Predicting and Controlling Tipping Point in Complex Networked Systems

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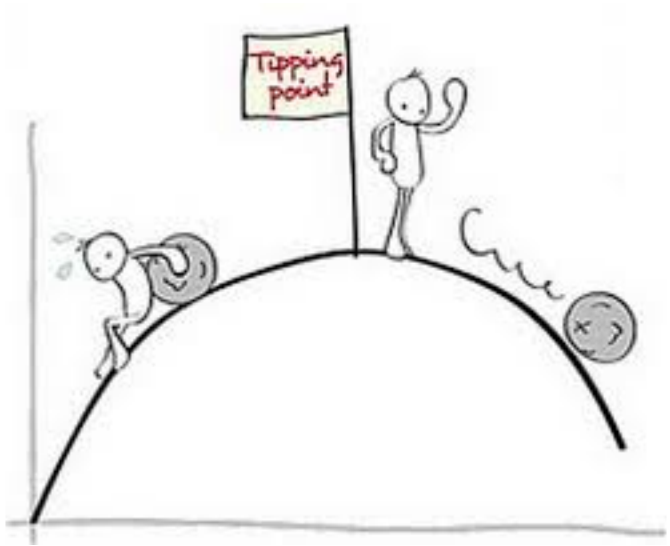
Tipping Point

Google: the point at which a series of small changes or incidents becomes significant enough to cause a larger, more important change.

Merriam-Webster: the critical point in a situation, process, or system beyond which a significant and often unstoppable effect or change takes place



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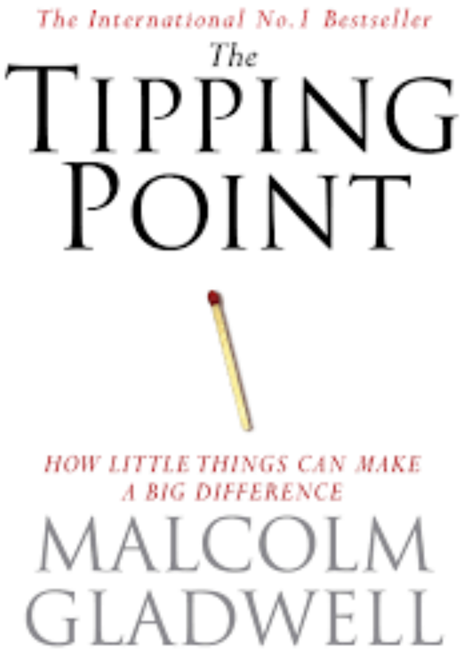
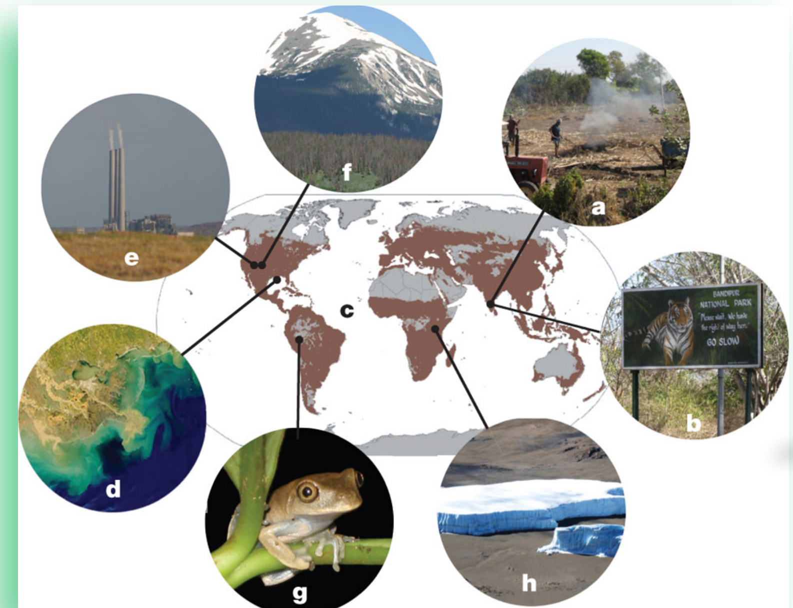
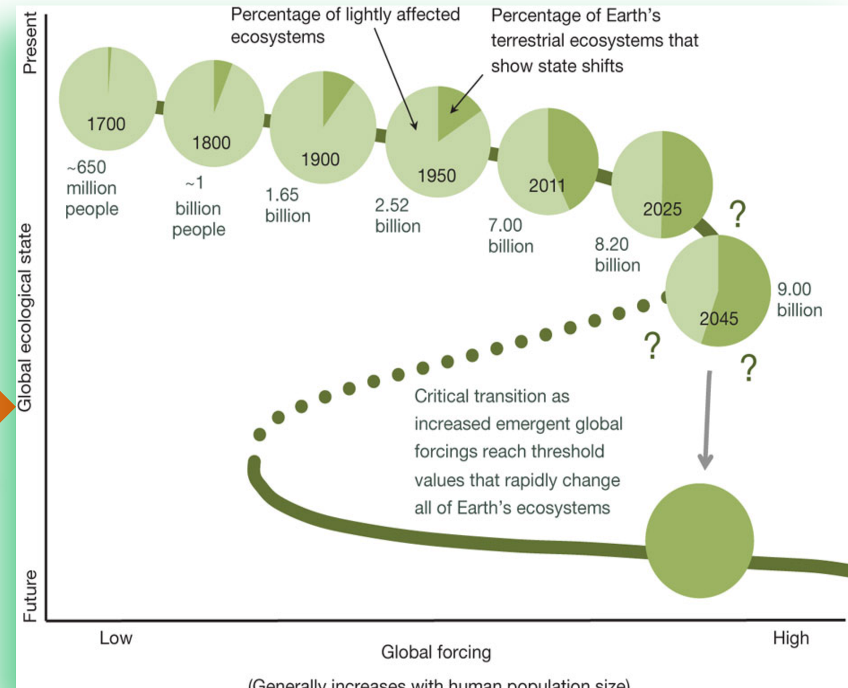


Tipping point: Prediction & Control?





Barnosky, Anthony D., et al. *Nature* **486**, 52-58 (2012).

Plant-pollinator network with complex mutualistic interactions

ECOLOGY LETTERS

Ecology Letters, (2014) **17**: 350–359 doi: 10.1111/ele.12236

LETTER

The sudden collapse of pollinator communities

J. Jelle Lever,[1,2]* Egbert H. van Nes,[1] Marten Scheffer,[1] and Jordi Bascompte[2]

Abstract

Declines in pollinator populations may harm biodiversity and agricultural productivity. Little attention has, however, been paid to the systemic response of mutualistic communities to global environmental change. Using a modelling approach and merging network theory with theory on critical transitions, we show that the scale and nature of critical transitions is likely to be influenced by the architecture of mutualistic networks. Specifically, we show that pollinator populations may collapse suddenly once drivers of pollinator decline reach a critical point. A high connectance and/or nestedness of the mutualistic network increases the capacity of pollinator populations to persist under harsh conditions. However, once a tipping point is reached, pollinator populations collapse simultaneously. Recovering from this single community-wide collapse requires a relatively large improvement of conditions. These findings may have large implications for our view on the sustainability of pollinator communities and the services they provide.

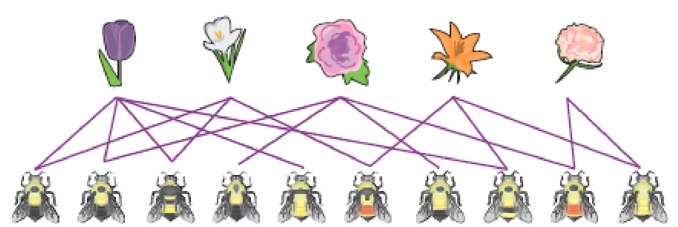
Keywords

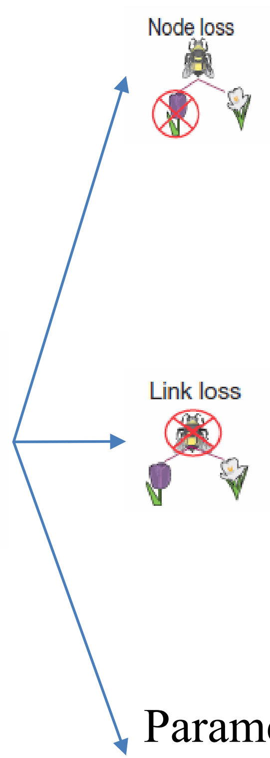
Critical transitions, hysteresis, mutualistic networks, nestedness, pollinator decline.

Perturbation Types

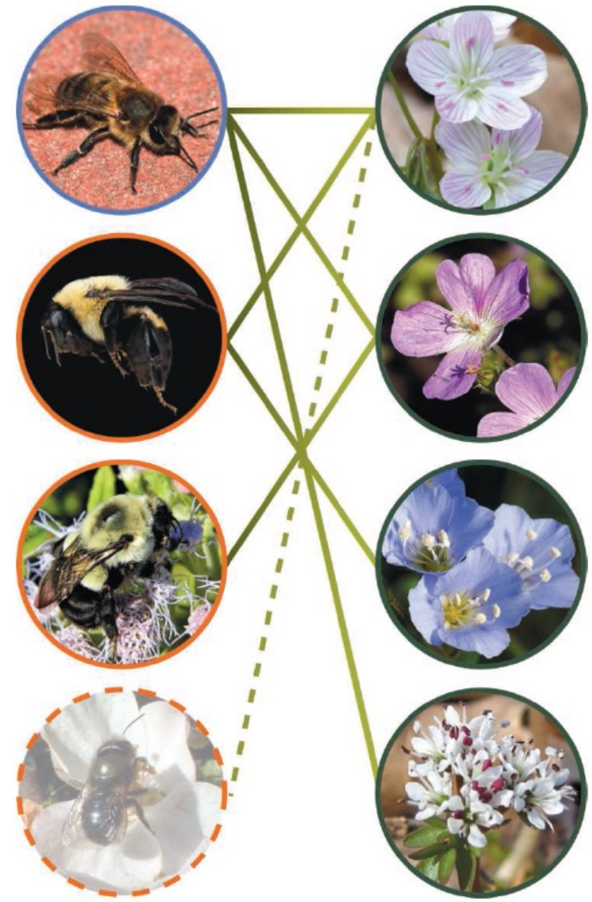
Cause of perturbation: global warming caused climate change, excessive use of pesticides leading to death of pollinators, loss of habitats due to pollution, etc.

