continuously, which is characteristic of second-order phase transitions.

Our main finding is that asymmetric interdependence can shift the critical point of the percolation-induced phase transition in a desirable way and, strikingly, can affect the nature of the transition. In particular, as the degree of asymmetry is systematically tuned, the system can undergo a switch from a first-order percolation transition to a second-order one. Qualitatively, this can be understood, as follows. When the nodes with large degrees in one network layer depend highly on the nodes of small degrees in the other network, the failure of a low-degree node in the latter can destroy a high degree node in the former. In this case, the whole system can be

quite fragile due to the relative abundance of the small degree nodes, where a first-order phase transition is expected. Quite the contrary, when nodes of large degrees in one network layer depend on the large degree nodes in the other layer, a second-order percolation transition arises and the system is robust. We have developed a theory to predict the phase transition points and provide strong numerical support with synthetic network models. To demonstrate the relevance of our work and finding to the real world, we have also studied the double-layer system of Internet and power grid with randomly assigned one-to-one interdependence and the rail-coach transportation system.

From the point of view of design and control, our finding implies that the degree of asymmetry (or symmetry) of interdependence can be exploited to enhance the robustness of multilayer networks against cascading failures. This can be especially meaningful in engineering design of complex infrastructure systems that are intrinsically multilayer structured, or in biological systems where the interdependent interaction strength may be tuned biochemically.

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