Conductance fluctuations in graphene systems: The relevance of classical dynamics

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Conductance fluctuations associated with transport through quantum-dot systems are currently understood to depend on the nature of the corresponding classical dynamics, i.e., integrable or chaotic. However, we find that in graphene quantum-dot systems, when a magnetic field is present, signatures of classical dynamics can disappear and universal scaling behaviors emerge. In particular, as the Fermi energy or the magnetic flux is varied, both regular oscillations and random fluctuations in the conductance can occur, with alternating transitions between the two. By carrying out a detailed analysis of two types of integrable (hexagonal and square) and one type of chaotic (stadium) graphene dot system, we uncover a universal scaling law among the critical Fermi energy, the critical magnetic flux, and the dot size. We develop a physical theory based on the emergence of edge states and the evolution of Landau levels (as in quantum Hall effect) to understand these experimentally testable behaviors.

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I. INTRODUCTION

A fundamental problem in quantum transport through nanoscale devices is conductance fluctuations. Consider, for example, a quantum-dot system. As the Fermi energy of the conducting electrons is varied, the conductance can exhibit fluctuations of distinct characteristics, depending on the geometrical shape of the dot. Research in the past two decades has demonstrated that the nature of the corresponding classical dynamics can play a key role in the conductance-fluctuation pattern.1-4 For example, when the classical scattering dynamics is integrable or has a mixed phase-space structure, there can be sharp resonances in the conductance curve. However, when the classical dynamics is fully chaotic, the conductance variations tend to be smoother.

There have been tremendous recent efforts in graphene5-8 due to its relativistic quantum physical properties and its potential for applications in nanoscale electronic devices and circuits. The study of transport in open graphene devices is thus a problem of vast interest.8 For example, the role played by disorder in conductance fluctuations in graphene was investigated, where anomalously strong fluctuations9 or suppression of the fluctuations10 were reported. A recent work has revealed that, in graphene quantum dots, the characteristics of conductance fluctuations also depend on the nature of the classical dynamics similar to those for conventional two-dimensional electron-gas (2DEG) quantum-dot systems.11 In these recent works, magnetic field is absent. The magnetic properties of graphene, however, are different from those associated with 2DEG systems. For example, in graphene the quantum Hall effect can be observed even at room temperature due to the massless Dirac fermion nature of the quasiparticles and significantly reduced scattering effects.12 Especially, the linear energy-momentum relation13 in graphene stipulates that the Landau levels are distributed according to $\pm \sqrt{N}$, where $N$ is the Landau index, as opposed to the proportional dependence on $N$ in 2DEG systems.14

In this paper, we study conductance fluctuations in graphene quantum-dot systems in the presence of magnetic field. We present two main results. First, in the parameter plane spanned by the perpendicular magnetic flux and the Fermi energy, there are regions of regular and random conductance oscillations, respectively. As the Fermi energy or the magnetic flux is changed, the fluctuations can be either regular or random, implying a kind of “coexistence” of regular and irregular conductance fluctuations as a single physical parameter is varied. Second, an experimentally significant issue is how conductance fluctuations are affected by the size of the quantum dot in the presence of a perpendicular magnetic field. In a previous experimental study15 of quantum dots of sizes ranging from 0.7 to 1.2 $\mu$m, the authors found nearly periodic conductance oscillations as the magnetic-field strength is varied. The frequency of the oscillation pattern, the so-called magnetic frequency, was found to follow a scaling relation with the edge size of the dot.16 In a recent study of the magnetic scaling behavior in graphene quantum dots,16,17 it was found that for small dots of edge size less than 0.3 $\mu$m, the magnetic frequency exhibits a scaling relation with the dot area. Here we shall focus on an important set of scarred orbits and examine the resulting conductance oscillations. We find that, for graphene quantum dots, below the first Landau level, the conductance exhibits periodic oscillations with the magnetic flux and with the Fermi energy. In fact, the magnetic frequency scales linearly with the dot size. However, the energy frequency, the inverse of the variation in the Fermi energy for the conductance to complete one cycle of oscillation, scales inversely with the dot size. Beyond the regime of periodic conductance oscillations, new sets of scarred orbits emerge and evolve as successive Landau levels are crossed, each with its own period, leading to random conductance fluctuations. The remarkable feature is that these scaling behaviors are independent of the nature of the underlying classical dynamics, i.e., regular or chaotic. Considering that a large body of existing literature points to the critical role played by the nature of the classical dynamics in conductance fluctuations,1-4 our finding that the presence of magnetic field can greatly suppress this sensitivity to classical dynamics is striking.
The rest of the paper is organized as follows. Section II describes briefly the tight-binding Hamiltonian and the nonequilibrium Green’s function method to calculate the conductance for graphene quantum dots. Extensive evidence of periodic conductance oscillations and the emergence of random conductance fluctuations is presented in Sec. III. In Sec. IV, we develop a theoretical understanding of the numerical results based on the emergence of edge states and semiclassical quantization. Conclusive remarks are presented in Sec. V.