Bipartite mutualistic network

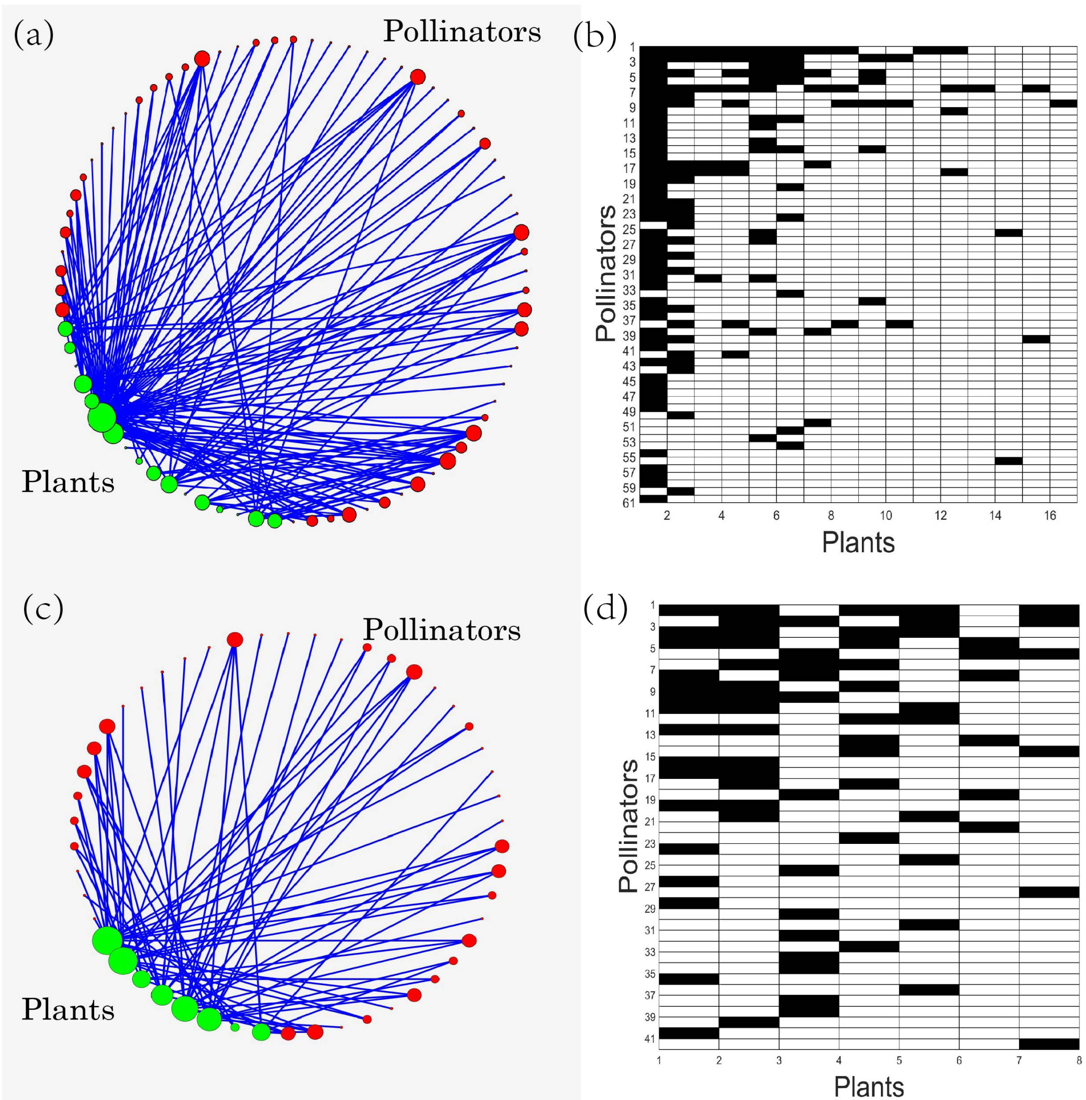




Parameter change



Empirical Data



Network A: Data from Hicking, Norfold, UK - 61 Pollinators, 17 plants, and 146 mutualistic interactions [L. Dicks, S. Corbet, and R. Pywell, "Compartmentalization in plant-insect flower visitor web," *J. Anim. Ecol.* **71**, 32-43 (2002)]

Network B: Data from Hestehaven, Denmark – 42 pollinators, 8 plants, and 79 mutualistic connections [A. C. Montero, "The ecology of three pollinator network," Master thesis, Aarhus University, Denmark (2005)]

Data from 59 such networks are currently available: ***http://www.web-of-life.es***

Nonlinear Network of Mutualistic Interactions

$$\frac{dP_i}{dt} = P_i\left(\alpha_i^{(P)} - \sum_{j=1}^{S_P}\beta_{ij}^{(P)}P_j + \frac{\sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j}{1 + h\sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j}\right)_i + \mu_P,$$

$$\frac{dA_i}{dt} = A_i\left(\alpha_i^{(A)} - \kappa_i - \sum_{j=1}^{S_A}\beta_{ij}^{(A)}A_j + \frac{\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j}{1 + h\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j}\right) + \mu_A,$$

Holling type-II dynamics

$\gamma_{ij} = \varepsilon_{ij}\dfrac{\gamma_0}{(k_i)^t}$, $0 \le t \le 1$ ($t = 0$: structure has no effect; $t = 1$: structure is important)

$\varepsilon_{ij} = 1$ if plant/pollinator i and pollinator/plant j are connected; 0 otherwise;

P_i, A_i – Abundance of ith plant and ith pollinator;

S_P, S_A – numbers of plants and pollinators;

$\alpha_i^{(P)}, \alpha_i^{(A)}$ – intrinsic growth rates of ith plant and ith pollinator;

β_{ii}, β_{ij} – intraspecific and interspecific competition strength ($\beta_{ii} >> \beta_{ij}$);

μ_P, μ_A – immigration of plants and pollinators;

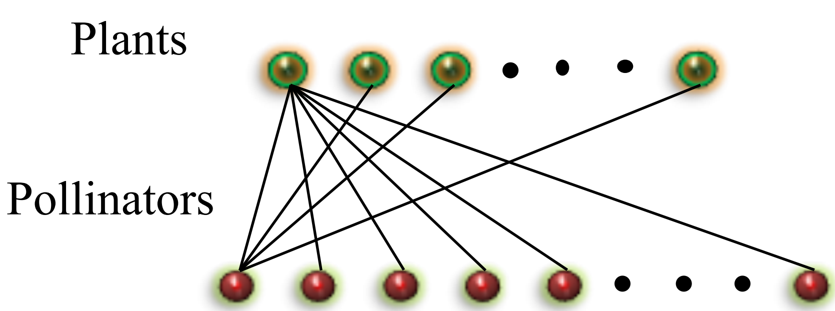
γ_0 – strength of mutualistic interaction;

κ_i – pollinator decay rate - bifurcation parameter

} Possible control parameters

- Lever, Nes, Scheffer, and Bascompte, "The sudden collapse of pollinator communities," *Ecol. Lett.* **17**, 350-359 (2014)
- Rohr, Saavedra, and Bascompte, "On the structural stability of mutualistic systems," *Science* **345**, 1253497 (2014).
- J.-J. Jiang, Z.-G. Huang, T. P. Seager, W. Lin, C. Grebogi, A. Hastings, and Y.-C. Lai, "Predicting tipping points in mutualistic networks through dimension reduction," *PNAS (Plus),* in press

Derivation of 2D Dynamical System (1)



$$\frac{dP_i}{dt} = P_i\left(\alpha_i^{(P)} - \sum_{j=1}^{S_P}\beta_{ij}^{(P)}P_j + \frac{\sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j}{1+h\sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j}\right)_i + \mu_P,$$

$$\frac{dA_i}{dt} = A_i\left(\alpha_i^{(A)} - \kappa_i - \sum_{j=1}^{S_A}\beta_{ij}^{(A)}A_j + \frac{\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j}{1+h\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j}\right) + \mu_A,$$

$$\gamma_{ij} = \varepsilon_{ij}\frac{\gamma_0}{(k_i)^t}$$

Step 1:

$$\alpha_i^{(P)}P_i \cong \alpha P_{eff}$$

$$\alpha_i^{(A)}A_i \cong \alpha A_{eff}$$

Step 2:

$$\beta_{ii}^{(A)} >> \beta_{ij}^{(A)},\ \beta_{ii}^{(P)} >> \beta_{ij}^{(P)}$$

$$\rightarrow \sum_{j=1}^{S_A}\beta_{ij}^{(A)}A_jA_i \approx \beta_{ii}^{(A)}A_i^2 \cong \beta A_{eff}^2$$

$$\sum_{j=1}^{S_P}\beta_{ij}^{(P)}P_jP_i \approx \beta_{ii}^{(P)}P_i^2 \cong \beta P_{eff}^2$$

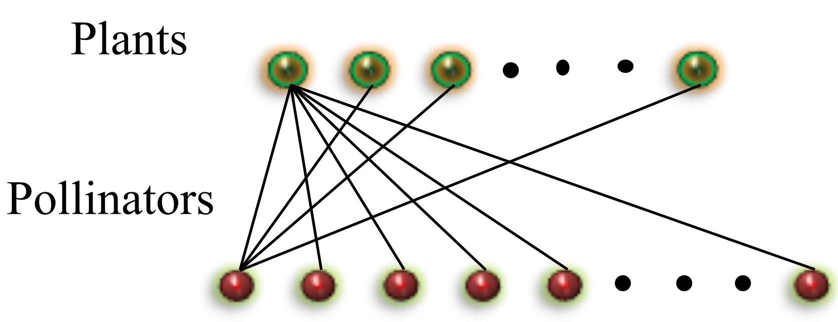
Step 3:

$$\gamma_{ij} = \varepsilon_{ij}\frac{\gamma_0}{(k_i)^t}$$

$$\rightarrow \sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j = \sum_{j=1}^{S_A}\frac{\gamma_0}{k_{P_i}^t}\varepsilon_{ij}A_j \cong \gamma_0 k_{P_i}^{1-t}A_{eff}$$

$$\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j = \sum_{j=1}^{S_P}\frac{\gamma_0}{k_{A_i}^t}\varepsilon_{ij}P_j \cong \gamma_0 k_{A_i}^{1-t}P_{eff}$$

Derivation of 2D Dynamical System (2)



$$\frac{dP_i}{dt} = \alpha P_{eff} - \beta P_{eff}^2 + \frac{\gamma_0 k_{P_i}^{1-t} A_{eff}}{1 + h\gamma_0 k_{P_i}^{1-t} A_{eff}} P_{eff} + \mu_P,$$

$$\frac{dA_i}{dt} = \alpha A_{eff} - \beta A_{eff}^2 - \kappa A_{eff} + \frac{\gamma_0 k_{A_i}^{1-t} P_{eff}}{1 + h\gamma_0 k_{A_i}^{1-t} P_{eff}} A_{eff} + \mu_A$$

Averaging - Method 1:

$$\langle \gamma_P \rangle = \frac{\sum_{i=1}^{S_P} \gamma_0 k_{P_i}^{1-t}}{\sum_{i=1}^{S_P} 1}, \quad \langle \gamma_A \rangle = \frac{\sum_{i=1}^{S_A} \gamma_0 k_{A_i}^{1-t}}{\sum_{i=1}^{S_A} 1}$$

Averaging - Method 2:

