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Observation-based control of rare intense events in the complex Ginzburg-Landau equation

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We demonstrate that rare intense events in the complex Ginzburg-Landau equation can be suppressed by using only observation-based control. Our control strategy eliminates the requirements of a precise system model and prediction. Analytic insights for guiding the control and numerical verification are obtained. The issue of time delay is also addressed. We expect our results to provide insight into the challenging problem of harnessing spatially extended complex systems.

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Spatially extended systems exhibit extremely rich dynamical phenomena such as nonlinear waves, synchronization, intermittency, and spatiotemporal chaos or turbulence. These have been studied extensively, due to their relevance to many branches of science and engineering. One important phenomenon that has received relatively less attention is extreme events in spatiotemporal systems that occur rarely but with intensities far greater than those associated with typical events in the system. Intuitively, rare intense events are due to nonlinear interactions among distinct spatial components in the system, leading to an extreme form of constructive interference. Consider, for example, a wave system. The wave amplitude at a particular spatial location at a particular time is the result of many wave packets generated at different spatial locations from previous times. If the phases of most wave packets happen to be coherent, a large amplitude event at that location and at that time can arise. It is, however, difficult for phase coherence over a large spatial scale to emerge, the occurrence of extreme-amplitude events thus cannot be expected to occur commonly. Nonetheless, once such an event actually occurs, the consequences can be severe in the sense that significant damage can incur to the system and its surroundings. Despite previous efforts to probe statistical behaviors of and scaling laws associated with rare intense events [1,2], their dynamical origins remain elusive.

Among the numerous spatially extended dynamical systems, the complex Ginzburg-Landau equation (CGLE) is one of the most studied. It models the dynamical behaviors of a broad class of systems including nonlinear waves, superconductivity, superfluidity, Bose-Einstein condensation, and liquid crystals [3–7]. The CGLE has also been used as a paradigmatic model for investigating rare intense events in spatially extended dynamical systems. Recently, an attempt has been made to control rare intense events in the CGLE [8]. The idea is that if a precise model of the underlying system is available so that the occurrence of some rare intense events can be predicted accurately in the sense that their precise locations and timings can be calculated in advance, some suitable control can be applied to these locations at those times so as to suppress the amplitudes of the events. Apparently, the success of this control strategy relies on one's ability to construct a precise model of the underlying physical system. Even when such a model is available, sensitive dependence on initial conditions can ruin any relatively long term prediction in a fundamental way, rendering difficult the implementation of such prediction-based control. In view of the ubiquity of spatially extended dynamical systems in nature [9] and the severe consequences of rare intense events, it is of broad interest to articulate control strategies that do not rely on model and prediction.

In this paper, we propose a *prediction-free, observation-based* scheme to control rare intense events in the CGLE. Given a system, our hypothesis is that its state is *qualitatively* observable in the sense that the time and space averaged values of some key state variables of the system are available. Practically, this can be accomplished by some proper imaging monitoring techniques (e.g., satellite image) that are nowadays employed commonly in many fields of science and engineering. At any given time, such an observation would give the intensity distribution of the physical variable of interest and, in particular, a small set of points in space for which the intensity values are significantly larger than those at typical points of the system. Such a small set of points can be regarded as the locations at which rare intense events are likely to occurr. We call such a set the *relevant set* (of points in space) for rare intense events, and we assume that the set is continuously available through observation. Our basic idea for control is then to apply spatially highly localized perturbations at the relevant set of points. The detailed control law depends, of course, on system details. For example, in the common situation where rare intense events are associated with large energies, some proper local damping control can be applied to the relevant set. We shall provide analytic insights into the choosing of control parameters in the CGLE and demonstrate computationally that rare intense events can be significantly suppressed by our control method. The important technical issue of time delay in applying the control will also be addressed. Although our analysis and demonstration are for the specific system of the CGLE, our strategy of applying local control at a relevant set

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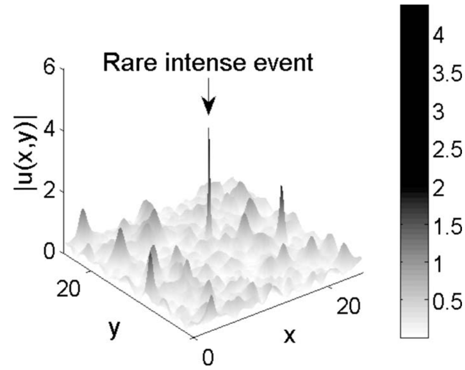

FIG. 1. An example of rare intense events at a specific instant of time in the CGLE for $\alpha=30$, $\beta=30$, $\varepsilon=1$, $\gamma=1$, and $\mu=1$.

of points through observation can be useful for mitigating rare intense events in other spatially extended systems.