II. GRAPHENE QUANTUM DOTS AND CONDUCTANCE CALCULATION

We use the standard tight-binding framework to compute the conductances through graphene quantum dots of various geometrical shapes, where $p_x$ orbitals and nearest-neighbor hopping are assumed. The tight-binding Hamiltonian has the form

$$H = \sum_{i,j} -t_{ij}(c_i^\dagger c_j + H.c.),$$

(1)

where the summation is over all nearest-neighbor pairs and $c_j^\dagger$ ($c_j$) is the creation (annihilation) operator, $t_{ij}$ is the hopping energy from atom $j$ and to atom $i$, and the onsite energy has been set as the reference energy as it is the same for all the atoms. In the absence of magnetic field, the nearest-neighbor hopping energy is $t_{ij} = t_0 = 2.7\,\text{eV}$. When a perpendicular uniform magnetic field $B$ with vector potential $A = (-By,0,0)$ is applied, the hopping energy is altered by a phase factor:

$$t_{ij} = t_0 \exp(-i2\pi \phi_{i,j}),$$

(2)

where $\phi_{i,j} = (1/\phi_0) \int_j^i A \cdot dl$, and $\phi_0 = h/e = 4.136 \times 10^{-15}\,\text{Tm}^2$ is the magnetic flux quanta. For convenience, we use magnetic flux through a hexagonal plaque of graphene, $\phi = BS$, as a control parameter characterizing variations in the magnetic-field strength, where $S$ is the area of the hexagonal plaque composed of six carbon atoms. Thus, $S_0 = 3\sqrt{3}a_0^2/2$, where $a_0 = 1.42\,\text{Å}$. Here, we treat graphene devices as flat two-dimensional systems. Large ripples modify the hopping and can induce localization and additional transport fluctuations.

At low temperature, the conductance $G$ of a quantum-dot device is approximately proportional to transmission $T$ and is given by the Landauer formula: $G(E) = (2e^2/h)T_\infty(E)$. The standard nonequilibrium Green’s function (NEGF) method can be used to calculate the transmission, which can be expressed by

$$T(E) = \text{Tr}(\Gamma_L G_D \Gamma_R G_D^\dagger),$$

(3)

where $G_D$ is the Green’s function of the device given by $G_D = (E I - H_D - \Sigma_L - \Sigma_R)^{-1}$, $H_D$ is the Hamiltonian of the closed device, the semi-infinite leads are accounted for by the self-energies $\Sigma_L$ and $\Sigma_R$, and $\Gamma_{L,R}$ are the coupling matrices given by

$$\Gamma_{L,R} = i(\Sigma_{L,R} - \Sigma_{L,R}^\dagger).$$

(4)

The local density of states (LDS) for the device is

$$\rho = -\frac{1}{\pi} \text{Im}[\text{diag}(G_D)].$$

(5)

To be representative, we consider graphene quantum dots of three different geometric shapes: hexagonal, square, and stadium, as shown in Fig. 1. Hexagonal geometry is interesting due to the graphene lattice symmetry, i.e., the boundaries consist of zigzag edges only. Thus, regardless of the device size, the boundaries remain to be zigzag. The square geometry has both zigzag and armchair boundaries along the two perpendicular directions, respectively. The classical dynamics in these two structures are integrable. The stadium-shaped quantum dot, however, has chaotic dynamics in the classical limit, which has been used as a paradigmatic system in the quantum-chaos literature to explore various quantum manifestations of classical chaos.

