Onset of traffic congestion in complex networks

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Free traffic flow on a complex network is key to its normal and efficient functioning. Recent works indicate that many realistic networks possess connecting topologies with a scale-free feature: the probability distribution of the number of links at nodes, or the degree distribution, contains a power-law component. A natural question is then how the topology influences the dynamics of traffic flow on a complex network. Here we present two models to address this question, taking into account the network topology, the information-generating rate, and the information-processing capacity of individual nodes. For each model, we study four kinds of networks: scale-free, random, and regular networks and Cayley trees. In the first model, the capacity of packet delivery of each node is proportional to its number of links, while in the second model, it is proportional to the number of shortest paths passing through the node. We find, in both models, that there is a critical rate of information generation, below which the network traffic is free but above which traffic congestion occurs. Theoretical estimates are given for the critical point. For the first model, scale-free networks and random networks are found to be more tolerant to congestion. For the second model, the congestion condition is independent of network size and topology, suggesting that this model may be practically useful for designing communication protocols.

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I. INTRODUCTION

Free, uncongested traffic flows on networks are critical for a modern society as its normal and efficient functioning relies on such networks as the internet, the power grid, and transportation networks, etc. To ensure free traffic flows on a complex network is naturally of great interest. The aim of this paper is to address this problem via modeling. Our particular interest is to understand under what conditions traffic congestion can occur on a complex network and to explore possible ways of control to alleviate the congestion. The models we have constructed are based on the setting of information transmission and exchange on the internet. There have been many previous works in this direction [1–14]. A basic assumption used in these studies is that the network possesses a regular and homogeneous structure. Recent works reveal, however, that many realistic networks including the internet are complex with scale-free and small-world features [15,16]. It is thus of paramount interest to study the effect of network topology on traffic flow, which is the key feature that distinguishes our work from the existing ones. While our model is for computer networks, we expect it to be relevant to other practical networks in general, such as the postal service network or the airline transportation network. Our studies may be useful for designing communication protocols for complex networks.

The structure and dynamics on complex networks have attracted a tremendous amount of recent interest [16–18] since the seminal works on scale-free networks by Barabási and Albert [15] and on the small-world phenomenon by Watts and Strogatz [19]. Large networks in nature are always evolving in that nodes and links are continuously added to and/or deleted from the network. Networks are growing if, on average, the numbers of nodes and links increase with time. Most large networks are sparse, that is, the average number of links per node is much smaller than the total number of nodes in the network. Growing and complex networks may be classified according to whether there exists a hierarchy of organized structures. In particular, in a scale-free network, the number of links of various nodes follows a power-law (or algebraic) probability distribution, indicating that nodes in the network are organized into a hierarchy of connected clusters in terms of their numbers of links. In a random network [20], nodes are connected to each other in a completely random fashion and, as such, there is no organized hierarchy of structures in links. Regular networks possess only a few types of linking structures, in contrast to scale-free networks that have an infinite number of possibilities of linking. In this sense, the class of small-world networks studied by Watts and Strogatz [19] is constructed by randomly rewiring only a small fraction of links in a regular network and, hence, they are only a perturbed version of the "backbone" regular network.

Mathematically, a way to characterize a complex network is to examine the degree distribution $P(k)$, where k is the realization of a random variable K measuring the number of links at a node. Scale-free networks are characterized by

$$P(k) \sim k^{-\gamma}, \tag{1}$$

where $\gamma > 0$ is the algebraic scaling exponent. For random networks, the degree distributions are exponential,

$$P(k) \sim \exp(-ak), \tag{2}$$

where $a > 0$ is a constant. The specific class of small-world networks proposed by Watts and Strogatz [19] also assumes the exponential distribution. It should be noticed that strictly scale-free networks are idealized. Realistic networks always contain both scale-free and random components. This "mixed" characteristic is the case for many networks in nature such as the scientific-collaboration network [18,21], the movie-actor network [22,23], and the conceptual network of languages [24].

Models of traffic flow on computer networks have been studied extensively [1–14]. In this context, the information processors are routers which have the same function as, say, workers in the postal service. Routers route the data packets to their destinations. In a computer network, a node may be a host or a router. A host can create packets with addresses of destination and receive packets from other hosts. A router finds, for each packet, the shortest path between the host and the destination and forward the packet along this path in each time step. Here, by "shortest" we mean the path with the smallest number of links. Previous studies focus on two different classes of computer network models. The first class treats all nodes as both hosts and routers [7,10–12], and for the second class [5,8,13,14], some nodes are hosts and others are routers. However, all existing models assume regular network topology, such as two-dimensional lattices [5,7,8,13] or Cayley trees [9–12]. In view of the recent evidence that the internet and many other realistic networks are complex to a significant extent [15,16,18], there is a need to investigate the dynamics of traffic flow on these networks.