The two-dimensional CGLE is

$$\frac{\partial u}{\partial t} = \varepsilon u - (\gamma + i\alpha)|u|^2 u + (\mu - i\beta)\nabla^2 u, \tag{1}$$

where ε, γ, μ, α, and β are parameters. For $\varepsilon=0$, $\gamma=0$, and $\mu=0$, Eq. (1) is reduced to the nonlinear Schrödinger equation (NLSE). It has been demonstrated that rare intense events can occur in the parameter region close to the NLS limit [1], namely, for $\alpha,\beta \gg \varepsilon,\gamma,\mu$. To illustrate this, we show in Fig. 1 a snapshot of $u(x,y,t)$, where a square region of side length $l=20\pi$ in the (x,y) plane is chosen and periodic boundary conditions are used. The solution $u(x,y,t)$ to Eq. (1) is obtained by using a standard finite-difference method with random initial conditions.

We develop a physical theory for the control of rare intense events in the CGLE. To illustrate the basic idea in an intuitive way, we assume a control parameter of the system is available in the sense that adjustments can be made to it about a nominal value in a small region in the space. Suppose at time t, an event of large amplitude has been observed to occur at the point (x^{max}, y^{max}). A small square region of side length r about this point, say, $\Omega:\{(x,y)||x-x^{max}| < r/2, |y-y^{max}| < r/2\}$, is then chosen as the control region for which the control parameter is changed from its nominal value. In our strategy, the region Ω varies with time, depending on the location of the observed event. Let $\Gamma \in \{\varepsilon,\gamma,\mu,\alpha,\beta\}$ be the control parameter. Mathematically the control law can be represented by

$$\Gamma = \begin{cases} (1+\delta)\Gamma^0, & \text{if } (x,y) \in \Omega, \\ \Gamma^0, & \text{otherwise}, \end{cases} \tag{2}$$

where Γ^0 is the nominal parameter value and δ is the control perturbance. A key question is how to choose a proper control parameter. While the answer in general depends on the details of the system to be controlled, for the CGLE we are able to address this question analytically by focusing on the dynamics of the system in the vicinities of locations where rare intense events are likely to occur.

Taking the real and imaginary parts of Eq. (1), we obtain

$$\frac{\partial u_r}{\partial t} = \varepsilon u_r - \gamma|u|^2 u_r + \alpha|u|^2 u_i + \mu\nabla^2 u_r + \beta\nabla^2 u_i,$$

$$\frac{\partial u_i}{\partial t} = \varepsilon u_i - \gamma|u|^2 u_i - \alpha|u|^2 u_r + \mu\nabla^2 u_i - \beta\nabla^2 u_r. \tag{3}$$

At a given time, the value of the control parameter in the small control region Ω is changed. While the control parameter is spatially discontinuous, it is constant in Ω. It is insightful to examine the evolution of the intensity $|u(x,y,t)|^2$ in Ω,

$$\begin{aligned}\frac{\partial |u|^2}{\partial t} &= 2u_r\frac{\partial u_r}{\partial t} + 2u_i\frac{\partial u_i}{\partial t} \\ &= 2\varepsilon|u|^2 - 2\gamma|u|^4 + 2\mu(u_r\nabla^2 u_r + u_i\nabla^2 u_i) \\ &\quad + 2\beta(u_i\nabla^2 u_r - u_r\nabla^2 u_i).\end{aligned} \tag{4}$$

In Ω, the system is locally smooth so that the evolution of the total intensity contained is determined by

$$\begin{aligned}\frac{\partial \int\int_\Omega |u|^2 dxdy}{\partial t} &= 2\varepsilon \int\int_\Omega |u|^2 dxdy - 2\gamma \int\int_\Omega |u|^4 dxdy \\ &\quad + 2\mu \int\int_\Omega (u_r\nabla^2 u_r + u_i\nabla^2 u_i) dxdy \\ &\quad - 2\beta \int\int_\Omega (u_i\nabla^2 u_r - u_r\nabla^2 u_i) dxdy.\end{aligned} \tag{5}$$

Due to the symmetry of the CGLE: $u \rightarrow u\exp(i\theta)$, spatial shapes of the real and imaginary parts of the CGLE are similar. Therefore, we can approximate u_r and u_i as

$$u_r = \exp[-A(x^2+y^2)^B]u_r^{max}(t),$$

$$u_i = \exp[-A(x^2+y^2)^B]u_i^{max}(t), \tag{6}$$

where A and B are constants, $u_r^{max}(t)$ and $u_i^{max}(t)$ are the respective maxima of $u_r(t)$ and $u_i(t)$ in Ω. Making use of the fact that rare intense events possess an approximately self-similar structure in space [1], we can regard A and B as constants. Inserting Eq. (6) in Eq. (5) yields