The geometrical parameters of the three types of devices are as follows. For the hexagonal geometry the height (the distance between the two parallel boundaries) is $10.934\,\text{nm}$. The width of the lead is $1.136\,\text{nm}$, which is chosen somewhat arbitrarily. For the square device the width is $10.934\,\text{nm}$ and the width of the lead is $1.136\,\text{nm}$ so that the overall size is comparable to the hexagonal dot. The size of the rectangular part of the stadium dot structure is $16.898 \times 10.988\,\text{nm}$ and its lead width is $1.136\,\text{nm}$.

III. NEARLY PERIODIC CONDUCTANCE OSCILLATIONS AND EMERGENCE OF RANDOM CONDUCTANCE FLUCTUATIONS

Figures 2–4 are representative examples of conductance variations either with the Fermi energy for fixed magnetic flux or with the magnetic flux for fixed Fermi energy, for the hexagonal, square, and stadium dot shape, respectively. In all cases, a critical point can be identified unequivocally (denoted by either $E_1$ or $\phi_1$), where the conductance variations are nearly periodic on one side of the point and random on the other side. In particular, for all three geometrical shapes, for fixed magnetic flux, the conductance varies quite regularly for $E < E_1$ but randomly for $E > E_1$. For fixed Fermi energy, the conductance variations are regular for $\phi > \phi_1$ and random for $\phi < \phi_1$. Better insights into the transition from regular to random conductance variations (or vice versa) can be gained by examining the typical LDS patterns about the critical point. For example, for the hexagonal geometry, there is a circularly localized pattern at $E_1 = 0.2350\,\text{eV}$, as shown.

FIG. 1. (Color online) Schematic illustration of hexagonal-, square-, and stadium-shaped graphene quantum dots in a perpendicular magnetic field. Note that the magnetic field exists only in the device region.
At this energy, there are Landau levels located at $E_\phi = 0.005\phi_0$. The energy values of the shown LDS patterns are those of the Landau levels: $E_1 = 0.2350t$, $E_2 = 0.3395t$, $E_3 = 0.4100t$, and $E_4 = 0.4730t$, respectively. (b) Conductance versus the magnetic flux $\phi$ for fixed Fermi energy $E = 0.35t$. At this energy, there are Landau levels located at $\phi_1 = 0.0115\phi_0$, $\phi_2 = 0.0057\phi_0$, $\phi_3 = 0.0037\phi_0$, and $\phi_4 = 0.0024\phi_0$ (from large to small). The corresponding LDS patterns are also shown.

in Fig. 2(a), where the conductance of the dot structure is effectively zero due to the localization of conducting electrons inside the device. Figure 2(a) also displays several similar, recurring LDS patterns at $E_2$, $E_3$, and $E_4$. The ratios among these energy values are $E_1 : E_2 : E_3 : E_4 = 1 : 1.44 : 1.74 : 2.01 = \sqrt{2} : \sqrt{3} : 2$. We observe that the energy values are approximately proportional to $\sqrt{N}$, where $N$ is the index of $E_N$. These behaviors have also been observed for the square and stadium-shaped quantum dots. For example, Fig. 3(b) shows, for the square geometry, occurrences of the transition between regular and random conductance fluctuations at $E_1 : E_2 : E_3 : E_4 = 0.2344t : 0.3289t : 0.4021t \approx 1 : \sqrt{2} : \sqrt{3} : 2$ for fixed magnetic flux $0.005\phi_0$. The ratio is also consistent with the Landau distribution as in Eq. (6) below. In Fig. 3(b), the Fermi energy is fixed at $E = 0.4t$, and the transition points are $\phi_1 : \phi_2 : \phi_3 : 0.01508\phi_0 : 0.00756\phi_0 : 0.00501\phi_0$.