In this paper, we construct two dynamical models, each with two parameters: the information creation rate λ and a control parameter β that measures the capacity of nodes to process information. In the first model, the capacity of packet delivery of each node is proportional to its degree, while in the second model, it is proportional to the number of shortest paths passing through the node (betweenness [21]). The quantity of interest is the critical rate λ_c of information generation (as measured by the number of packets created within the network in unit time) at which a phase transition occurs from free to congested traffic flow. In particular, for $\lambda < \lambda_c$, the numbers of created and delivered packets are balanced, resulting in a steady state, or free flow of traffic. For $\lambda > \lambda_c$, congestions occur in the sense that the number of accumulated packets increases with time, due to the fact that the capacities of nodes for delivering packets are limited. We are interested in determining the phase-transition point λ_c, given a network topology, in order to address which kind of network is more susceptible to phase transition and therefore traffic congestion. For this purpose, we study four kinds of networks: Cayley trees, regular, random, and scale-free networks. Our main result is that, in model I, λ_c is larger for networks that have a larger connectivity to betweenness ratio for the small set of nodes with the largest betweenness. Specifically, congestion is easier to occur in Cayley trees, then regular networks, then scale-free networks, and random networks are most tolerant to congestion. We give a theoretical argument to explain this phenomenon, based on identifying the existence of a subset of relatively heavily linked nodes in a network as the key. This is further supported by examining the effect of enhancing the capacities of these nodes to process information. From another standpoint, this result suggests a way to alleviate traffic congestions for scale-free networks: making heavily linked nodes [12] as powerful and efficient as possible for processing information. In the second model, we find that the congestion condition is independent of network size and topology and it thus represents a more useful protocol for alleviating traffic congestion on networks, especially for trees and regular networks.

One recent work that is particularly relevant to our study is the one addressing optimal network topologies for local search on networks [12]. This paper addressed the problem of searchability in complex networks with or without congestion. The focus was on optimal network configurations in terms of search cost, with the conclusion that there are only two classes of optimal networks: starlike or homogeneous-isotropic configurations, depending on the number of parallel searches. Our interest here is in the phase transition from free traffic to congestion and how it occurs with respect to the most representative types of complex networks found in realistic applications: regular, random, and scale-free networks. Despite the difference in the objective, the idea about the definition and analysis of congestion in Ref. [12] is very useful, which we have adopted here.

In Sec. II, we describe our traffic flow models. In Sec. III, we present a theoretical analysis for estimating the critical point for phase transition. Simulation results are given in Sec. IV and a discussion is offered in Sec. V.

II. TRAFFIC-FLOW MODELS

Our traffic-flow model is based on the routing algorithm in computer networks. To account for the network topology, we assume that the capacities for processing information are different for different nodes, depending on the numbers of links (model I) or the number of shortest paths (model II) passing through them. Our routing algorithm consists of the following steps.

(1) At each time step, the probability for node i to generate a packet is λ.

(2) At each time step, a node i delivers C_i packets one step toward their destinations, where $C_i = (1 + \mathrm{int}[\beta k_i])$ in model I and $C_i = (1 + \mathrm{int}[\beta B_i / N])$ in model II, $0 < \beta < 1$ is a control parameter, k_i is the degree of node i, and B_i is its betweenness. A packet, once reaching its destination, is removed from the traffic.

(3) Once a packet is created, it is placed at the end of the queue if this node already has several packets waiting to be delivered to their destinations. The existing packets may be created at some previous time steps or they are transmitted from other nodes. At the same time, a destination node, different from the original one, is chosen at random in the network. The router finds a shortest path between the node with the newly created packet and its destination and, the packet is forwarded along this path during the following time steps. If there are several shortest paths for one packet, the one is chosen whose next station (selected node) has the smallest

number of waiting packets or the shortest queueing length.

