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Dynamical origin of low frequency fluctuations in external cavity semiconductor lasers

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Abstract

Low-frequency fluctuations (LLFs) in semiconductor lasers subject to time-delayed optical feedback from external reflectors have been experimentally observed and theoretically studied for the past two decades. Yet recently a question has come into focus: whether these fluctuations are only a transient phenomenon. This Letter addresses this issue from the viewpoint of deterministic chaotic transitions. In particular, we investigate the single-mode single-delay Lang–Kobayashi equations by constructing a scheme that allows for a detailed bifurcation analysis and, consequently, an understanding of the coexistence of LFFs and stable emission on the *maximum gain mode*, the latter's being the subject of recent experiments [Phys. Rev. A 60 (1999) 634]. Our computations suggest that LFFs can be either *transient* or *sustained*, depending on the dynamical interplay between the maximum gain mode and a chaotic set. © 2000 Elsevier Science B.V. All rights reserved.

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A well-known phenomenon in the study of semiconductor lasers subject to optical feedback is *low-frequency fluctuations* (LFF) [1–12], which take place when a semiconductor laser in biased close to the solitary laser threshold. In such a case, the power of the laser exhibits irregular dropout events at an average frequency that is about three orders of magnitude smaller than that of oscillations of the laser field. Typically, the fast dynamics take place at GHz frequencies and the average frequency of the dropout events is of the order of a few MHz. Although LFF appear to be random, for which modeling based on stochastic assumptions may be natural, a first key step is to seek an understanding of the phenomenon by exploring its deterministic origin [6]. In particular, chaos is expected to occur commonly because, a semiconductor laser in the presence of optical feedback represents a highly nonlinear dynamical system. LFF pose a serious problem in applications where a constant mean square power output from the laser is

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desired. The paramount interest in recent years in these systems and associated phenomena comes form efforts to apply stabilization techniques [13] and from applications in communications [14].

One important recent observation in the study of semiconductor lasers is that, in some regions of the parameter space, LFF coexist with stable emission on the *maximum gain mode* (MGM) [15–17]. The implication is then that LFF may only be a transient phenomenon in the sense that the laser output would eventually be stabilized after suffering an initial time period of power drops. Apparently, if this were true, then LFF would not be a serious problem in applications. We then ask: (1) whether or not LFF can be transient, and (2) if yes, how common do we expect LFF to be transient?

The aim of this Letter is to address the above questions by investigating the Lang–Kobayashi equations [18]. These equations combine a phenomenological description of the laser gain medium with a first principles wave equation for the electric field. In the past, they have been successfully used in modeling a variety of laser structures exposed to optical feedback, Fabry–Perot [19], distributed-feedback [20], and vertical-cavity surface-emitting devices [21]. Our approach will be to perform a detailed numerical bifurcation analysis, in conjunction with qualitative arguments, to understand LFF from the perspective of deterministic chaotic dynamics. Our principal result is that LFF can be either transient or sustained, depending on the dynamical interplay between invariant sets that physically correspond to different operational modes of the laser.

We begin by briefly describing the Lang–Kobayashi equations [18] and discussing their suitability for modeling the phenomenon of LFF in semiconductor lasers subject to optical feedback. The equations are:

$$\frac{\mathrm{d}\mathscr{E}(t)}{\mathrm{d}t} = \frac{1+\mathrm{i}\alpha}{2}\left(G(N,\mathscr{E}) - \tau_{\mathrm{p}}^{-1}\right)\mathscr{E}(t) + \gamma \mathrm{e}^{-\mathrm{i}\omega_0\tau_{\mathrm{ext}}}\mathscr{E}(t-\tau_e)\,,$$
$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = J - \frac{N(t)}{\tau_{\mathrm{s}}} - G(N,\mathscr{E})\,|\mathscr{E}(t)|^2\,, \qquad (1)$$

where $\mathscr{E}(t)$ and $N(t)$ are the laser intracavity complex electric field and the carrier population, respectively, α is the linewidth enhancement factor [22,23], ω_0 is the laser emission frequency in the absence of feedback, τ_{p} is the photon lifetime, τ_{s} is the carrier lifetime, and J is the injected current density. The external cavity parameters are the feedback strength γ that measures the intensity of the light reflected back into the laser cavity, and the delay time $\tau_{\mathrm{ext}} = 2L_{\mathrm{ext}}/c$, which is the round-trip time of the light in the external cavity of length L_{ext}. This model assumes single-mode operation of the solitary semiconductor laser and neglects multiple reflections from the external mirror.