FIG. 2. (Color online) Conductances of the hexagonal-shaped quantum dot. The height of the dot is $W_D = 10.934$ nm and the lead width is $W_L = 1.136$ nm. The device region contains 4158 carbon atoms. (a) Conductance versus the Fermi energy $E_F$ for fixed magnetic field $\phi = 0.005\phi_0$. The energy values of the shown LDS patterns are those of the Landau levels: $E_1 = 0.2350t$, $E_2 = 0.3395t$, $E_3 = 0.4100t$, and $E_4 = 0.4730t$, respectively. (b) Conductance versus the magnetic flux $\phi$ for fixed Fermi energy $E = 0.35t$. At this energy, there are Landau levels located at $\phi_1 = 0.0115\phi_0$, $\phi_2 = 0.0057\phi_0$, $\phi_3 = 0.0037\phi_0$, and $\phi_4 = 0.0024\phi_0$ (from large to small). The corresponding LDS patterns are also shown.

This formula can be verified by noting that, for example, as shown in Fig. 2(b), for fixed Fermi energy $E = 0.35t$ in the hexagonal dot, varying the magnetic field also partitions the conductance curve into different regions with regular and random conductance fluctuations. The critical magnetic fluxes are $\phi_1 = 0.0115\phi_0$, $\phi_2 = 0.0057\phi_0$, $\phi_3 = 0.0037\phi_0$, and so on.
$\phi_4 = 0.0024\phi_0$, leading to the approximate ratios of $1 : 1/2 : 1/3 : 1/4$, which is consistent with Eq. (7).

IV. SEMICLASSICAL THEORY OF REGULAR CONDUCTANCE OSCILLATIONS AND UNIVERSAL TRANSITION TO RANDOM CONDUCTANCE FLUCTUATIONS

Our numerical computations indicate strongly that the emergence and properties of the Landau levels are key to understanding the origin of regular conductance oscillations in the presence of magnetic field. In fact, significant physical insights can be gained from the phenomenon of integer quantum Hall effect in semiconductor 2DEG systems, which is a direct manifestation of the evolution of the Landau levels. In that case, when the magnetic field strength is fixed and the Fermi energy is increased, the conductance reaches minimum when the Fermi energy is at a Landau level and takes on a much larger value when the Fermi energy is in between two neighboring Landau levels. This is contrary to the behavior of the density of the states, which is appreciable only at the Landau levels. The basic reason is that, for a quantum dot, at the Landau level the charge carriers tend to be localized in the central region of the dot and so cannot participate in the transport process. However, when the Fermi energy is in between two adjacent Landau levels, edge states arise which circulate around the boundary of the quantum dot, facilitating a strong coupling with the propagating modes in the semi-infinite leads and resulting in a large conductance. In our case, there is a new feature. Between two neighboring Landau levels, the energy difference $\Delta E_h$, where the subscript “h” stands for Hall effect, is enormous so that, besides the formation of the circular edge states associated with the quantum Hall effect, another class of circular edge states can be formed, as stipulated by the semiclassical Bohr-Sommerfield quantization condition. This introduces another energy period $\Delta E_q$, where “q” stands for quantization, in which the Bohr-Sommerfield edge states form and disappear. Since the circular edge states facilitate transport through the quantum dot and since $\Delta E_q$ is typically smaller than $\Delta E_h$, the fulfillment of the semiclassical quantization condition contributes to fine-scale oscillations in the conductance curve.

To exploit the Bohr-Sommerfield quantization condition for the edge states in graphene, it is convenient to modify the size of the device but keep the geometric shape unchanged. Without loss of generality, we focus on the hexagonal geometry that possesses zigzag boundaries. We choose (somewhat arbitrarily) several heights of the hexagonal devices: $W_{D1} = 19.454$ nm, $W_{D2} = 10.934$ nm, and $W_{D3} = 6.674$ nm with the relative ratio $W_{D1} : W_{D2} : W_{D3} = 2.9 : 1.7 : 1$. Figure 5 shows, for these devices, periodic conductance oscillations below the first Landau level.