(4) At each time step, the first C_i packets at the top of the queue of node i, if it has more than C_i packets in its queue, are forwarded one step toward their destinations and placed at the end of the queues of the selected nodes. Otherwise, all packets in the queue are forwarded one step. This procedure applies to every node at the same time. As a result, the delivering time that a packet needs to reach its destination is related not only to the distance (number of time steps) between the source and the destination, but also to the number of existing packets along its path. Note that, here, the quantity C_i measures the forwarding capacity of node i.

Since N is the total number of nodes in the network, the total number of created packets at each time step is λN, and the total number of delivered packets at each time step is approximately $\Sigma_{i=1}^{N} C_i$ if every node has a sufficient number of packets, which is greater than the total number of created packets provided that $\lambda < 1$. Due to the network complexity, packets are more likely to be routed to the nodes with higher betweenness on their way to the final destinations. As a result, packets are more likely to be accumulated at these nodes, resulting in traffic congestion.

Qualitatively, the dynamics of traffic flow on a network is then as follows. For small values of the creation rate λ, the number of packets on the network is small so that every packet can be processed and delivered in time. Typically, after a short transient time, a steady state for the traffic flow is reached in which the instantaneous number $\langle n(t)\rangle$ of packets, averaged over all nodes in the network, fluctuates about a constant. That is, on average, the total numbers of packets created and delivered are equal, resulting in a free-flow state. This is in fact the well-known Little's law in queueing theory [25]. For larger values of λ, the number of packets created is more likely to exceed that which can be processed in time. In this case, $\langle n(t)\rangle$ grows in time and traffic congestion becomes possible. As λ is increased from zero, we thus expect to observe two phases: free flow for small λ and a congested phase for large λ, with a phase transition from the former to the latter at λ_c. To observe the phase transition and to determine λ_c, given a network structure, are main goals of this paper.

III. THEORETICAL ESTIMATION OF CRITICAL POINT

Here we give a heuristic theory for determining the phase-transition point λ_c, given a particular network structure. Because the node with the largest betweenness can be easily congested and the congestion can quickly spread to the entire network, it is necessary to consider only the traffic balance of this node. Since the packets are transmitted along the shortest paths from the source to the destination, the probability that a created packet will pass through the node with the largest betweenness i is $B_i/\Sigma_{j=1}^{N} B_j$. At each time step, on average, λ packets are generated. Thus, the average number of packets that the node with the largest betweenness receives at each time step is

$$Q_{in} = \lambda N D \frac{B_{L_{max}}}{\sum_{j=1}^{N} B_j}, \tag{3}$$

where D is the average shortest path length of the network and L_{max} is the index of the node with the largest betweenness. On the other hand, the total number of packets that the node with the largest betweenness can deliver at each time step is

$$Q_{out} = C_{L_{max}}. \tag{4}$$

Congestion occurs when the number of incoming packets is equal to or larger than the outgoing packets at the node with the largest betweenness, i.e.,

$$Q_{in} \geqslant Q_{out}. \tag{5}$$

Then,

$$\lambda_c N D \frac{B_{L_{max}}}{\sum_{j=1}^{N} B_j} = C_{L_{max}}. \tag{6}$$

Since $\Sigma_{j=1}^{N} B_j = N(N-1)D$, Eq. (6) can be simplified to

$$\lambda_c = \frac{C_{L_{max}}(N-1)}{B_{L_{max}}}. \tag{7}$$

Equation (7) can be applied to general networks, which is the same result as in Ref. [12].

For model I, Eq. (7) turns out to be

$$\lambda_c = \frac{(1+\text{int}[\beta k_{L_{max}}])(N-1)}{B_{L_{max}}}. \tag{8}$$

To gain insight, we consider two special cases, regular networks and Cayley trees. First, for regular networks, all nodes have the same structure and the same number of links. We thus have

$$B_{L_{max}} = (N-1)D. \tag{9}$$

The congestion condition can then be estimated by the following equation:

$$\lambda_c \approx \frac{1+\beta k}{D}. \tag{10}$$

For regular networks $D = N/2k$, we have

$$\lambda_{c,reg} \approx \frac{2k(1+\beta k)}{N}. \tag{11}$$

Now consider a special kind of regular network, square lattices with periodic boundary condition. For such a lattice with $L \times L$ nodes, $D = L/2$, we have