$$\mathcal{P}\frac{\partial(|u|^{max})^2}{\partial t} = 2\varepsilon\mathcal{P}(|u|^{max})^2 - 2\gamma\mathcal{Q}(|u|^{max})^4 + 2\mu\mathcal{K}(|u|^{max})^2,$$

where $\mathcal{P} \equiv \int\int_\Omega \exp(-2A(x^2+y^2)^B)dxdy$, $\mathcal{Q} \equiv \int\int_\Omega \exp[-4A(x^2+y^2)^B]dxdy$, and $\mathcal{K} \equiv \int\int_\Omega [4A^2B^2(x^2+y^2)^{2B-1} - 4AB^2(x^2+y^2)^{B-1}]\exp[-2A(x^2+y^2)^B]dxdy$ are constants in the control region Ω. Combining terms of the same order, we obtain

$$\mathcal{P}\frac{\partial(|u|^{max})^2}{\partial t} = 2(\varepsilon\mathcal{P} + \mu\mathcal{K})(|u|^{max})^2 - 2\gamma\mathcal{Q}(|u|^{max})^4 \tag{7}$$

from which we see that the maximum intensity $(|u|^{max})^2$ exhibits a stable equilibrium

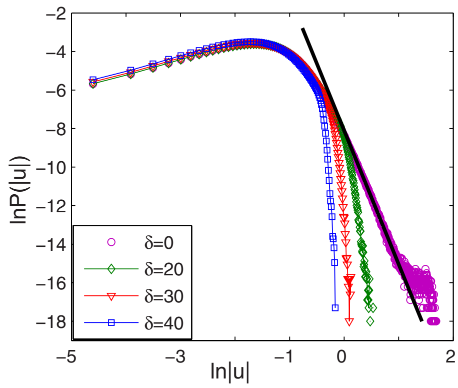

FIG. 2. (Color online) Probability distribution functions of spatial oscillation amplitudes of the CGLE with and without control. Each curve is obtained by using 4000 snapshots separated by $\Delta t = 0.05$.

$$(|u|^{max})^2 = \frac{\varepsilon \mathcal{P} + \mu \mathcal{K}}{\gamma \mathcal{Q}}. \tag{8}$$

Equation (8) indicates that the intensities of rare events are determined by the system parameters ε, μ, and γ only; the intensities are in fact independent of the NLSE parameters α and β. For the CGLE, although the occurrence of rare intense events requires that the parameters $(\varepsilon, \mu, \gamma)$ be much smaller than α and β, the triplet of parameters is critical for rare intense events [10]. Thus choosing any one from the triplet as the control parameter can be effective for suppressing intense events in the system. Or, we can adjust all three parameters so as to make the right-hand side of Eq. (8) small, thereby reducing the maximum intensity of the local bursts. For instance, since $\mathcal{P} > 0$, $\mathcal{Q} > 0$, and $\mathcal{K} < 0$, we can decrease ε, increase μ and γ to reduce significantly the intensities of the bursts.

We now demonstrate our method by choosing ε as the control parameter. (Similar results have been obtained by choosing μ and/or γ as control parameters.) Figure 2 shows, on a logarithmic scale, the probability distribution function $P(|u|)$ for different values of δ in the control law (2), where the side length of the local control region is set to be $r = 20\pi/16$. Note that the area of the control region is over two orders of magnitude smaller than the region of interest. In the absence of control (the rightmost curve), the amplitudes of intense events apparently obey a power-law distribution, as indicated by the thick straight-line fit, which has been shown previously as a characteristic feature of rare intense events in the CGLE [11,12]. When control is applied, a significant reduction in the maximum possible amplitude of the solution occurs. There is an indication that the control results in an exponential cutoff from the original power-law distribution in $P(|u|)$. Another feature is that, since at any time our control is targeted at a small local area in the space that contains some high-amplitude bursts and since such bursts are rare, the typical system dynamics as represented by relatively small-amplitude oscillations are largely unaffected. We thus expect the distribution $P(|u|)$ for small values of $|u|$ to be invariant under control. This has indeed been observed, as in Fig. 2.

The control scheme discussed so far assumes tacitly that at any time parameter perturbation can be applied instantaneously at the location where an event of relatively large amplitude has been observed. In physical reality the issue of time delay in applying the control must then be addressed. Intuitively, if the time delay is small, the control region is still under the influence of some event so that the control can be effective. However, a large time delay would render our control strategy ineffective because, when such a control is applied, the target region is already free of any intense activities, reducing the likelihood of suppressing such activities. Some specific questions can be, for example, how much delay can our control scheme tolerate while still achieving the goal of suppressing rare intense events? Would it be possible to compensate the effect of time delay by increasing the magnitude of the control perturbations? To address these questions, we consider the following time-delayed version of the CGLE (assuming ε is the control parameter):