The modal gain per unit time $G(N,\mathscr{E}) = G_0(N)(1 - \varepsilon|\mathscr{E}|^2)$ contains the linear gain $G_0(N) = G_N(N - N_0)$ where G_N is the gain constant and N_0 is the carrier density at transparency, and intensity reduction of the gain due to spatial and spectral hole burning and carrier heating, with ε being the gain saturation coefficient [24,25]. The effect of nonlinear gain saturation was extensively studied by Masoller [25] and was found to lead to the stabilization of the external cavity modes (ECM), so that the onset of chaos and chaotic transitions is shifted towards higher feedback levels. Since the purpose of this work is to investigate chaotic transitions in the system with delayed feedback, we will ignore the saturation effects and set $\varepsilon = 0$.

The electric field is normalized so that $V_{\mathrm{c}}|\mathscr{E}(t)|^2$ is the total photon number in the laser wave guide, where V_{c} is the volume of the active region. The parameter $J_{\mathrm{th}} = N_{\mathrm{th}}/\tau_{\mathrm{s}}$ is the lasing threshold current density of a solitary laser and N_{th} is the threshold carrier density. Typical values for the above parameters are: (1) linewidth enhancement factor $\alpha = 3$–7, (2) photon lifetime $\tau_{\mathrm{p}} \sim 1$ ps, (3) carrier lifetime $\tau_{\mathrm{s}} \sim 1$ ns, (4) threshold current $J_{\mathrm{th}} \approx 2.0 \times 10^{33}$ m^{-3} s^{-1}, (5) modal gain coefficient $G_N \approx 1.0 \times 10^{-12}$ m^3 s^{-1}, (6) carrier density at transparency $N_0 \approx 1.0 \times 10^{24}$ m^{-3}, (7) volume of the active region $V_{\mathrm{c}} \sim 1.0 \times 10^{-16}$ m^3, (8) feedback level $\gamma = 0$–30×10^9 s^{-1}), and (9) delay time $\tau_{\mathrm{ext}} = 1$–10 ns. Despite recent discussions about the importance of multimode behavior for the description of LFF [11], we think that Eq. (1) is suitable for our study, since in the experiments LFF persist under single mode operation as it was described in a recent experiment where a distributed-feedback semiconductor was used to ensure single-mode operation [20].

In our numerical simulations, we measure time in units of the photon lifetime and introduce the normalized excess carrier number density $n(t) \sim N(t) - N_{th}$. The dynamical system, Eqs. (1) can then be written in the following normalized form [26]:

$$\frac{d\mathscr{E}(t)}{dt} = (1 + i\alpha) n(t)\mathscr{E}(t) + \eta e^{-i\omega_0\tau}\mathscr{E}(t-\tau),$$

$$T\frac{dn(t)}{dt} = P - n(t) - [2n(t) + 1]|\mathscr{E}(t)|^2, \qquad (2)$$

where, $\eta = \tau_p\gamma$, $T = \tau_s/\tau_p$, and $P \sim (J/J_{th}) - 1$, $\tau = 2L_{ext}/c\tau_p$. The above equations describe dynamics of three independent real variables, since $\mathscr{E}(t) = \mathscr{E}_R(t) + i\mathscr{E}_I(t)$. However, the phase space associated with Eq. (2) is of infinite dimension, since, to integrate these equations starting at $t = 0$, it is necessary to define initial conditions for the three variables everywhere in the time interval $-\tau \le t < 0$.

