Multi-Carrier Differential Chaos Shift Keying System With Subcarriers Allocation for Noise Reduction

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***Abstract*—In this brief, a multi-carrier differential chaos shift keying system capable of mitigating channel noise is proposed. In this system, subcarriers are allocated to the reference and data signals in an optimal way, while noise reduction is achieved through reference diversity. To verify its performance, the bit error rate of the proposed system over multipath Rayleigh fading channels is derived, together with a detailed discussion about the determination of the optimal number of reference-carrying subcarriers. Extensive simulations are carried out for performance evaluation, which are further compared to other state-of-the-art systems. Results confirm the superior bit error rate performance of the proposed system, which has only small loss in energy and spectral efficiencies.**

***Index Terms*—Chaos, MC-DCSK, subcarriers allocation, noise reduction, bit error rate.**

I. Introduction

THE THEORETICAL foundation for exploiting chaos for communications was laid more than two decades ago [1]–[3]. Since then, continuous efforts have been made to explore potential applications of the chaos theory, while a recent focus has been on the design of efficient chaos-based digital modulation (CDM) systems [4]–[10]. This class of systems performs signal spectrum spreading and digital modulation simultaneously by embedding data bits into chaotic signals. The intrinsic unpredictability of chaotic signals and the simplicity to generate them (e.g., by simple and low-cost electronic circuits) make CDM systems appealing. Such systems not only possess all the merits of traditional spread-spectrum systems (e.g., low probability of detection, anti-jamming and mitigation of multi-path fading), but also have high data security [11].

Among the existing CDM systems, differential chaos shift keying (DCSK) [12] and its variants [13] are promising due to, e.g., their low level of design complexity. DCSK systems, however, have low bit rates and are sensitive to noises in both reference and data channels due to the sequential transmission protocol in-use [14]. To achieve a high bit rate, multi-carrier DCSK (MC-DCSK) system was recently proposed [15], in which a single reference together with multiple data signals is sent simultaneously via multiple subcarriers. Although the MC-DCSK system obtains higher energy efficiency and a lower bit error rate (BER) than DCSK, the difficulty in dealing with channel noise persists. For example, in an additive white Gaussian noise (AWGN) channel, the BER performance of MC-DCSK is worse than that of the coherent binary phase shift keying (BPSK) scheme. Noise reduction DCSK (NR-DCSK) system utilizing duplicated reference signals was subsequently proposed [16], in which all received replicas are averaged to diminish noise so as to achieve enhanced BER performance. In [17], each reference is shared by multiple data signals, resulting in further reduction in noise and improvement in BER performance. In all the existing noise reduction schemes [16], [17], the need of multiple duplicates of the samples leads to a strong similarity among the reference and data signals, compromising communication security undesirably.

This brief articulates a subcarriers allocated MC-DCSK (SA-MCDCSK) system in order to mitigate channel noise as well as to improve BER performance. In particular, several subcarriers are explored for reference transmission to achieve noise reduction through signal averaging. To optimize performance, the number of subcarriers to be allocated to the reference is critical. It is resolved by deriving an analytical formula for the BER over a multipath Rayleigh fading channel, for which a criterion can be obtained to determine the optimal number of reference-carrying subcarriers, which is confirmed by numerical validation. The main result is that the proposed scheme with an optimal number of reference-carrying subcarriers can effectively eliminate noise, thereby achieving much better BER performance than all other existing DCSK schemes over both AWGN and multipath fading channels. Moreover, with a large number of subcarriers, SA-MCDCSK can obtain

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a BER performance comparable to that of the coherent BPSK over AWGN channels.

II. Proposed Subcarriers Allocated MC-DCSK Scheme

The initial step in MC-DCSK is to convert a serial high rate bit stream into M-1 parallel low rate sub-streams. To eliminate the need of channel estimation, MC-DCSK sends the reference through one subcarrier and allocates the other M-1 subcarriers to M-1 sub-streams, respectively. Inevitably, noise would affect reference samples, deteriorating the performance of MC-DCSK. To reduce the noise effect, the proposed SA-MCDCSK system adopts multiple subcarriers to send replicas of the reference and then averages all corrupted reference copies at the receiver side.

A simple subcarrier allocation method is designed for the SA-MCDCSK as shown in Figure 1, where the power spectral density (PSD) of MC-DCSK is also given for comparison. One can see that, in both systems, all subcarriers are divided into M equal but disjoint bands. In SA-MCDCSK, N ($M > N \geq 1$) subcarriers of respective frequencies $f_1, \ldots, f_N$ are utilized to carry N copies of the current reference signal, while the others are allocated to M-N low rate bit sub-streams sharing this reference signal. To better adapt to wireless channels, reference copies in the proposed system can be distributed in a comb-type way (similar to the design in [18]) in the frequency spectrum, and the separation between reference-carrying subcarriers can be increased or decreased depending on the channel profile. As a result, it is possible to demodulate data bits carried by M-N subcarriers correctly even if the channel gain changes from the center to the edge frequencies. It is noted that, for the case of $N = 1$, the SA-MCDCSK system reduces to the MC-DCSK system.