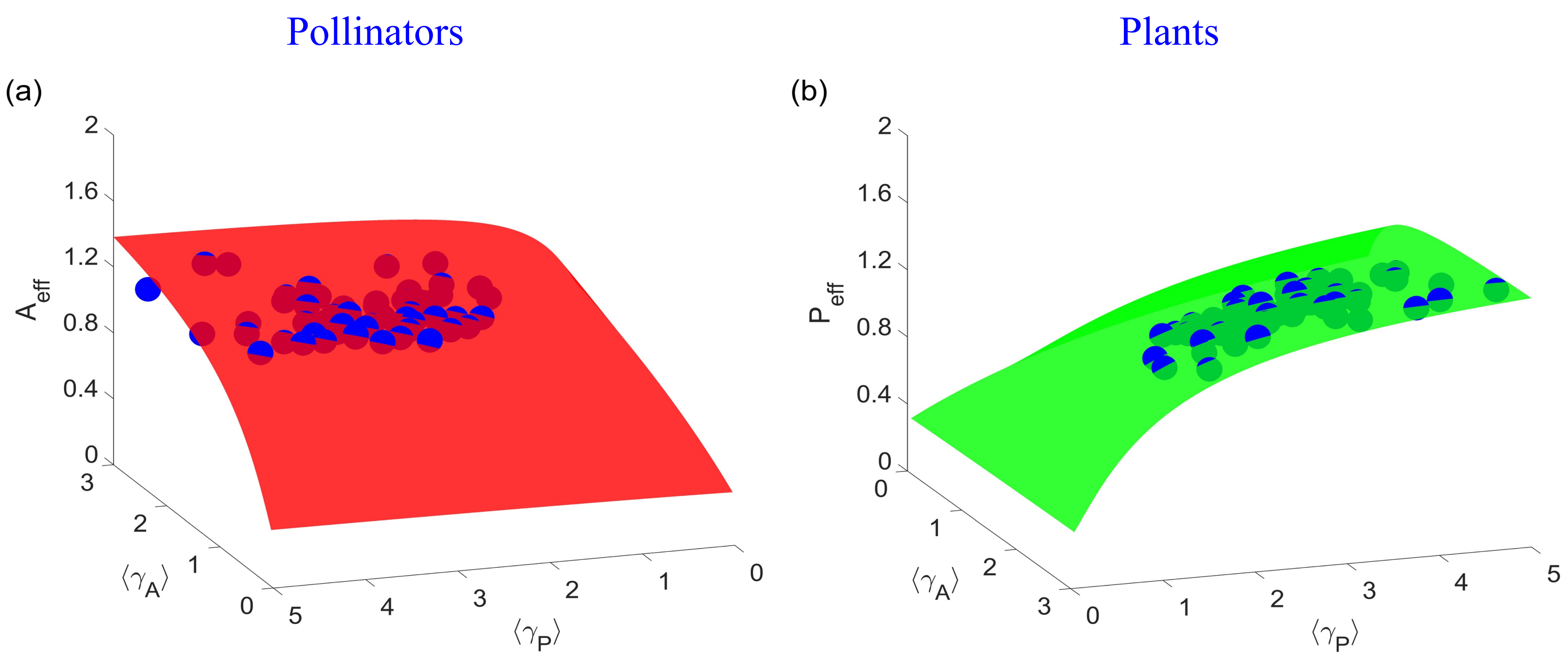
$$\langle \gamma_P \rangle = \frac{\sum_{i=1}^{S_P} \gamma_0 k_{P_i}^{1-t} \cdot k_{P_i}}{\sum_{i=1}^{S_P} k_{P_i}}, \quad \langle \gamma_A \rangle = \frac{\sum_{i=1}^{S_A} \gamma_0 k_{A_i}^{1-t} \cdot k_{A_i}}{\sum_{i=1}^{S_A} k_{A_i}}$$

Effective, two-dimensional dynamical system:

$$\begin{cases} \dfrac{dP_{eff}}{dt} = \alpha P_{eff} - \beta P_{eff}^2 + \dfrac{\langle \gamma_P \rangle A_{eff}}{1 + h\langle \gamma_P \rangle A_{eff}} P_{eff} + \mu, \\[2ex] \dfrac{dA_{eff}}{dt} = \alpha A_{eff} - \beta A_{eff}^2 - \kappa A_{eff} + \dfrac{\langle \gamma_A \rangle P_{eff}}{1 + h\langle \gamma_A \rangle P_{eff}} A_{eff} + \mu \end{cases}$$

J.-J. Jiang, Z.-G. Huang, W. Lin, T. Seager, C. Grebogi, A. Hastings, and Y.-C. Lai, "Predicting tipping points in mutualistic networks through dimension reduction," *PNAS (Plus),* in press.

Universality of 2D Model



Red surface: stable steady states of pollinator from effective system

Green surface: stable steady states of plants from effective system

Blue dots: corresponding stable steady states from 59 available real-world networks

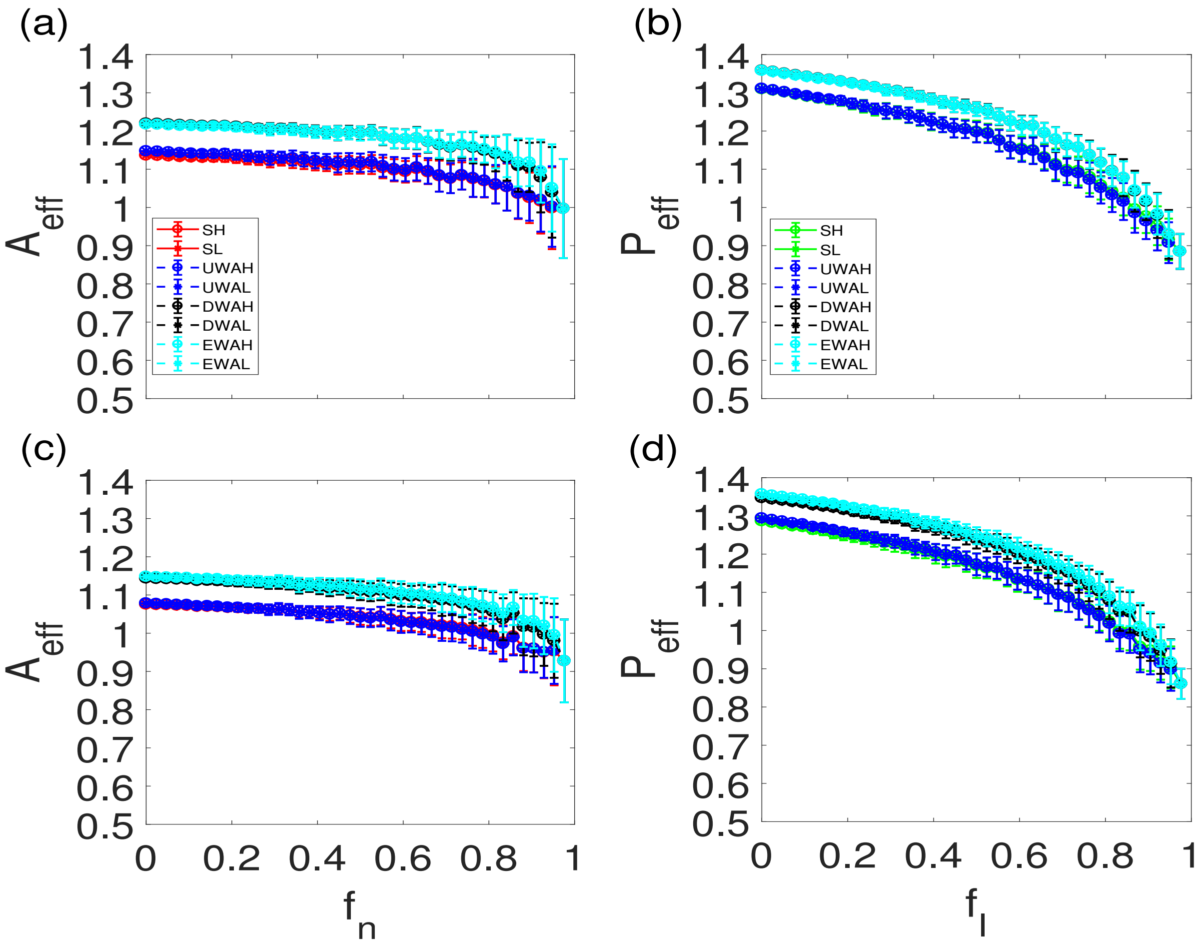
Average Abundance Predicted by Effective Dynamical System