The stationary solutions of Eq. (2) can be written as: $\mathscr{E}(t) = E_s e^{i\omega_s t}$ and $n(t) = n_s$. Writing $\mathscr{E}(t) = E(t)e^{i\phi(t)t}$ and introducing the phase delay variable: $\Delta(t) = \phi(t) - \phi(t-\tau) + \omega_0\tau$, we see that the stationary solutions are the fixed points in the configuration space of the three variables $E(t)$, $n(t)$, and $\Delta(t)$. We have: $n_s = -\eta\cos\Delta_s$ and $E_s^2 = (P - n_s)/(1 + 2n_s)$, where $\Delta_s \equiv (\omega_s + \omega_0)\tau$ are the solutions to the implicit equation:

$$\Delta_s - \omega_0\tau = -\eta\tau\sqrt{\alpha^2 + 1}\sin(\Delta_s + \tan^{-1}\alpha). \qquad (3)$$

At low feedback levels this equation has only one solution, which is close to the solitary laser frequency ω_0. With increasing η, additional solutions appear in pairs. For low pumping $(P < \eta)$ some of the solutions are not allowed because of the condition $E_s \ge 0$. A stability analysis [27] shows that one of the fixed points is stable and is thus identified as an external cavity mode (ECM) of the laser, while another is an unstable saddle point, often called an *antimode*, which physically corresponds to destructive interference between the external cavity and laser fields [7].

We study the dynamics of Lang–Kobayashi equations using Adams–Bashford–Moulton predictor-corrector method [28]. We set parameter values $\alpha = 6$, $\tau = 1000$, $\omega_0\tau = -1$, $T = 1000$, $P = 0.001$ and vary the feedback parameter η. The choice of these parameters is consistent with a recent experiment in which bifurcation cascades were recorded for an external cavity of 15 cm [15]. A large variety of different chaotic transitions occur with increasing feedback level as shown in Fig. 1. Each new ECM is stable upon creation, but then becomes unstable due to Hopf bifurcation at higher η and is replaced by a limit cycle. As the feedback is further increased, the cycle undergoes a period-doubling bifurcation at $\eta \approx 13.25 \times 10^{-4}$ [Fig. 1(a)] followed by a cascade of bifurcations leading to a chaotic attractor at $\eta = 13.5 \times 10^{-4}$, as shown in Fig. 1(b). This attractor grows in size, occupying more space between the neighboring saddle points until at $\eta \approx 14.08 \times 10^{-4}$ it merges with the attractor ruins of another ECM, which were inaccessible at a lower feedback level [Fig. 1(c)]. At the same time, the attractor gets closer to the saddle separating it from the basin of the maximum gain mode (MGM), and at $\eta \approx 14.34 \times 10^{-4}$ a boundary crisis [29,30] occurs: after spending some time on the chaotic attractor, the trajectory escapes across the saddle to the MGM [Fig. 1(d)]. Thus, at this feedback level the chaotic behavior of the system is *transient* and, after a finite time interval, is replaced by a limit cycle around the MGM.

This sequence of transition events (stable ECM → ECM attractor → merging with lower gain attractor ruins → transient towards MGM) repeats qualitatively for higher feedback levels until a large attractor containing many ECMs is created. LFF, also known as the Sisyphus effect [7], occur when the system evolves on this large attractor made of the attractor ruins associated with many ECMs. We call this large attractor the *Sisyphus attractor*. In order to distinguish between sustained and transient LFF, we need to consider the basin of attraction of both MGM and Sisyphus attractors: if the basin of the MGM attractor contains part of the Sisyphus attractor, then LFF will only be transient. Strictly speaking, in this case, the term ‘Sisyphus attractor’ is no longer valid because it is only a transient. In chaos theory, the event when two previously isolated basins begin to connect with each other is called *crisis* [29,30]. If, on the other hand, the MGM attractor is unstable and becomes part of the Sisyphus attractor then the corresponding LFF are sustained.