Bohr-Sommerfield quantization theory stipulates that the action integral for two successive edge states satisfies the condition $\Delta L = h$, where $h$ is the Planck constant and $L = \int p \cdot dq$. In the presence of a magnetic field with vector potential $A$, the generalized momentum is $p = \hbar k + eA$ and the wave vector $k$ has the same direction as $dq$. For a given periodic orbit of length $L$, we have

$$I = |p|L = \hbar|k|L + eBS,$$

where $S$ is the area that the periodic orbit encloses in the physical space. For a fixed magnetic-field strength, we then have $\Delta L = 2\pi$, where $L$ is the length of the periodic orbit. For graphene, we have $E = h\nu_Fk$ near the Dirac point, so the relationship between the energy interval $\Delta E_q$ due to the quantization condition and the length of the periodic orbit is

$$\Delta E_q = h\nu_F/L.$$  (9)

Due to the different boundary conditions in two dimensions, we only test the ratio of the energy interval. In Figs. 5(a), 5(c), and 5(e), the energy intervals can be determined, giving the ratios $\Delta E_{q1} : \Delta E_{q2} : \Delta E_{q3} = 1/L1 : 1/L2 : 1/L3 = 1 : 1.76 : 2.92$, which are quite close to the inverse ratios of the device size $1/W_{D1} : 1/W_{D2} : 1/W_{D3} = 1 : 1.7 : 2.9$. Moreover, for $L = W_0$, we can estimate the Fermi velocity $v_F = \Delta E_qW_D/h \approx 10^6$ m/s, which is close to the Fermi velocity calculated from the dispersion curve. This means that the length of the circulating orbit is comparable to the device height, indicating that the effective diameter of the orbit is smaller than that of the device. We thus see that the regular conductance oscillations are a consequence of the Bohr-Sommerfield quantization of the edge states between two Landau levels. In particular, when the quantization condition is satisfied, a strong LDS pattern emerges at the edge of the device, as shown in Fig. 6, which bridges with the transmitting modes in the two leads and leads to the peak value $2e^2/h$ for the conductance. On the contrary, when the quantization condition is violated, edge states cannot form, giving rise to minimal conductance. Similarly, for fixed Fermi energy, or equivalently, fixed wave-vector (from the dispersion relation), the quantization condition becomes $\Delta(eBS) = h$, or

$$\Delta \phi = \Delta BS = \phi_0.$$  (10)

where $\phi_0 = h/e$ is the magnetic flux quanta. Since the edge states typically circulate the device boundaries, $S$ is
red dash lines are four Landau-levels, which divide the atoms. Colors of the contour lines represent conductance

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transmission or conductance. The ratio of the cyclotron related to the Fermi energy, the magnetic-field strength, and the magnetic field (above the first Landau level), edge states can emerge for both armchair and zigzag boundaries. In the absence of magnetic field or if the field is weak, edge states occur only at zigzag boundaries. However, under a strong magnetic field (above the first Landau level), edge states can emerge for both armchair and zigzag boundaries.

From the above analysis of the Bohr-Sommerfield quantization condition, we find that the conductance oscillations are related to the Fermi energy, the magnetic-field strength, and the size of the device. To obtain a quantitative scaling relation among those parameters, we develop the following physical analysis. Theoretically, the size of a device is related to the electron cyclotron radius at the Fermi energy, because only the electrons near the Fermi surface contribute to device transmission or conductance. The ratio of the cyclotron surrounding area and perimeter is given by

\[ S/L = k_F e^2/h, \]

(11)

where \( S/L \) can be regarded as a single parameter characterizing the device size. In a graphene system, the energy near a Dirac point is proportional to the Fermi wave-vector \( k_F \):

\[ E_F = \hbar v_F k_F \]

or in a different form as (for a given, fixed device size)

\[ S/L = \frac{2\hbar S_0}{3\hbar} \frac{\Delta E}{\Delta \phi} \].