Pollinators

Plants

Network A

Network B



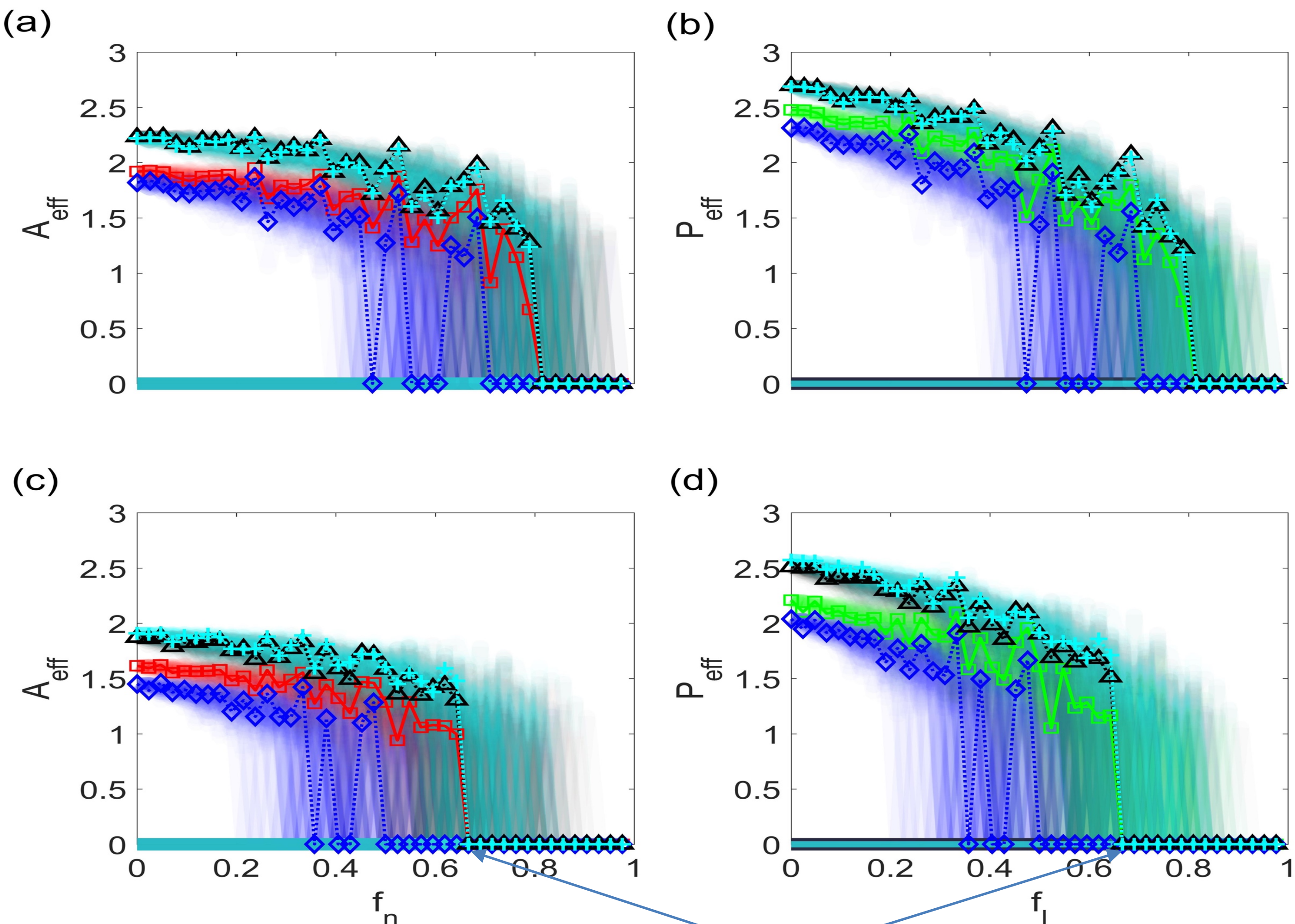
Red – from original system

Blue – from effective system with unweighted average

Cyan – from weighted average

One realization

Predicting network tipping point from effective dynamical system



Red – from original system

Blue – from effective system with unweighted average – not good agreement

Cyan – from weighted average

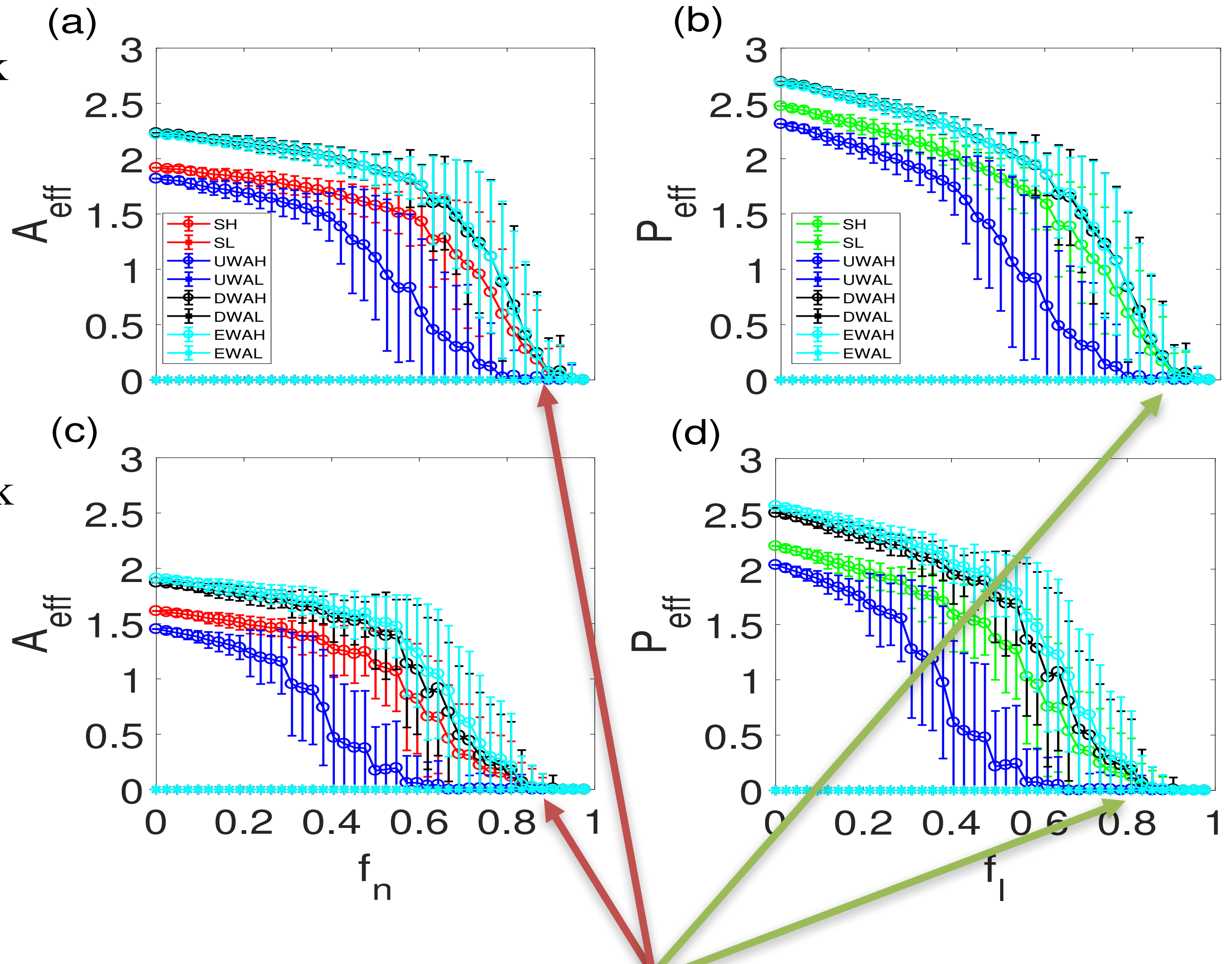
Symbols - individual realizations

Example of successful prediction of tipping point (from one realization)

Predicting network tipping point from effective dynamical system

Network A

Network B



Red, Green – from original system

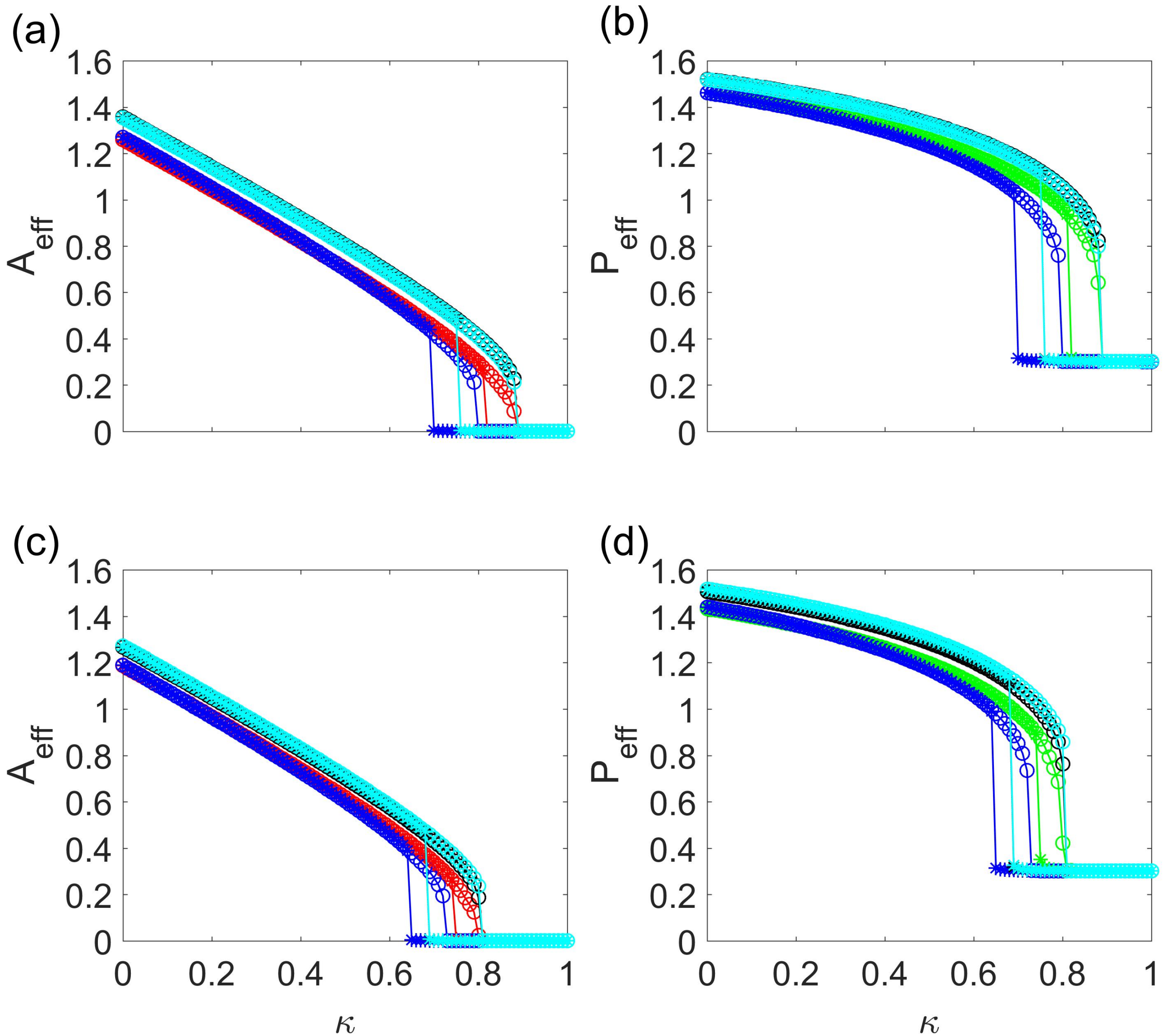
Blue – from effective system with unweighted average

Cyan – from weighted average

Ensemble averaged results

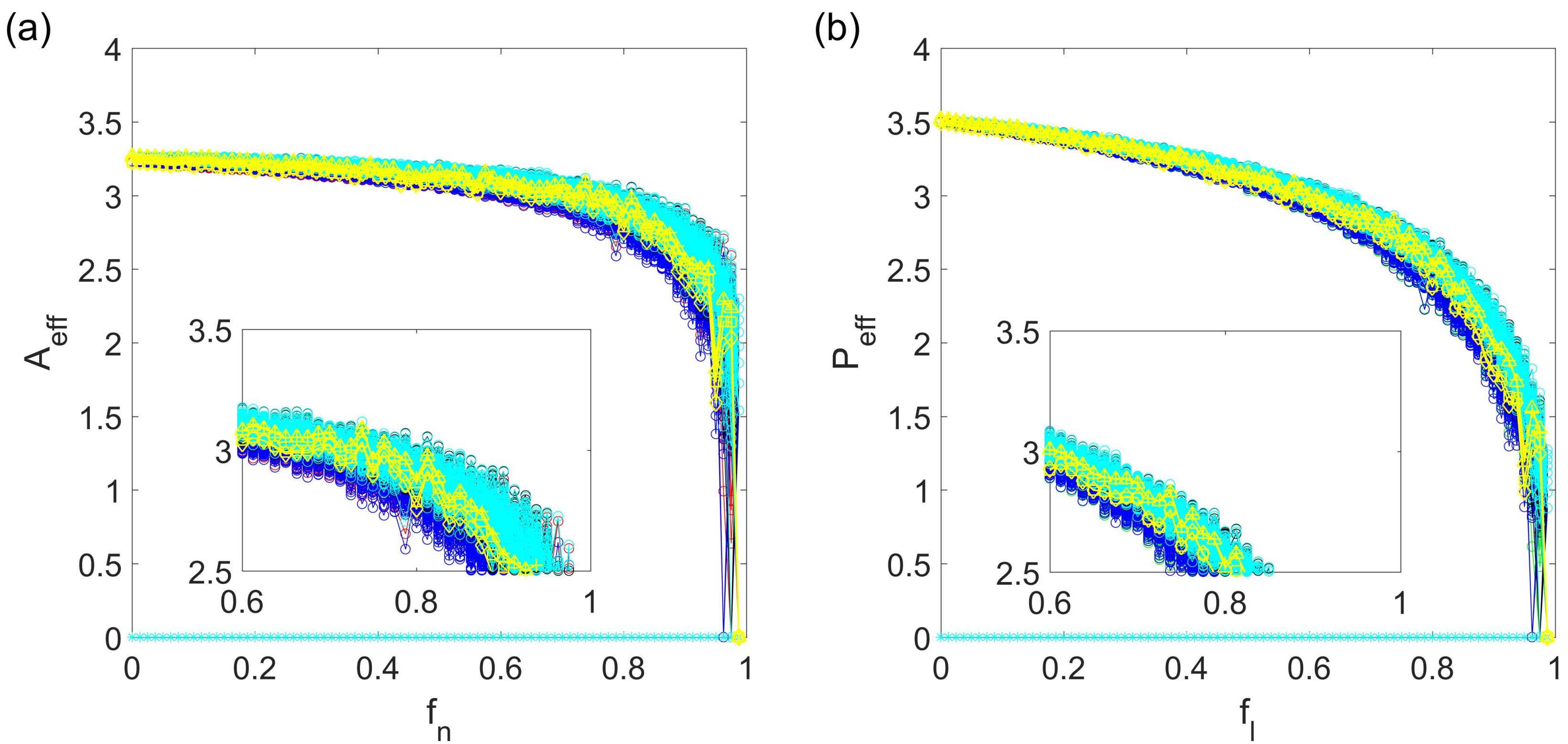
Example of successful prediction of a tipping point (many realizations)