To determine where in parameter space the LFF phenomenon is sustained or transient, the most direct

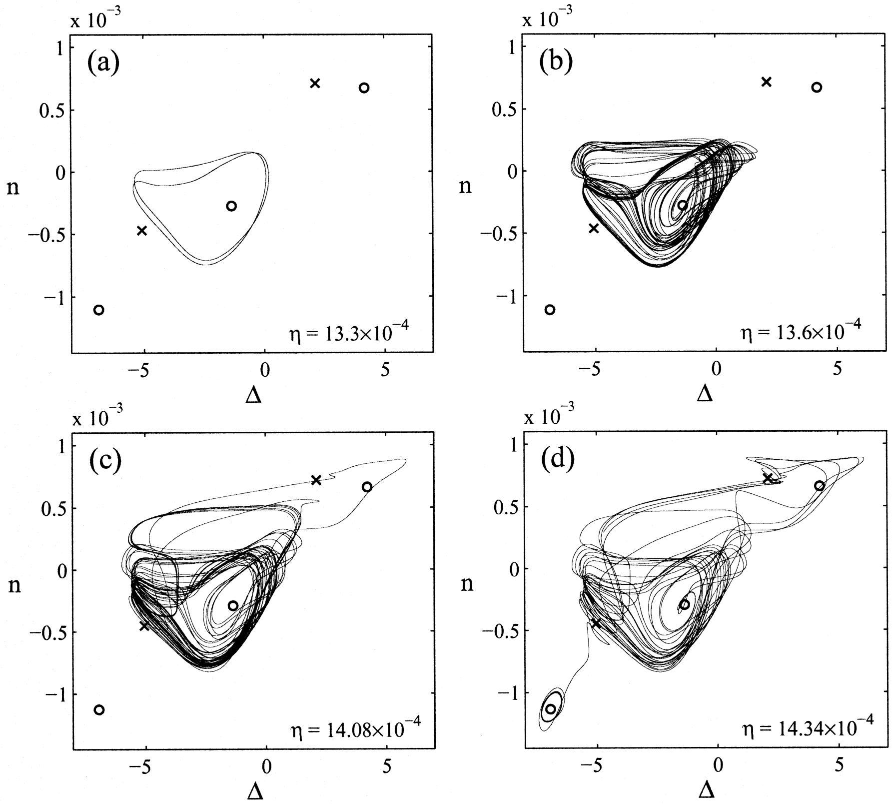

Fig. 1. Different stages in the evolution of the external cavity mode (ECM) attractor: (a) limit cycle after a period-doubling bifurcation; (b) chaotic attractor; (c) merger of the ECM attractor with attractor ruins of the neighboring ECM; (d) boundary crisis of the chaotic attractor. The crosses (×) designate saddles, and the circles (∘) denote nodes of the external cavity. In all cases the integration is initialized at the ECM and the trajectory between 100 and 250τ is plotted. The duration of the transient in (d) is 207τ.

way is to study the bifurcation diagram of the attractors. Recall that in low-dimensional chaotic systems, a bifurcation diagram typically contains parameter regions for chaotic attractors and for periodic windows [31]. Chaos on attractors is sustained but it is only transient in periodic windows. We thus seek to construct bifurcation diagrams for the Lang–Kobayashi equations. A major difficulty is the extremely high dimensionality involved in the system. It is thus necessary to carefully select a Poincaré surface of section and a variable that can adequately represent the system evolution. Since our main focus is on transitions between attractor ruins of different ECMs, we define the Poincaré section in such a way, that it passes through all the fixed points of the system. Namely, the point (E_i, n_i, Δ_i) is on the surface of section if $E_i^2 = (P - n_i)/(1 + 2n_i)$. The variable Δ_i provides the best view of the Sisyphus attractor since ECMs are separated by approximately 2π. Another complication in the construction of a bifurcation diagram is that, depending on the feedback level, the laser dynamics may have several attractors separated by the basin boundaries. The appearance of the bifurcation diagram depends then on the choice of the initial condition. That is, only attractors whose basin contains the initial point will appear on the bifurcation diagram.