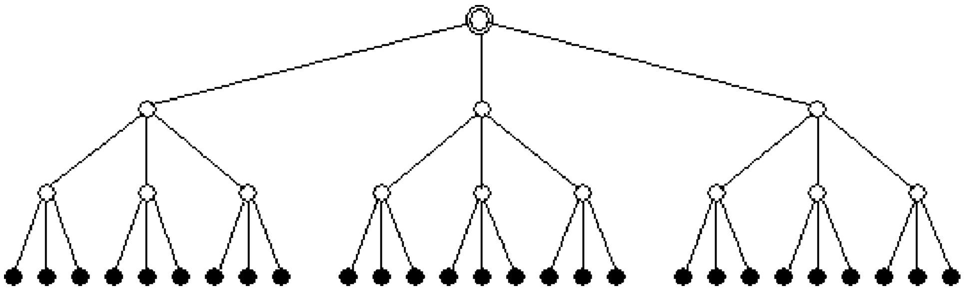

FIG. 1. Illustration of a Cayley tree with the branching factor $z=3$ and the level of the leaves $l=3$. The root is on level 0 and the depth of the tree is thus $l+1=4$. Solid circles, circles, and double circle represent leaves, intermediate nodes, and root, respectively.

$$\lambda_{c,lattice} \approx \frac{2(1+\beta k)}{L}. \tag{12}$$

If $\beta=0$, Eq. (12) recovers the estimate of Ref. [13] and the estimate from the mean field model of Fŭks and Lawniczak [7].

A schematic illustration of a Cayley-tree network is shown in Fig. 1. Since the root has the highest betweenness and a small number of links, it is easy for congestion to occur at the root, which has a major impact on the whole tree. For these reasons, λ_c can be conveniently estimated by only considering the traffic flow through the root.

The total number of nodes in the tree is

$$N = z^0 + z^1 + z^2 + \cdots + z^l = \frac{z^{l+1}-1}{z-1}. \tag{13}$$

The betweenness B_r of the root can be calculated by counting the routes from any node in the tree to different first-order subtrees, which must pass through the root. We obtain

$$B_r = \frac{1}{2}\frac{(N-1)}{z}\frac{(z-1)(N-1)}{z}z \tag{14}$$

$$= \frac{z(z^l-1)^2}{2(z-1)}. \tag{15}$$

In Eq. (14), the factor $(N-1)/z$ is the number of nodes in one chosen first-order subtree and the factor $(N-1)(z-1)/z$ is the number of nodes in all other $z-1$ first-order subtrees. The number of shortest paths from the chosen first-order subtree to any other first-order subtree is the multiplication of these two factors. The factor z means that we have precisely z ways to choose a first-order subtree. The factor $1/2$ is included because each shortest path has been counted twice.

Since the number of links of the root is z, the number of packets that the root can deliver per unit time is

$$Q_{out} = C_r \approx 1 + \beta z. \tag{16}$$

Putting B_r and C_r in Eq. (7), $\lambda_{c,Cayley}$ is estimated to be

$$\lambda_{c,Cayley} \approx \frac{2(1+\beta z)}{z^l-1}. \tag{17}$$

For model II, the delivery capacity of each node is proportional to its betweenness, i.e., $C_i=1+\text{int}[\beta B_i/N]$. In this case, the critical generating rate for general networks becomes

$$\lambda_c = \frac{(1+\text{int}[\beta B_{L_{max}}/N])(N-1)}{B_{L_{max}}} \tag{18}$$

$$\approx \frac{(N-1)}{B_{L_{max}}} + \beta \tag{19}$$

$$\approx \beta, \tag{20}$$

where, because $B_{L_{max}} \gg N$ in all networks considered in this work, the second term in Eq. (19) dominates. Equation (20) shows that the critical generating rates are roughly independent of the network size and topology.

By comparing Eq. (20) to Eqs. (11) and (17), we see that, although model II makes no significant improvement on random and scale-free networks, it can increase λ_c for regular networks and Cayley trees. A practical significance is that protocols designed based on model II can generally be robust against traffic congestion, regardless of the network topology.