Tipping Point Prediction



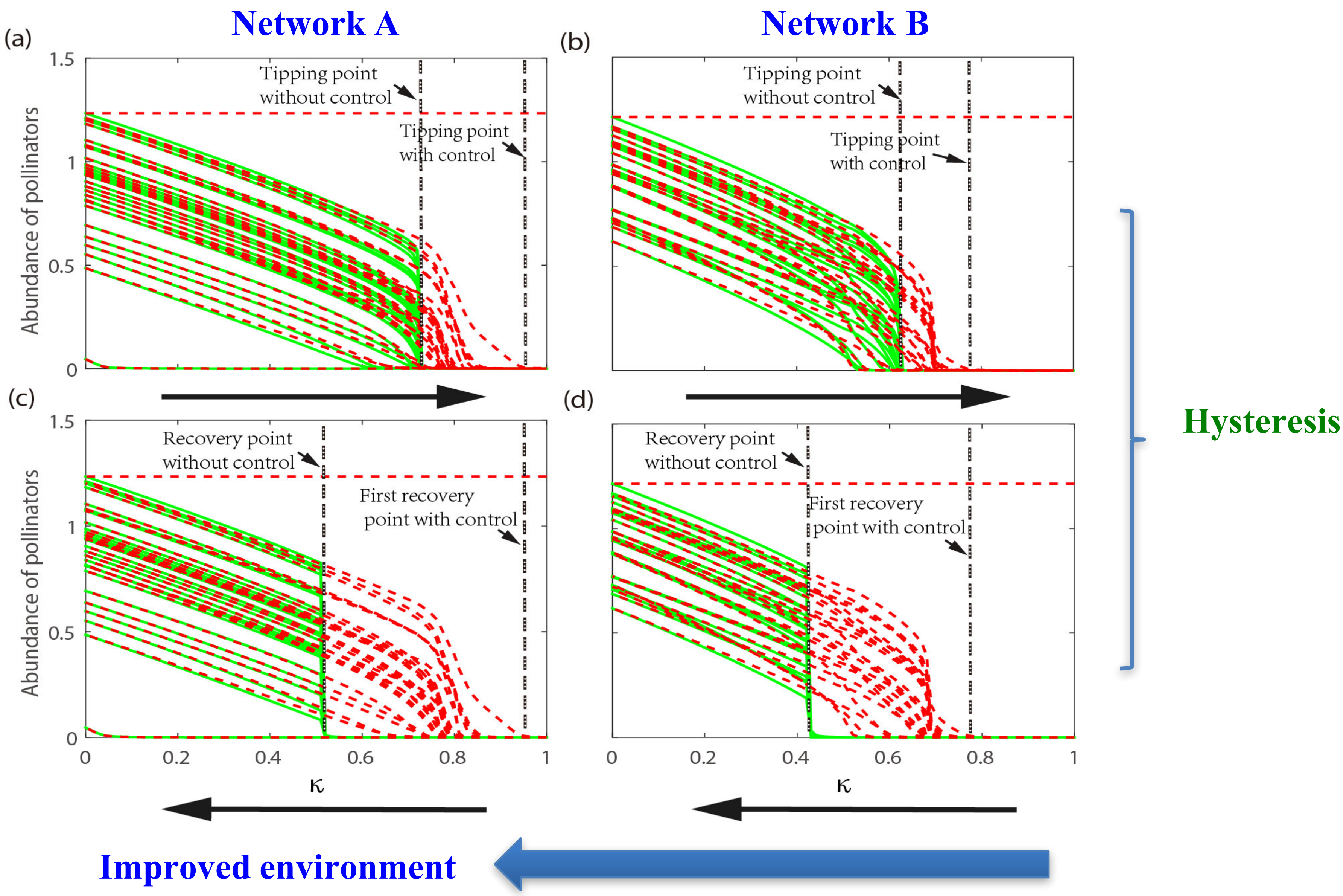
Red – pollinator abundance from original system;
Green – plant abundance from original system
Cyan – results from reduced 2D model

Random bipartite networks



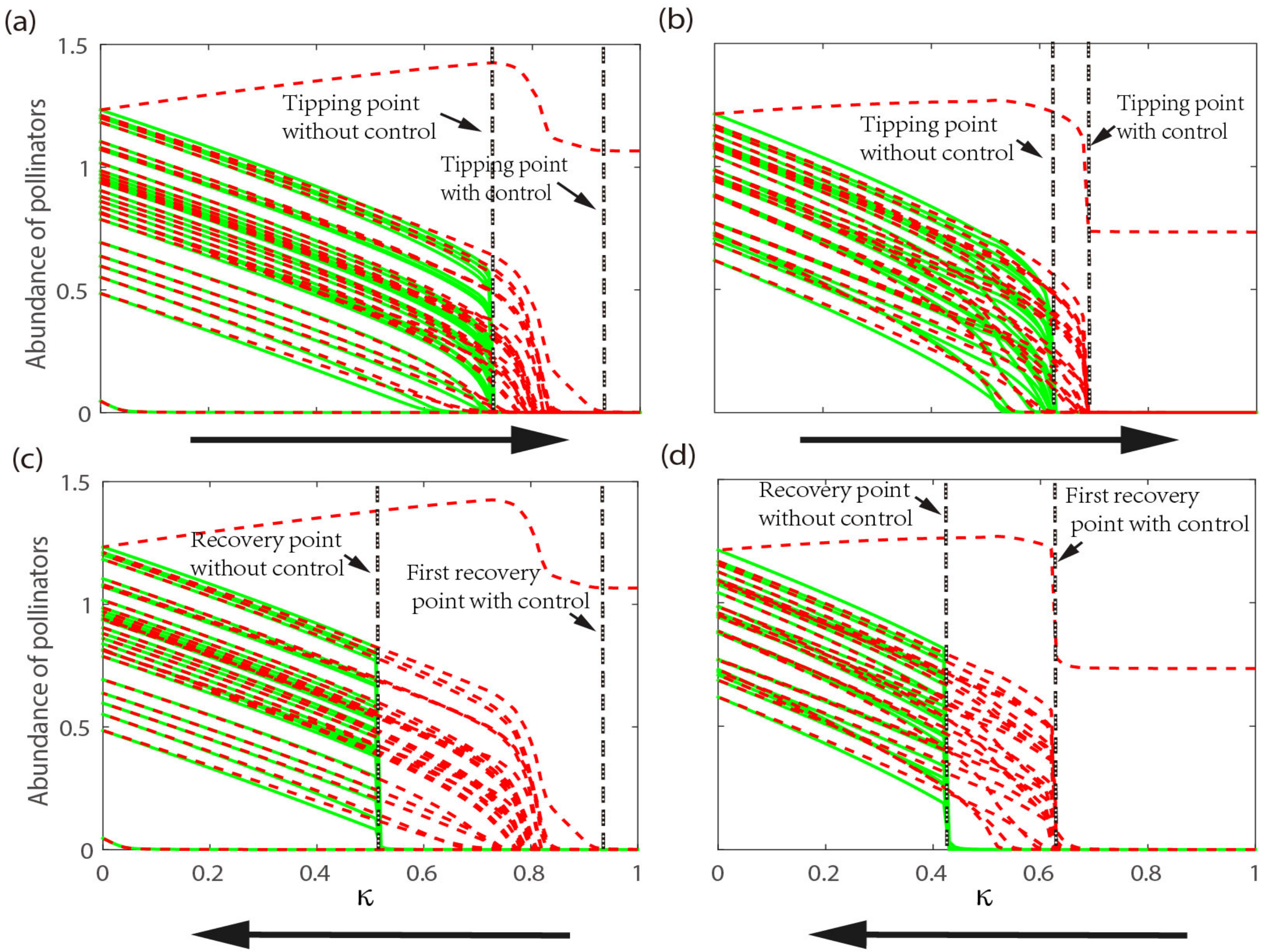
- Both weighted and unweighted averaging methods give good results.
- Realistic mutualistic networks are far from random – weighted averages are necessary!