The bifurcation diagram shown in Fig. 2 was constructed as follows. We fix the parameters $\alpha = 4$, $T = 1000$, $P = 0.001$, $\omega_0\tau = -1$, $\tau = 1000$, and vary η in increments of 1.0×10^{-5} in the range from zero to the value at which 20 ECMs exist. At a given

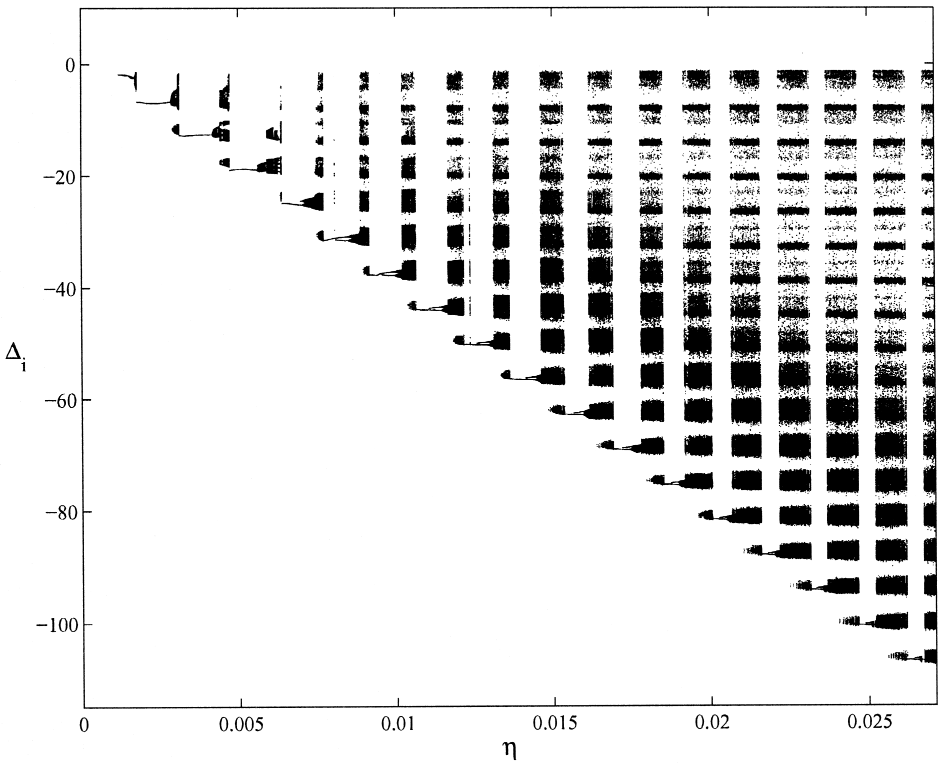

Fig. 2. Bifurcation diagram of the delay coordinate on the Poincaré surface. The system parameters are $\tau = 1000$, $\omega_0\tau = -1$, $T = 1000$, $P = 0.001$, and $\alpha = 4$. The 'window-like' structure reveals alternating regions of sustained and transient LFF.

value of η we find all stationary solutions and initialize the integration of Lang–Kobayashi equations at one of the ECMs inside the Sisyphus attractor. After a 500τ pre-iteration run, we record values of the phase delay coordinate Δ_i at points of intersection with the Poincaré surface during the following time period of 2000τ. The key feature of Fig. 2 is the appearance of 'window-like' structure in the diagram. In these 'windows' the attractor is localized around the MGM and LFF are transient. In each window, the MGM attractor goes through a cascade of period-doubling bifurcations, becomes chaotic, and eventually collides with the transient chaotic set, leading to LFF. After this, the MGM becomes part of a larger Sisyphus attractor, and LFF are then sustained until the a new maximum gain mode becomes stable.

In order to contrast the transient and sustained characters of the Sisyphus effect at different values of the feedback parameters, we show in Fig. 3 two cases from the opposite sides of one of the transient windows in Fig. 2. The dynamics in both cases appear similar, except that for Fig. 3(a), a trajectory initiated in the middle of the Sisyphus 'attractor' gets trapped around MGM after 226τ. In this case, the corresponding LFF are transient. Fig. 3(b) shows the Sisyphus attractor at a slightly higher feedback level. We see that even though the trajectory gets close to the MGM, it does not remain there, but returns back to the Sisyphus attractor, thus sustaining LFF and the power dropout events. For this level of feedback, the MGM is part of the Sisyphus attractor. Consequently, trajectory starting from most initial conditions remain on the Sisyphus attractor so that LFF are sustained.