IV. SIMULATION RESULTS

The primary goal of our simulation is to understand the behavior of the phase transition, which leads to traffic congestion, with respect to the network topology. Thus we focus on examining the value of the critical point λ_c for Cayley trees, regular, random, and scale-free networks. Another goal is to explore the effect of adjusting the capacity parameter β. In particular, we are interested in the possibility of increasing the capacities of a small subset of nodes with higher betweenness to improve the network's tolerance to traffic congestions. In order to characterize the transition, we use the order parameter introduced in Ref. [9]:

$$\eta = \lim_{t\to\infty}\frac{\langle\Delta\Theta\rangle}{\lambda\Delta t}, \tag{21}$$

where $\Delta\Theta=\Theta(t+\Delta t)-\Theta(t)$, $\Theta(t)$ is the total number of packets in the network at time t, and $\langle\cdots\rangle$ indicates the average over time windows of Δt. When $\lambda<\lambda_c$, the network is in the free-flow state; then $\Delta\Theta\approx 0$ and $\eta\approx 0$. For $\lambda>\lambda_c$, $\Delta\Theta$ increases with Δt.

In our simulations, the networks are generated as follows. For all kinds of networks, each node points to a linked list, which contains its nearest neighbors. For a Cayley tree with depth $l+1$ and branching factor z, there are $N=\text{int}[(z^{l+1}-1)/(z-1)]$ nodes, which are labeled as

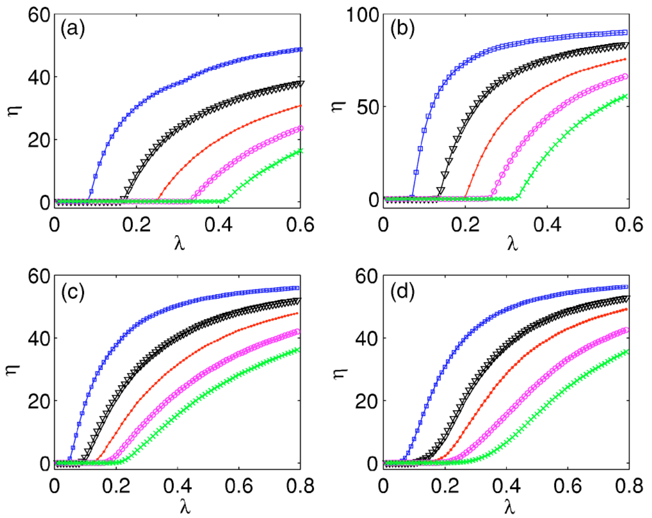

FIG. 2. (Color online) For model I, the order parameter η versus the packet-generating rate λ for the following. (a) Cayley tree, $z=3$, $l=6$, $\langle k\rangle=2$, and thus $N=1093$. Square, triangle, dot, circle, and cross curves correspond to the simulations of $\beta=10,20,30,40,50$, respectively. (b) Regular network, $N=1000$, $\langle k\rangle=4$, square, triangle, dot, circle, and cross curves correspond to the simulations of $\beta=2,4,6,8,10$, respectively (c) Scale-free network and (d) random network, $N=1000$, $\langle k\rangle=4$, where square, triangle, dot, circle, and cross curves correspond to the simulations of $\beta=0.1,0.2,0.3,0.4,0.5$, respectively. In all simulations, 50 realizations are averaged.

$\{0,1,2,\ldots,N-1\}$. Thus the list pointed to by node i at level $j\in\{0,1,\ldots,l-1\}$ contains the following node labels as children: $\{i\times Z+1, i\times Z+2,\ldots,i\times Z+Z\}$. At the same time, node i is inserted in the lists pointed to by each of its children. This process begins from the root with node label 0 and ends at the last node at level $l-1$. To generate a regular network with degree k and the label set $\{1,2,\ldots,N\}$, each node i points to a list containing the node labels $\{i-k/2,\ldots,i-2,i-1,i+1,i+2,\ldots,i+k/2\}$. However, if $i+j>N$, the label is replaced by $i+j-N$, and if $i-j<1$, it is replaced by $i-j+N$. Scale-free and random networks are generated by using the general network model proposed in Ref. [26].

First, we present simulation results with model I. Figure 2 shows the order parameter η versus λ for different capacity parameters β for the (a) Cayley tree, (b) regular network, (c) scale-free network, and (d) random network. We see that, for all cases considered, η is approximately zero when λ is small; it suddenly increases when λ is larger than a critical value λ_c. We also observe that λ_c increases with β, which means that enhanced capacity for processing packets can help alleviate possible congestions that are most likely to occur at the heavily linked nodes. As a result, phase transition can be delayed in the sense that the network can be more tolerant to traffic congestions for larger values of λ. Figure 2 also indicates that, in order to get the same order of λ_c, a large value of the capacity parameter β is required for Cayley trees and regular networks; however, small β is needed for random and scale-free networks. This means that Cayley trees and regular networks are significantly more susceptible to traffic congestion. This is because, in Cayley trees and regular networks, the most congested nodes have large betweenness, but very small number of links, i.e., the ratio $k_{L_{max}}/B_{L_{max}}$ is much smaller than those in random and scale-free networks. Equation (8) then indicates that the λ_c for Cayley trees and regular networks is much smaller than that for random and scale-free networks.