Control Method 1: Maintaining the Abundance of a Single Pollinator



Control Method 2: Setting Decay Parameter = 0 for a Single Pollinator

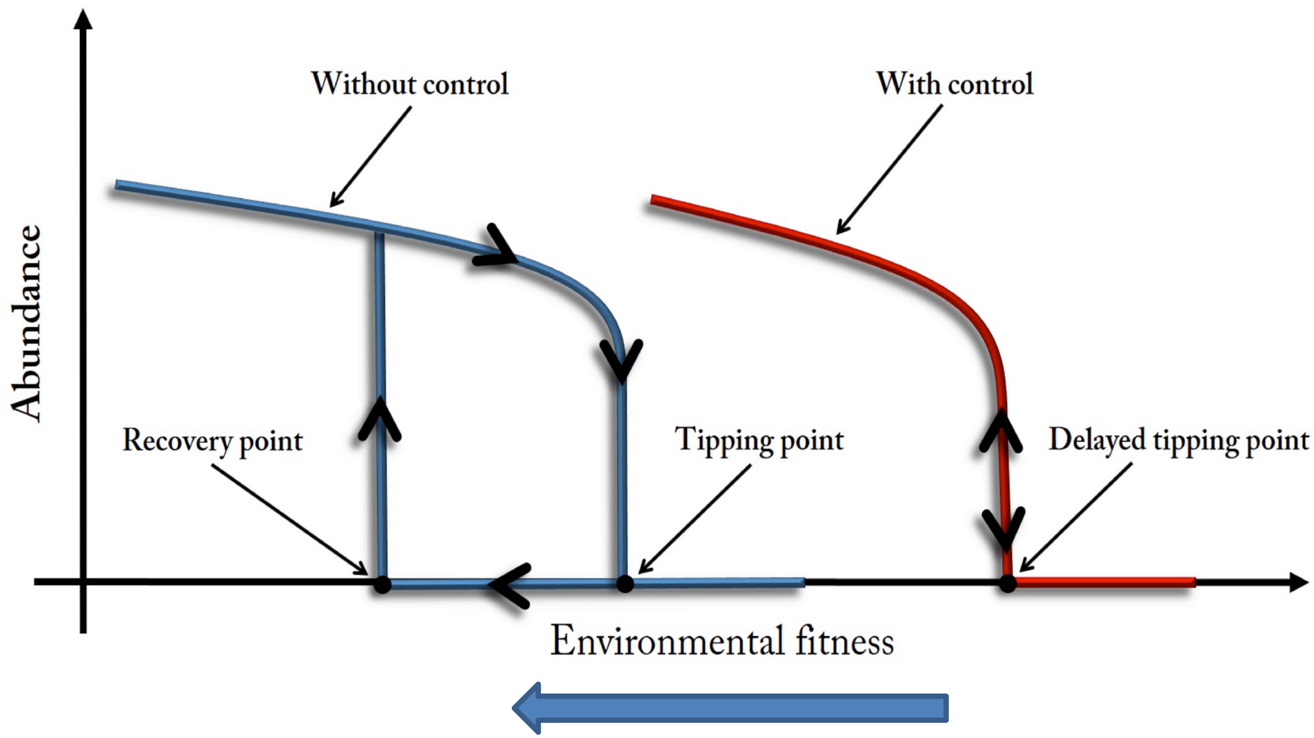
Network A **Network B**



Hysteresis

Improved environment

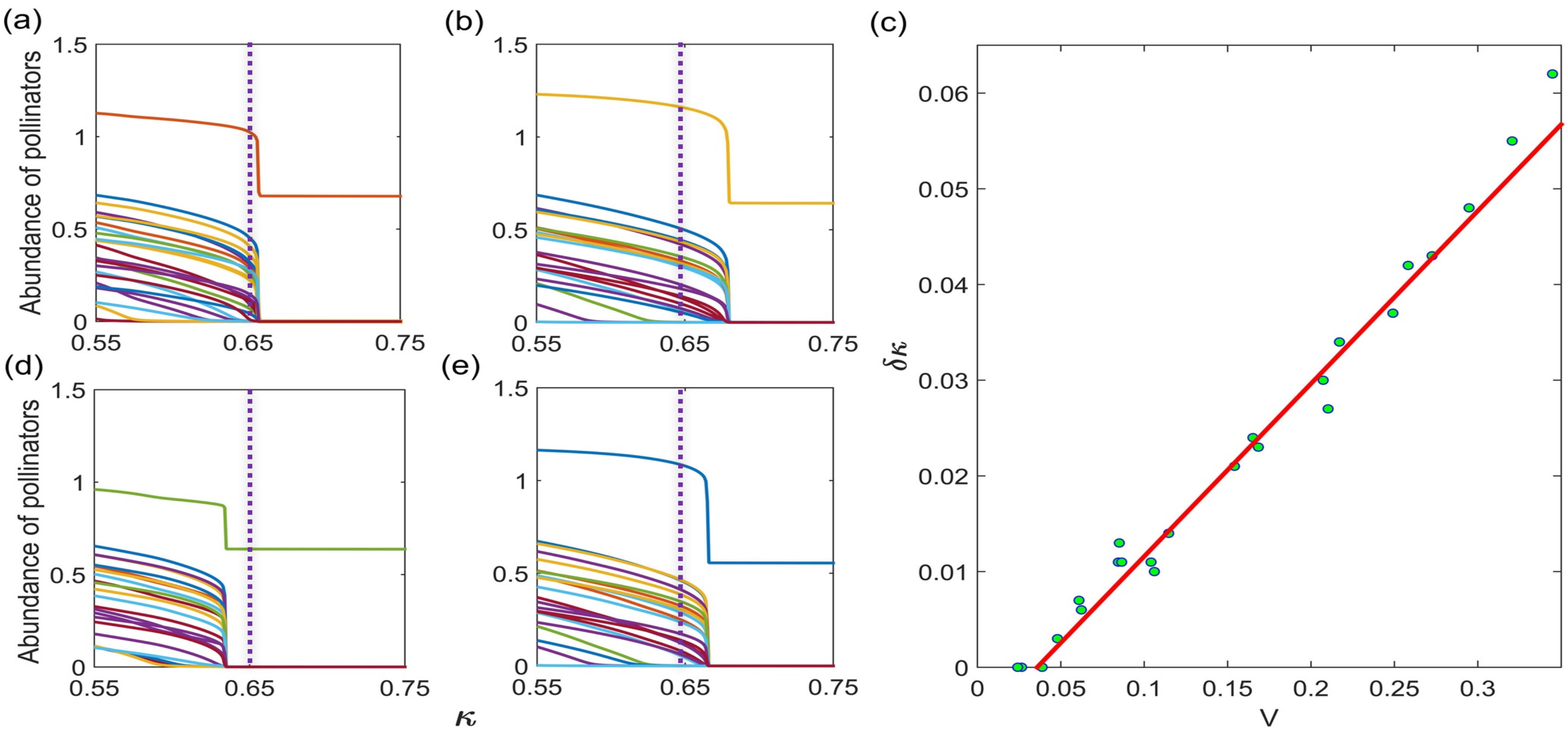
Hysteresis Loop and Benefit of Control



- Once the tipping point is reached, one must pay a higher price to bring the system back.
- Control can effectively remove the hysteresis, greatly facilitating system recovery from the tipping point.

Controllability Ranking of Pollinators

Network B (controlling pollinators 2, 3, 5, and 8)



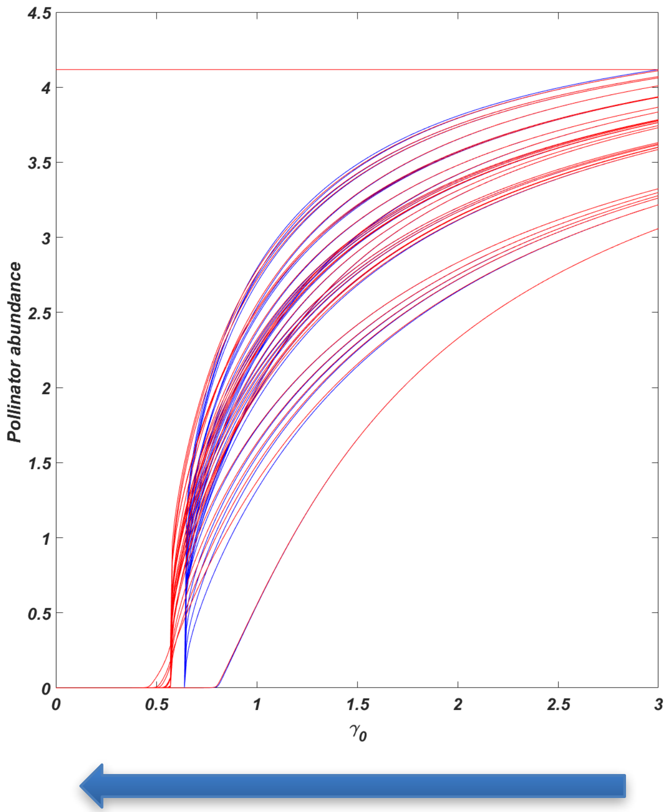
$M_{m \times n}$ – weighted matrix of original bipartite network

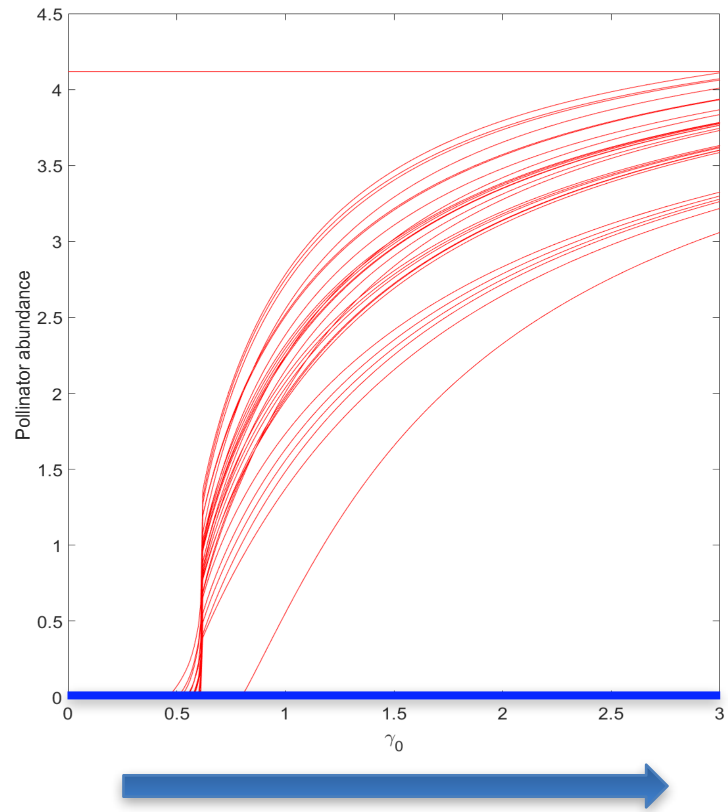
m – # of pollinators, n – # of plants

$M_P = M \cdot M^T$ – Projection matrix of pollinators

V – component of eigenvector associated with the largest eigenvalue of M_P

Benefit of Control: Enabling Species Recovery





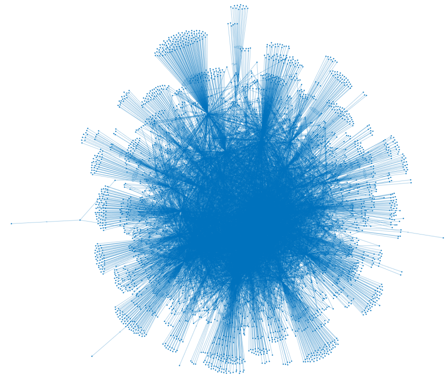
- **Blue – without control**

1. *Collapse abruptly and simultaneously*
2. *Unable to recover*

- **Red – with control**

1. *Collapse not as abrupt*
2. *Able to recover*

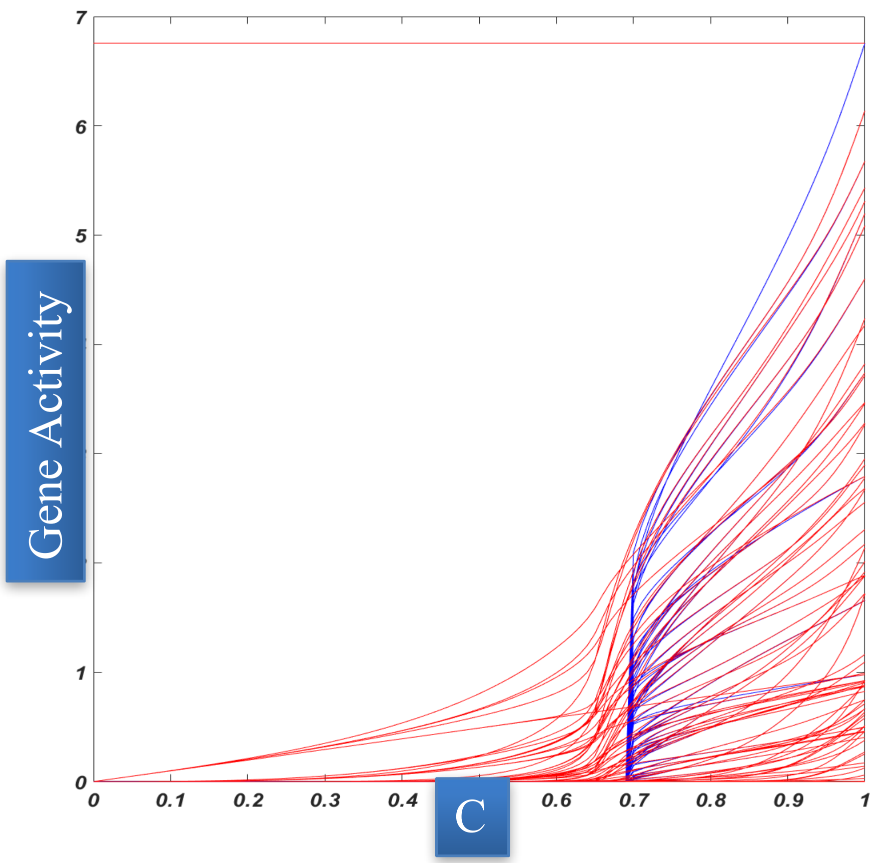
Controlling Tipping Point in a Gene Regulatory Network