In summary, we have proposed a bifurcation analysis scheme for studying deterministic chaotic dynamics of semiconductor lasers subject to optical

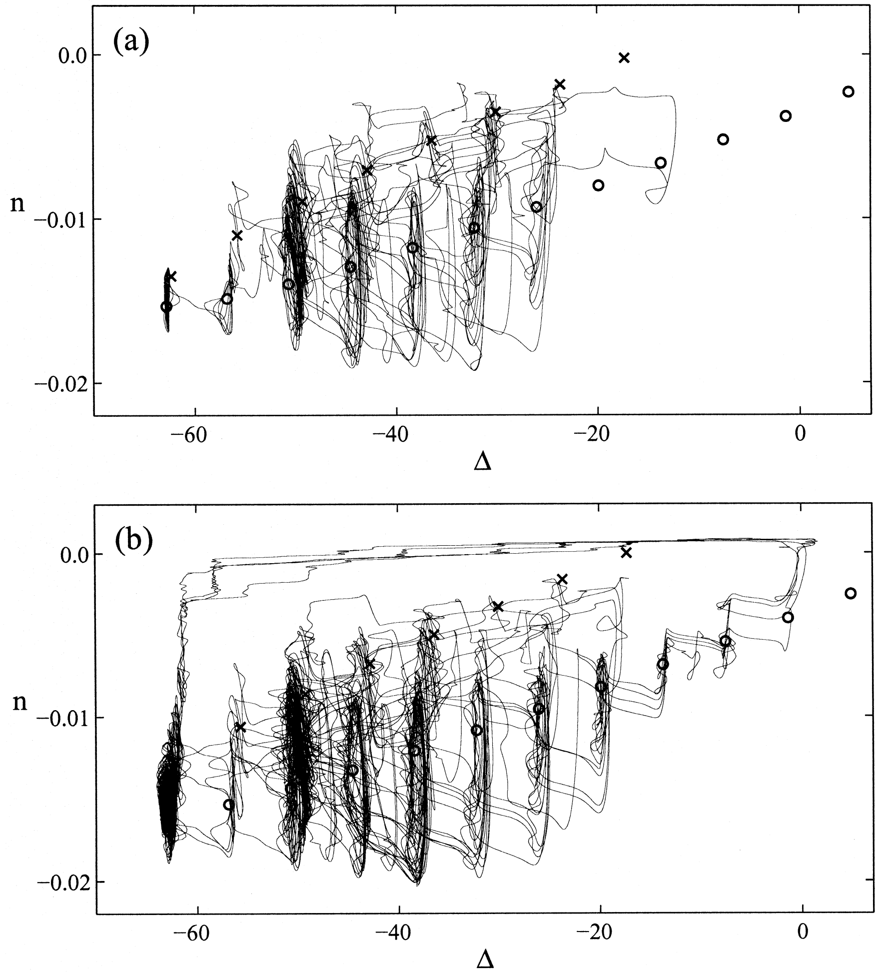

Fig. 3. Transient and sustained LFF: (a) $\eta = 0.01536$, the trajectory is initialized in the Sisyphus attractor and escapes to the maximum gain ECM after 226τ; (b) $\eta = 0.01624$, the trajectory is initialized at the MGM and remains on the Sisyphus attractor. Here also, the crosses ($\times$) designate saddles, and the circles ($\circ$) denote nodes. In both cases the trajectory between 100τ and 500τ is shown.

feedback. By analyzing the regime of coexistence of stable emission and low-frequency fluctuations, we find a window-like structure in which sustained LFF regions alternate with transient LFF regions, in latter case the asymptotic dynamical state is stable emission generated by the maximum gain mode. Evidently, the probability of observing LFF or stable emission depends on the relative widths of the windows. The study of how the window width depends on the system parameters will be the subject of future investigation.

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