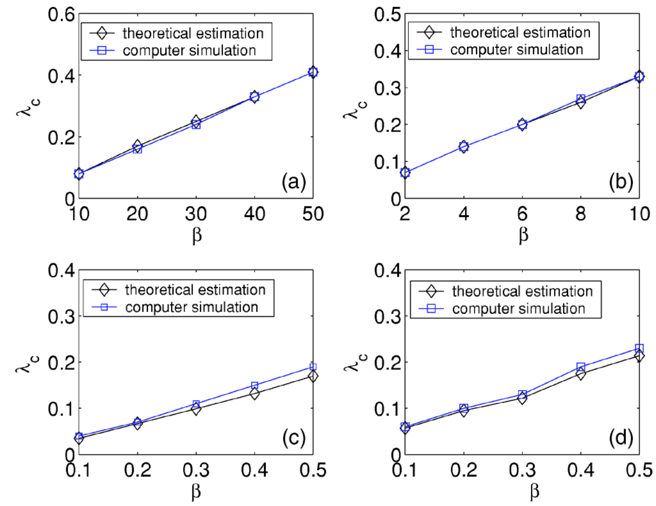

FIG. 3. (Color online) For model I, comparison of theoretical prediction (diamond curves) and simulation (square curves) of λ_c values for (a) Cayley trees, $z=3$, $l=6$, $\langle k\rangle=2$; for (b) regular networks, (c) scale-free networks, and (d) random networks, $N=1000$, $\langle k\rangle=4$.

Figure 3 shows the critical generating rate λ_c from theoretical predictions and from simulations. The theoretical results are obtained by Eqs. (17), (11), and (8) for Cayley trees, regular, random, and scale-free networks, respectively. In all cases, a good agreement is observed. From Fig. 2, we see that the critical packet generation rates λ_c of scale-free and random networks are of the same order. However, a direct comparison of simulation results (Fig. 4) shows that λ_c for random networks is actually larger than that for scale-free networks. As mentioned, scale-free networks are heterogeneous in links, which causes heterogeneity in betweenness.

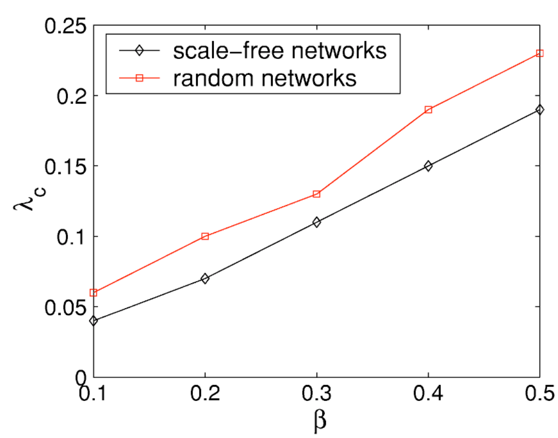

FIG. 4. (Color online) For model I with $N=1000$, $\langle k\rangle=4$, simulation results of λ_c versus β for scale-free and random networks.

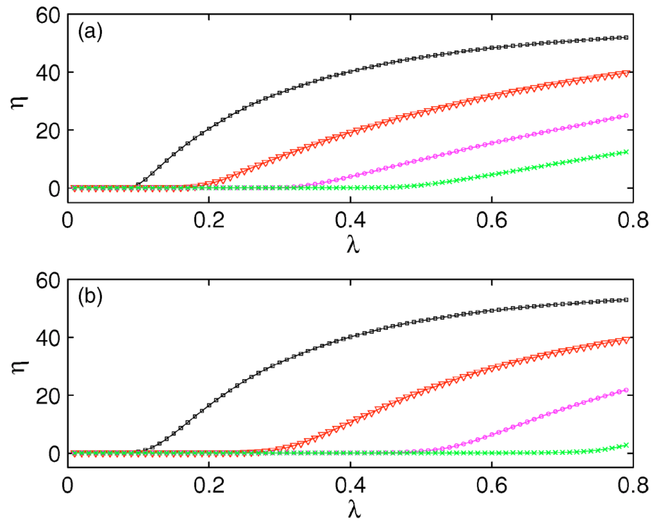

FIG. 5. (Color online) For model I with $N=1000$, $\beta=0.2$, order parameter η versus the packet-generating rate λ for (a) scale-free networks, (b) random networks. Square, triangle, circle, and cross curves correspond to the simulations of $\langle k\rangle=4,6,8,10$, respectively. In all simulations, 50 realizations are averaged.