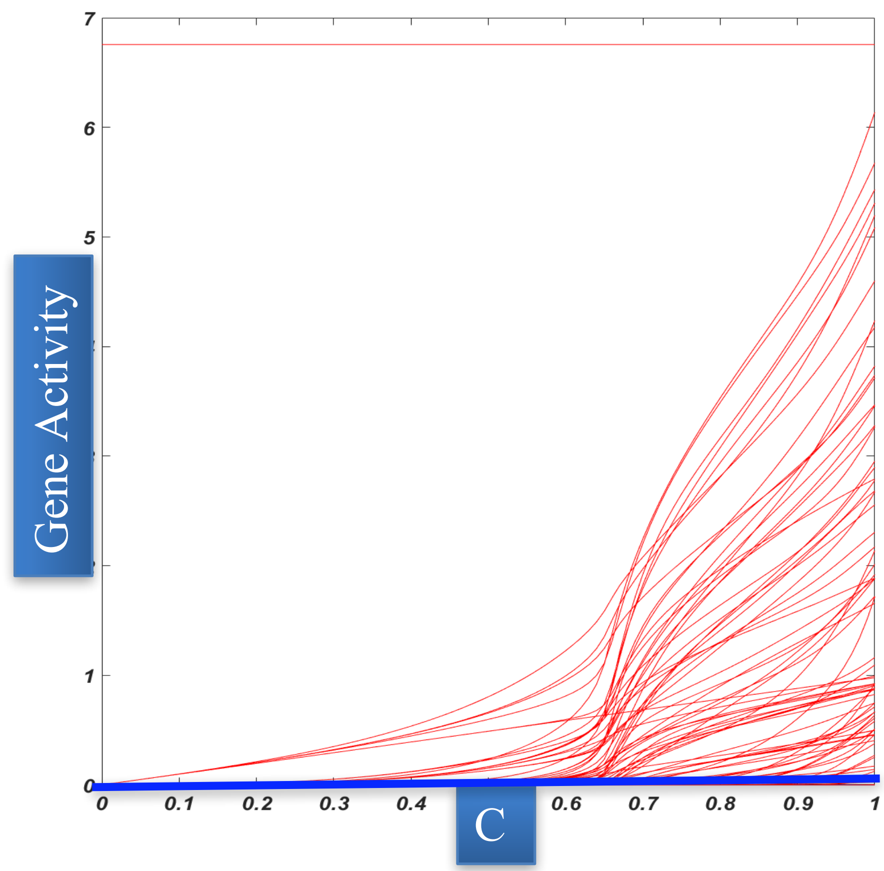


Gene regulatory network of *Saccharomyces cerevisiae*
- *4441* genes

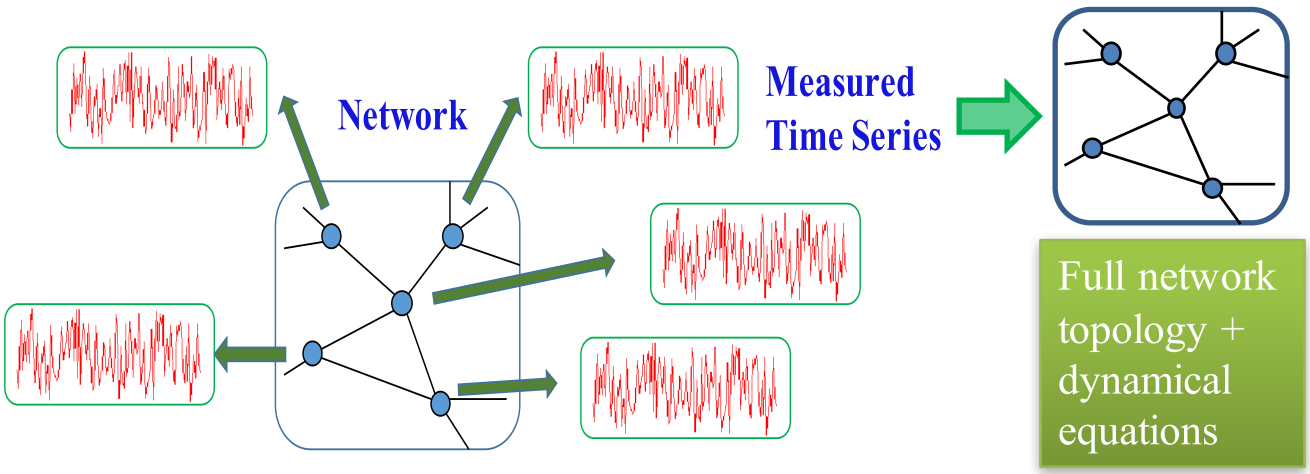
$$\frac{dx_i}{dt} = -Bx_i^f + C\sum_{j=1}^{N} A_{ij}\frac{x_j^h}{x_j^h + 1}$$

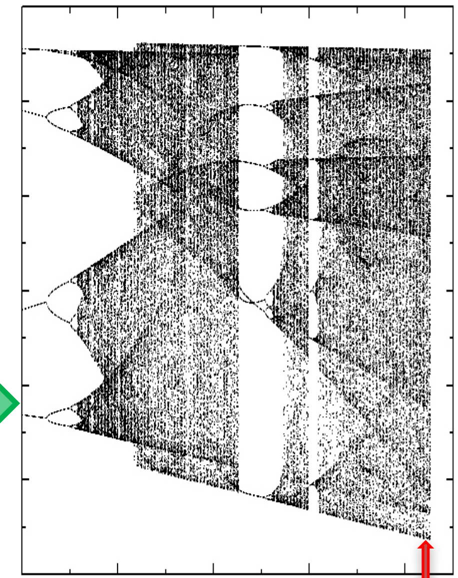
Holling type-III dynamics





Predicting Tipping Point: Data-Driven Method





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Basic Idea (1)

Dynamical system: $d\mathbf{x}/dt = \mathbf{F}(\mathbf{x})$, $\mathbf{x} \in R^m$

Goal: to determine $\mathbf{F}(\mathbf{x})$ from measured time series $\mathbf{x}(t)$!

Power-series expansion of jth component of vector field $\mathbf{F}(\mathbf{x})$

$$[\mathbf{F}(\mathbf{x})]_j = \sum_{l_1=0}^{n}\sum_{l_2=0}^{n}\cdots\sum_{l_m=0}^{n}(a_j)_{l_1 l_2 \ldots l_m} x_1^{l_1} x_2^{l_2} \ldots x_m^{l_m}$$

x_k – kth component of $\mathbf{x}$; Highest-order power: n

$(a_j)_{l_1 l_2 \ldots l_m}$ - coefficients to be estimated from time series

- $(1+n)^m$ coefficients altogether

If $\mathbf{F}(\mathbf{x})$ contains only a few power-series terms, most of the coefficients will be zero.

Basic Idea (2)

Concrete example: m = 3 (phase-space dimension): (x,y,z)

n = 3 (highest order in power-series expansion)

total $(1 + n)^m = (1+3)^3 = 64$ unknown coefficients

$$[\mathbf{F}(\mathbf{x})]_1 = (a_1)_{0,0,0}x^0y^0z^0 + (a_1)_{1,0,0}x^1y^0z^0 + \ldots + (a_1)_{3,3,3}x^3y^3z^3$$

Coefficient vector $\mathbf{a}_1 = \begin{pmatrix} (a_1)_{0,0,0} \\ (a_1)_{1,0,0} \\ \ldots \\ (a_1)_{3,3,3} \end{pmatrix}$ - 64×1

Measurement vector $\mathbf{g}(t) = [x(t)^0y(t)^0z(t)^0, x(t)^1y(t)^0z(t)^0, \ldots, x(t)^3y(t)^3z(t)^3]$

1×64

So $[\mathbf{F}(\mathbf{x}(t))]_1 = \mathbf{g}(t) \bullet \mathbf{a}_1$

Basic Idea (3)

Suppose $\mathbf{x}(t)$ is available at times $t_{0,}t_1,t_2,\ldots,t_{10}$ (11 vector data points)

$$\frac{dx}{dt}(t_1) = [\mathbf{F}(\mathbf{x}(t_1))]_1 = \mathbf{g}(t_1) \bullet \mathbf{a}_1$$

$$\frac{dx}{dt}(t_2) = [\mathbf{F}(\mathbf{x}(t_2))]_1 = \mathbf{g}(t_2) \bullet \mathbf{a}_1$$

...

$$\frac{dx}{dt}(t_{10}) = [\mathbf{F}(\mathbf{x}(t_{10}))]_1 = \mathbf{g}(t_{10}) \bullet \mathbf{a}_1$$

$$\text{Derivative vector } d\mathbf{X} = \begin{pmatrix} (dx/dt)(t_1) \\ (dx/dt)(t_2) \\ \ldots \\ (dx/dt)(t_{10}) \end{pmatrix}_{10\times 1} ; \text{ Measurement matrix } \mathbf{G} = \begin{pmatrix} \mathbf{g}(t_1) \\ \mathbf{g}(t_2) \\ \vdots \\ \mathbf{g}(t_{10}) \end{pmatrix}_{10\times 64}$$

We finally have $d\mathbf{X} = \mathbf{G} \bullet \mathbf{a}_1$ or $d\mathbf{X}_{10\times 1} = \mathbf{G}_{10\times 64} \bullet (\mathbf{a}_1)_{64\times 1}$

Basic Idea (4)

$$d\mathbf{X} = \mathbf{G} \bullet \mathbf{a}_1 \quad \text{or} \quad d\mathbf{X}_{10\times 1} = \mathbf{G}_{10\times 64} \bullet (\mathbf{a}_1)_{64\times 1}$$

Reminder: $\mathbf{a}_1$ is the coefficient vector for the first dynamical variable x.

To obtain $[\mathbf{F}(\mathbf{x})]_2$, we expand

$$[\mathbf{F}(\mathbf{x})]_2 = (a_2)_{0,0,0} x^0 y^0 z^0 + (a_2)_{1,0,0} x^1 y^0 z^0 + \ldots + (a_2)_{3,3,3} x^3 y^3 z^3$$

with $\mathbf{a}_2$, the coefficient vector for the second dynamical variable y. We have

$$d\mathbf{Y} = \mathbf{G} \bullet \mathbf{a}_2 \quad \text{or} \quad d\mathbf{Y}_{10\times 1} = \mathbf{G}_{10\times 64} \bullet (\mathbf{a}_2)_{64\times 1}$$

where

$$d\mathbf{Y} = \begin{pmatrix} (dy/dt)(t_1) \\ (dy/dt)(t_2) \\ \ldots \\ (dy/dt)(t_{10}) \end{pmatrix}_{10\times 1} .$$

Note: measurement matrix **G** is the same.

Similar expressions can be obtained for all components of the velocity field.