$$\frac{\partial u}{\partial t} = \varepsilon_c u - (\gamma + i\alpha)|u|^2 u + (\mu - i\beta)\nabla^2 u,$$

$$\varepsilon_c = \begin{cases} (1+\delta)\varepsilon^0, & \text{if } (x,y) \in \Omega(t-\tau), \\ \varepsilon^0, & \text{otherwise}, \end{cases} \tag{9}$$

where ε^0 is a nominal value of ε, and time delay occurs in the time-dependent control area $\Omega(t)$. We ask what the most relevant time scale is in the system to which the time delay can be compared. Physically, a rare intense event can last for a finite amount of time at a given space location, and this time can be regarded as the lifetime of the event at that space point. For intense events occurring at different space points, their lifetimes are typically random. An average lifetime $\langle t_e \rangle$ can then be defined. To compute $\langle t_e \rangle$, we locate the maximum value $|u|^{max}$ of $|u|$ in the region of interest and plot it as a function of time, as shown in Fig. 3(a) for the unperturbed system ($\delta=0$). Since $|u|^{max}$ can last for a finite amount of time at any particular space point, in the time evolution of $|u|^{max}$ there exists a set of times when $|u|^{max}$ switches from one space point to another. These time instants are illustrated by the open circles in Fig. 3(a). Any time interval between two adjacent open circles can be regarded effectively as the lifetime of an intense event at a particular space point, from which the average lifetime $\langle t_e \rangle$ can be calculated. When control is applied, the value of $|u|^{max}$ is reduced but the switching of the space point for $|u|^{max}$ occurs more frequently, as shown in Fig. 3(b). Intuitively, this can be understood by noting that the effect of control is basically to direct the "energy" associated with events of large amplitude to other space points and to "smooth out" the variation of $|u|$ in a desirable way.

We can now assess the effect of time delay on control. Figure 4 shows, for five different values of the time delay τ, the average lifetime $\langle t_e \rangle$ as a function of the magnitude of the control perturbation δ. We see that, for $\tau=0$, $\langle t_e \rangle$ reduces to near-zero values as δ is increased from zero. A general observation is that for $\tau \neq 0$, but small, $\langle t_e \rangle$ still decreases as δ is increased, however, it plateaus when δ becomes large. A reduction in $\langle t_e \rangle$ indicates that the control is effective. How-

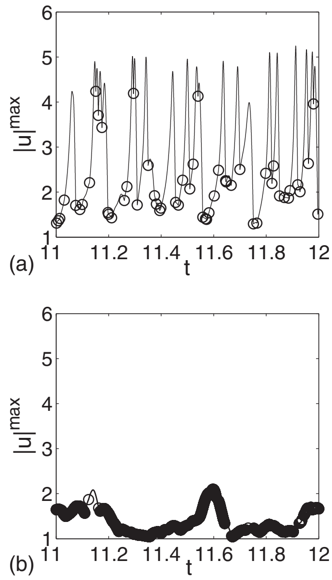

FIG. 3. A representative example of time evolution of $|u|^{max}$ for (a) $\delta=0$ (absence of control) and (b) $\delta=10$. Open circles indicate the time instants when $|u|^{max}$ switches its location in space.

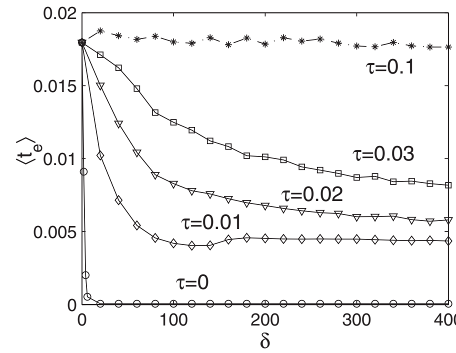

FIG. 4. For the locally time-delayed CGLE (9), the average lifetime $\langle t_e \rangle$ of intense events as a function of the magnitude of the control perturbation δ for five different values of the time delay. A significant reduction in $\langle t_e \rangle$ as δ is increased indicates the effectiveness of control.

ever, if τ is too large (e.g., for $\tau=0.1$), there is essentially no change in $\langle t_e \rangle$ as δ is increased, indicating the ineffectiveness of the control. Examining the behavior of $\langle t_e \rangle$ as in Fig. 4 thus allows us to obtain an estimate of the allowed time delay for effective control: it should be of the same order of magnitude as the average lifetime of the intense events in the unperturbed system.

In conclusion, we have articulated a strategy for controlling rare intense events in the CGLE without the need of prediction. Our method is local and based on observation only, and it is tolerant to time delay. We have gained analytic insights and carried out numerical experiments to demonstrate the working of our method. Controlling rare intense events is an important problem in many areas of science and engineering where spatially extended dynamics arise, and we hope our work can be stimulating for addressing this challenging problem.

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