This means that there is a small group of nodes which have large betweenness but majority of nodes in the network have small betweenness. Thus, most generated packets will have a high probability to pass through this small number of high betweenness nodes, making them vulnerable to congestion. Qualitatively, we may think that the packet transmission routes are relatively better distributed for random networks than for scale-free networks.

How does the congestion condition change with the network's average degree $\langle k\rangle$? Figure 5 shows that in both scale-free and random networks, λ_c increases as the average degree increases. This is because increasing the average degree makes nodes in the networks more connected and hence the shortest paths are less dependent on the heavily linked nodes. Consequently, congestion on the heavily linked nodes can be delayed. As mentioned, betweenness homogeneity is an important factor for traffic congestion. In order to characterize this feature, we calculate the standard deviation of betweenness defined as

$$\delta_B = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(B_i - \langle B\rangle)^2}, \tag{22}$$

where $\langle B\rangle$ is the average betweenness of the network in consideration. Figure 6 shows the decreasing of the standard deviation of betweenness for both the scale-free and random networks as the average degree increases, indicating that the distribution of betweenness is more homogeneous with increasing $\langle k\rangle$. Thus, packet loads of the nodes with the largest betweenness are reduced and congestion triggered by these nodes is delayed. From the same figure, we see that, except for $\langle k\rangle=2$, the betweenness deviation in random networks is smaller than that in scale-free networks. That is, the betweenness distribution in random networks is in general more homogeneous. This is another supporting factor for the

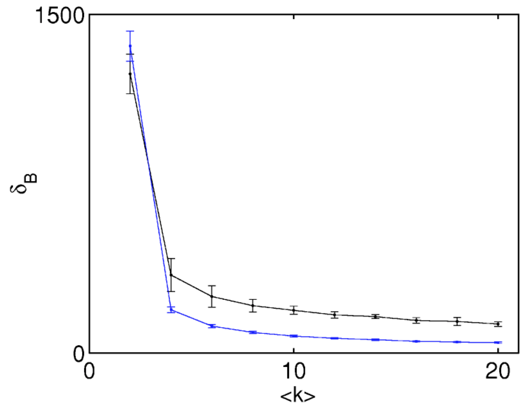

FIG. 6. (Color online) For $N=1000$ and 50 realizations, the standard deviation of betweenness δ_B versus the average degree $\langle k\rangle$ for scale-free (upper trace) and random (lower trace) networks.

explanation as to why random networks are more tolerant to congestion than scale-free networks.

We now present simulation results with model II. Here, the delivery capacity of each node is proportional to its betweenness, i.e., $C_i=1+\mathrm{int}[B_i/N]$. Figure 7 shows the order parameter η versus λ for different capacity parameters β for (a) Cayley tree, (b) regular, (c) scale-free, and (d) random network. We see that values of λ_c are roughly the same for all kinds of networks considered here, confirming our prediction by Eq. (20).

Figure 8 shows the critical generating rate λ_c from theoretical predictions and simulations for the four kinds of networks. In all cases, good agreement is observed. These simu-