(13)

This relation can be used to infer the characteristic size \( D \) of the device from the conductance oscillations. For example, for the hexagonal geometry, \( S = \sqrt{3}D^2/2 \) and \( L = 2\sqrt{3}D \). The scaling relation can be modified to

\[ D_{\text{hex}} = \frac{12\hbar S_0}{\sqrt{3}e\hbar} \frac{\Delta E}{\Delta \phi}, \]

(14)

which can be readily verified numerically. In particular, since the curves shown in Fig. 5 are for the edge states circulating the device, we can use \( \Delta E \) and \( \Delta \phi \) from the figure to infer the corresponding values of \( D \), which yields \( D_1 = 13.926 \text{ nm}, \)

\[ D_2 = 7.836 \text{ nm}, \text{ and } D_3 = 4.734 \text{ nm}. \]

Comparing with the actual size of the dot \( W_D \) as described in the caption of Fig. 5, we observe somewhat large discrepancies. However, if we compare the ratios, we have \( D_1 : D_2 : D_3 = 2.94 : 1.655 : 1 \), which are extremely close to the ratios of the actual dot sizes \( W_D : W_D/2 : W_D/3 = 2.915 : 1.640 : 1 \). We also see that, for the three dot sizes, the ratio \( W_D/D \) is the same, which is about 1.4. The discrepancies in the actual size are caused by the approximation in Eq. (11) and by the assumption that the diameter of the circulating orbits is equal to the device size. Nevertheless, since the estimated values of \( D \) and \( W_D \) are of the same order of magnitude, it can be used to infer the dot size from the conductance oscillations versus the Fermi energy and the magnetic flux, which can be used as corroborative evidence and be compared with other direct/indirect measurements.

The scaling relation (14) may be feasibly observed experimentally in graphene quantum dots because, for low Fermi energy, the underlying phenomenon emerges even when the applied magnetic field is weak, i.e., \( \phi \to 0 \). For conventional semiconductor 2DEG systems with a parabolic energy-momentum relation, similar scaling can in principle be observed but only for enormous magnetic field, as we have verified numerically. In particular, for a graphene quantum dot of size \( D \sim 1 \mu \text{m} \), the minimally required magnetic-field strength to observe the periodic conductance oscillations is about \( 3T \). While for a 2DEG device of the same size as \( 1 \mu \text{m} \) made of GaAs/AlGaAs heterostructure, the minimum magnetic field required is about \( 10T \).

To obtain a global view of the conductance oscillations/fluctuations in terms of a combination of Eqs. (6) and (7), we overlay the Landau levels on top of the contour plot of the conductance versus both energy \( E \) and magnetic flux \( \phi \) for the hexagonal dot, as shown in Fig. 6. We see that the Landau levels divide the whole parameter space of \((E, \phi) \) into different regions with behaviors ranging from regular, parallel line patterns to complicated irregular patterns. We have analyzed the case that the Fermi energy is below the first Landau level, where the edge states recur with the period \( \Delta E_q \), leading to regular conductance oscillations of the same energy period. For \( E_F > E_1 \), there are two sets of edge states, leading to two uncorrelated repetitive patterns, each with its own period \( \Delta E_q \). This is also manifested in Fig. 6 for the hexagonal dot that, in region 2 (between the first and the second Landau levels), there are two sets of conductance lines: one with the same slope as in region 1 (the overlapped gray lines) and

\[ S/L = \frac{2\hbar S_0}{3\hbar} \frac{E}{\phi}, \]

(12)

FIG. 6. (Color online) A hexagonal geometry device with 4158 atoms. Colors of the contour lines represent conductance \( G/G_0 \). The red dash lines are four Landau-levels, which divide the atoms. Colors of the contour lines represent conductance.
another with a larger slope (brown lines) that originates in this region but persists in regions between higher Landau levels. In region 3 a new pattern appears, as indicated by the blue dashed lines in Fig. 6. The corresponding edge states are also shown in Fig. 6 for these typical line segments. We see that, for a fixed magnetic flux, as the Fermi energy is increased across a Landau level, a new set of edge states appears, adding a new set of line segments in the conductance plot. Since the energy period \( \Delta E_\phi \) is uncorrelated for different types of edge states, as can be seen from Fig. 6, the conductance will fluctuate randomly when there are many sets of edge states. This explains the transition from regular conductance oscillations to random conductance fluctuations, as shown in Figs. 2(a), 3(a), and 4(a). A similar analysis can be carried out when the magnetic flux is varied [Figs. 2(b), 3(b), and 4(b)]. Since the transition is caused by the crossing of Landau levels and the variation of the edge states, it holds regardless of the detailed geometric shape of the quantum dot and the nature of the underlying classical dynamics, i.e., integrable or chaotic. The transition can thus be characterized as universal.