Compressive Sensing (1)

Look at

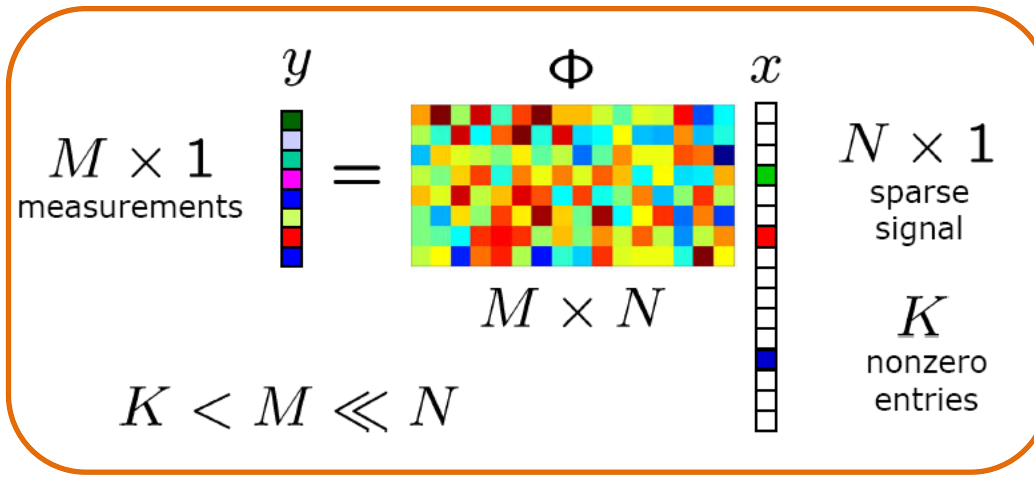
$\mathrm{d}\mathbf{X} = \mathbf{G} \bullet \mathbf{a}_1$ or $\mathrm{d}\mathbf{X}_{10\times 1} = \mathbf{G}_{10\times 64} \bullet (\mathbf{a}_1)_{64\times 1}$

Note that $\mathbf{a}_1$ is sparse - Compressive sensing!

Data/Image compression:

Φ : Random projection (not full rank)

x - sparse vector to be recovered

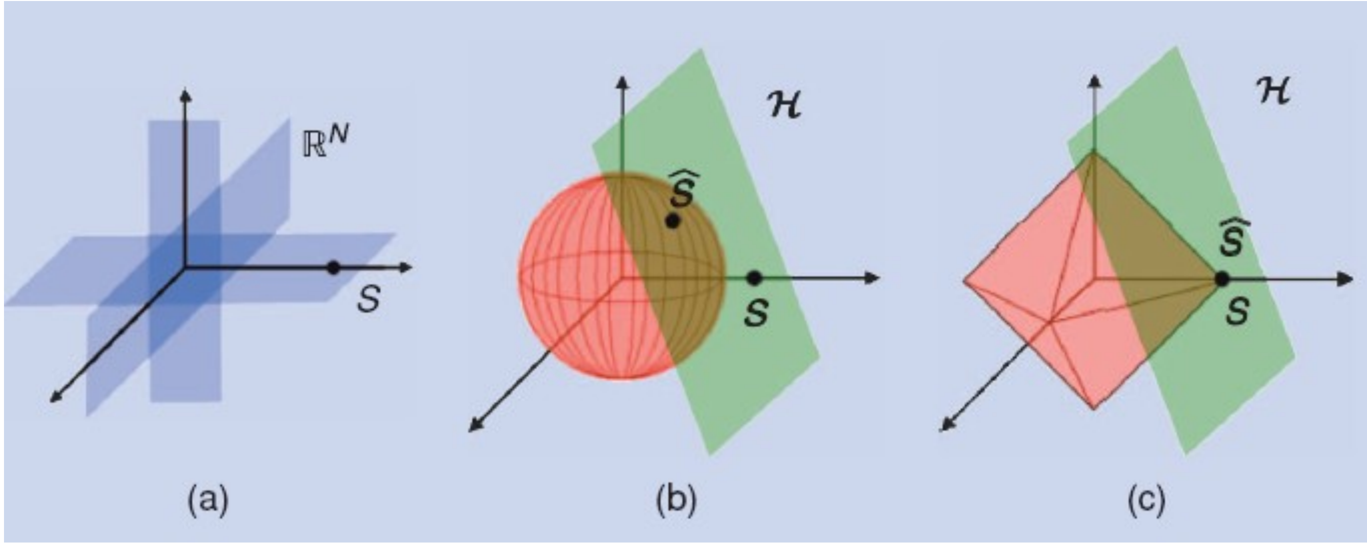


Goal of compressive sensing: Find a vector x with minimum number of entries subject to the constraint $y = \Phi \bullet x$

Compressive Sensing (2)

Find a vector x with minimum number of entries subject to the constraint $y = \Phi \bullet x$: $l_1 - \text{norm}$

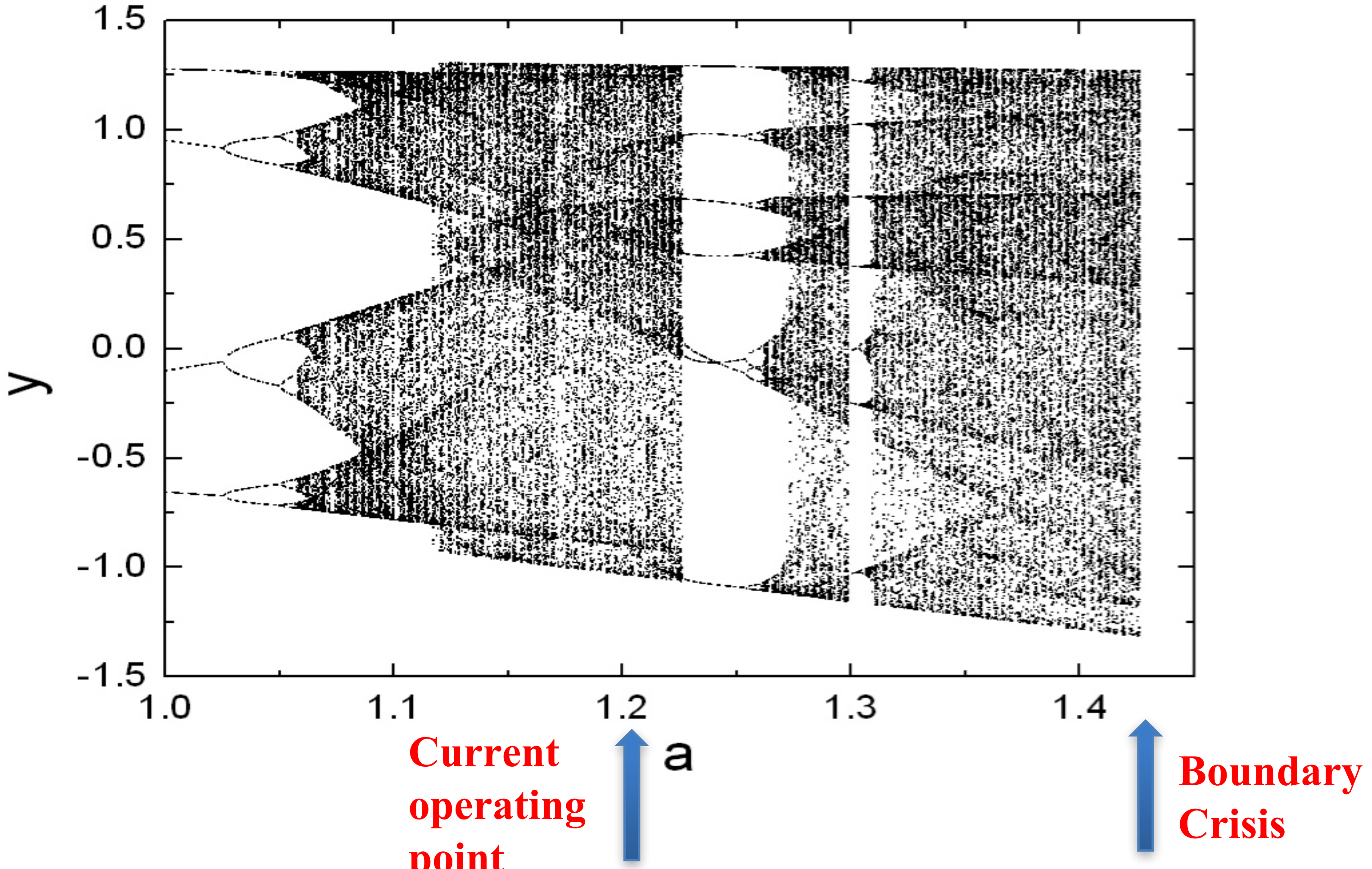
Why $l_1 - \text{norm}$? - Simple example in three dimensions



E. Candes, J. Romberg, and T. Tao, *IEEE Trans. Information Theory* **52**, 489 (2006),
Comm. Pure. Appl. Math. **59**, 1207 (2006);
D. Donoho, *IEEE Trans. Information Theory* **52**, 1289 (2006));
Special review: *IEEE Signal Process. Mag*. **24**, 2008

Predicting Tipping Point (2)

Step 2: Performing numerical bifurcation analysis



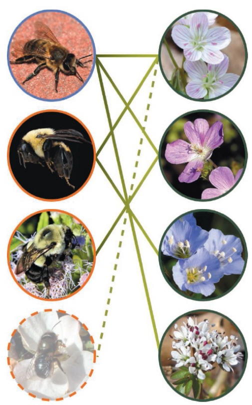
W.-X. Wang, R. Yang, Y.-C. Lai, V. Kovanis, and C. Grebogi, *PRL* **106**, 154101 (2011).

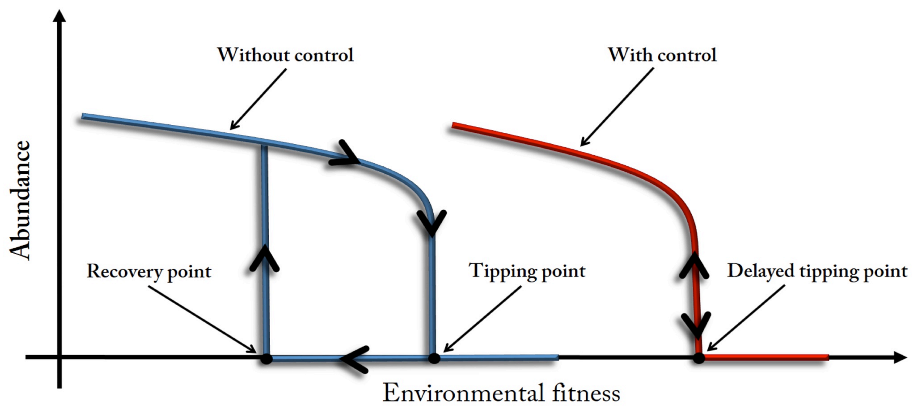
Take Home Message

1. An effective two-dimensional model to predict tipping point in mutualistic networks

J.-J. Jiang, Z.-G. Huang, W. Lin, T. Seager, C. Grebogi, A. Hastings, and Y.-C. Lai, "Predicting tipping points in mutualistic networks through dimension reduction," *PNAS (Plus)*, in press.

2. Control delays tipping point, eliminates hysteresis loop, and enables recovery that is not possible without control





3. Compressive sensing based identification and prediction of complex and nonlinear systems