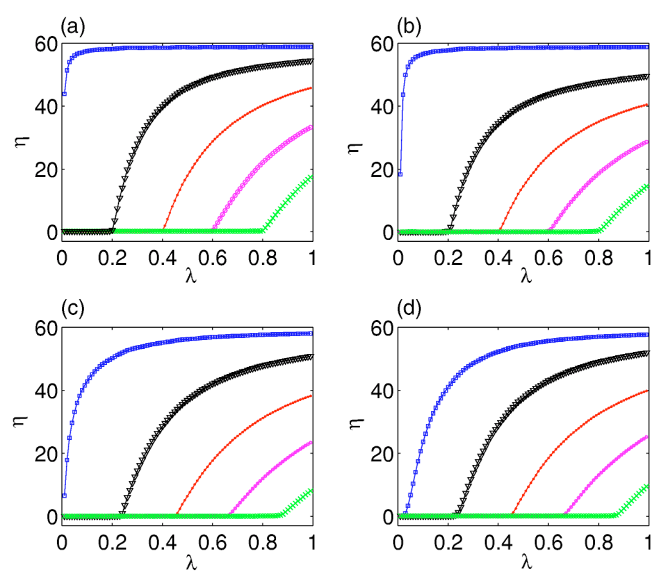

FIG. 7. (Color online) For model II, order parameter η versus the packet-generating rate λ for (a) Cayley tree, $z=3$, $l=6$, $\langle k\rangle=2$, thus $N=1093$; for (b) regular network, (c) scale-free network, and (d) random networks, $N=1000$, $\langle k\rangle=4$. In all of the four cases, square, triangle, dot, circle, and cross curves correspond to the simulations of $\beta=0.0,0.2,0.4,0.6,0.8$, respectively. The capacity of delivery of each node is proportional to its betweenness. In all simulations, 50 realizations are averaged.

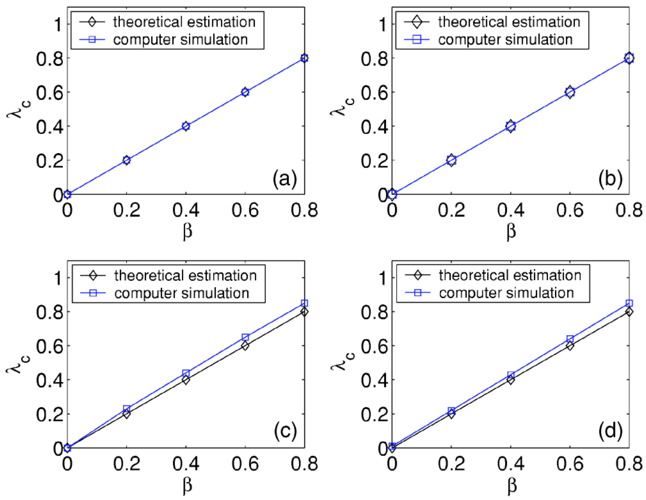

FIG. 8. (Color online) For model II, comparison of theoretical prediction (diamond curves) and computer simulation (square curves) of λ_c versus the capacity parameter β for (a) Cayley trees, $z=3$, $l=6$, $\langle k \rangle=2$; for (b) regular, (c) scale-free, and (d) random networks, $N=1000$, $\langle k \rangle=4$.

lation results thus show that protocols based on our model II are more tolerant to congestion for all kinds of networks studied here, especially for Cayley trees and regular networks.

V. DISCUSSION

We live in a modern world supported by large, complex networks. Examples range from financial markets to internet, communication, and transportation systems. Recently there has been a tremendous effort to study the general structure of these networks [16–18]. Universal features such as the small-world [19] and scale-free [15] properties, which can be characterized at a quantitative level, have been discovered in almost all realistic networks. The discoveries suggest that, to understand the dynamics on complex networks, their structures have to be taken into account.

This paper addresses the dynamics of traffic flow on complex networks. Our motivation comes from the desire to understand the influence of topological structure on the traffic dynamics on a network, as existing works in this direction often assume regularity and homogeneity for the underlying network [1–14]. We consider general network structures to couple with simple traffic-flow models determined by the rate of information generation and a parameter to describe the average capacity of nodes to process information. Our study indicates that phase transition can generally occur in the sense that free traffic flow can be guaranteed for low rates of information generation but large rates above a critical value can result in traffic congestions. Our models enable the critical value for the phase transition to be estimated theoretically and computed, given a particular network topology. We present computational results and analysis, which indicate that, in case the delivery capacity of each node is proportional to its degree, the critical value is smaller for networks of smaller ratio of degree to betweenness for the set of most easily congested nodes (the set of nodes with the largest betweenness). In this case, random and scale-free networks are more tolerant to congestion than trees and regular networks. This is further supported by examining the effect of enhancing the capacities of these nodes to process information on the global traffic flow. These results suggest a way to alleviate traffic congestions for protocol based on model I for networks with a significant heterogeneous component: making nodes with large betweenness as powerful and efficient as possible for processing and transmitting information. For protocol based on model II, the capacity of delivery of each node is proportional to its betweenness. In this case, the critical value λ_c is independent of the network topology. Compared with model I, while model II can improve a little the performance for scale-free and random networks, it can improve significantly the performance for trees and regular networks against congestion.

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