While our discussion has been focused on the hexagonal dot, here we briefly show that the same mechanism leading to regular conductance oscillations and the transition to random fluctuations holds for other geometries as well. To demonstrate this in a comprehensive manner, we show in Fig. 7 the conductance in the \((\phi, E)\) plane for all three cases. We see that the conductance is symmetric with respect to reversal of the magnetic flux \( \{T(\phi) = T(-\phi)\} \) due to the two-terminal characteristic of our device. The patterns of the conductance oscillations and fluctuations for the three cases are apparently similar, due to the fact that the patterns are all partitioned by the Landau levels [e.g., Eq. (6)] that do not depend on the geometric details of the device. However, the fine structures can be different. First, below the first Landau level, the slopes of the line patterns indicate the size of the device because the edge states are exactly circulating the “edge” of the device (Fig. 6), which are slightly different for the three cases. Second, above the first Landau level, the details of the conductance patterns are more distinct. This is because, in contrast to the edge states below the first Landau level, these states are now more dispersive and also depend on the characteristic energy scale in the conductance-fluctuation pattern of the underlying quantum dot tends to be much larger, leading to a smoother variation. For quantum dots with integrable or mixed dynamics, there are sharp resonances in the conductance-fluctuation curves. This can be seen, e.g., from the sudden change of the color scale from blue to red, or vice versa, in Fig. 7(b) for \( \phi \sim 0 \). [In the chaotic case, the change in the color scale is much more smooth, as shown in Fig. 7(c)]. In addition, in the chaotic graphene quantum dot, there is level repelling, which can also be seen from Fig. 7(c) in the \( \phi \sim 0 \) regime, where the conductance lines tend to avoid each other, a feature that is absent in both Figs. 7(a) and 7(b).

V. CONCLUSION

Previous works on conductance fluctuations associated with transport through nanoscale, quantum-dot systems emphasized the difference between situations where the underlying classical dynamics are chaotic or integrable. A general understanding is that Fano-type\(^{31}\) of sharp resonances typically occur in dot systems with integrable classical dynamics, and chaos can effectively smooth out these resonances quantum-mechanically. This picture holds for both 2DEG and graphene systems in which the quantum dynamics are nonrelativistic and can be relativistic, respectively, and it has been suggested recently\(^{32}\) that altering classical chaos can effectively modulate quantum transport in terms of conductance-fluctuation patterns.

We find that the presence of magnetic field can alter the existing understanding of the quantum manifestations of classical chaos in that the difference in the quantum transport as caused by different types of classical dynamics can diminish. As a result, universal behaviors emerge. The remarkable phenomenon has been observed in graphene quantum dots of integrable and chaotic geometries. In particular, the conductance curves contain both regular oscillations and random fluctuations, and the transition is caused by the emergence of new edge states when crossing the Landau levels. In the region of regular oscillation, the periods in the Fermi energy and in magnetic flux are related to the size of the device in a universal
manner, regardless of the nature of the corresponding classical dynamics. The key to this universal scaling is the quantization of classically circulating edge orbits, which does not depend on the specific details of the geometrical shape of the dot. The details do appear in the fine-scale variations, where the random conductance fluctuations are typically smoother when the classical dynamics is chaotic.


27For semiconductor 2DEG systems, we have $E = \hbar^2 k^2 / 2m^*$ and $D = k^2 x_0^2$. The scaling relation becomes $D = \sqrt{2m^* E / e B}$, where $m^*$ is the effective mass of the electron.