In the new system, data bits in all low rate bit sub-streams are spread by multiplication in time with the same chaotic reference signal $x(t)$, that can be written as

$$x(t) = \sum_{k=1}^{\beta} x_k h_T(t - kT_c), \tag{1}$$

where β is the spreading factor, x_k is the k-th sample of the chaotic sequence, and $h_T(t)$ is the impulse response of a square-root-raised cosine filter of duration T_c with a roll-off factor $\alpha$and normalized energy. To meet the Nyquist criterion, the frequency spectrum of $h_T(t)$ is limited to [-B_c/2, B_c/2] with $B_c = (1 + \alpha)/T_c$.

The transmitted signal of SA-MCDCSK is

$$s(t) = \sum_{j=0}^{N-1} x(t) \cos\left(2\pi f_j t + \theta_j\right) + \sum_{j=1}^{M-N} a_j x(t) \cos\left(2\pi f_{j+N-1} t + \theta_{j+N-1}\right) \tag{2}$$

where θ_j is a phase angle introduced in the carrier modulation process and a_j is the data bit carried by the $(j + N - 1)$-th subcarrier.

Figure 2 presents a possible configuration of SA-MCDCSK system in its simplest form for clarity. The architecture is similar to that of MC-DCSK, except for some parts in the transmitter and the newly added average block in the receiver. This block collects all received duplicates of the reference from different subcarriers and then averages them for noise smoothing. The smoothed signal is stored in the reference matrix A for correlation computation. To retrieve the reference and data signals from the subcarriers, the receiver needs to synchronize all the subcarrier frequencies and phases with those of the transmitter. This, in fact, is a general and essential requirement for demodulation in all multi-carrier communication systems.

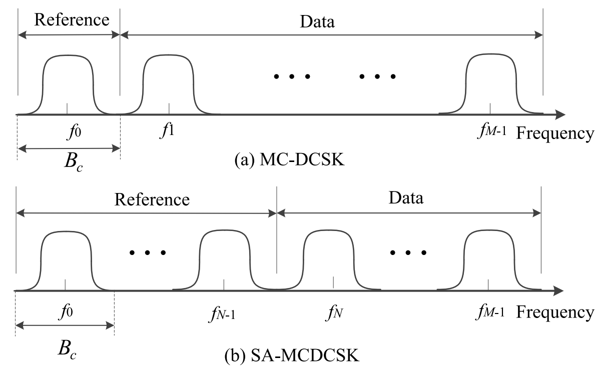


Fig. 1. PSD of MC-DCSK and SA-MCDCSK systems.

Consider the realistic condition for which only AWGN corrupts the received signal, the reference matrix A is

$$A = \begin{pmatrix} x_1 + \varepsilon_1 & \cdots & x_\beta + \varepsilon_\beta \end{pmatrix} \tag{3}$$

where

$$\varepsilon_k = \frac{1}{N} \sum_{j=0}^{N-1} n_{j,k} \tag{4}$$

with $n_{j,k}$ being the k-th sample of noise that affects the signal transmitted on the j-th subcarrier. The data matrix B containing all data signals is

$$B = \begin{pmatrix} b_{1,1} & \cdots & b_{1,\beta} \\ & \vdots & \\ b_{M-N,1} & \cdots & b_{M-N,\beta} \end{pmatrix} \tag{5}$$

where

$$b_{i,k} = a_i x_k + n_{i+N-1,k} \tag{6}$$

Finally, the transmitted M-N data bits can be recovered in a parallel way according to the signs of the elements of the matrix product of A and B':

$$\begin{pmatrix} a_1 & \ldots & a_{M-N} \end{pmatrix} = sign\left(AB'\right) \tag{7}$$

where sign(·) is an element-wise function, returning the sign of each element in a matrix, and ’ is the matrix transpose operator.

III. Performance Analysis

In this section, the BER performance of SA-MCDCSK is investigated. A commonly used multipath fading channel [4], [14], [15], consisting of L independent Rayleigh slow-fading paths, is considered. The output of this channel is given by

$$r(t) = \sum_{l=1}^{L} \alpha_l s(t - \tau_l T_c) + n(t) \tag{8}$$

where τ_l and α_l are the time delay and the propagation gain of the l-th path, respectively, and $n(t)$ is AWGN with PSD of $N_0/2$.

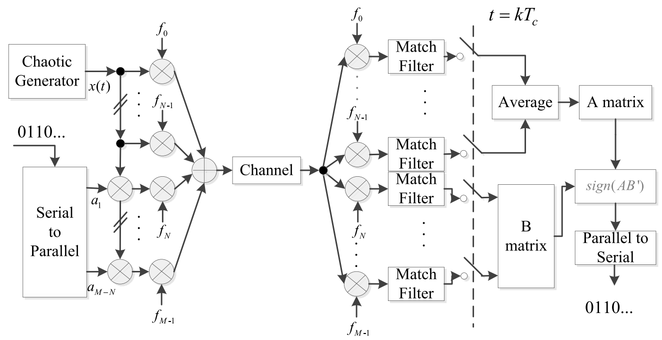


Fig. 2. System configuration of SA-MCDCSK.

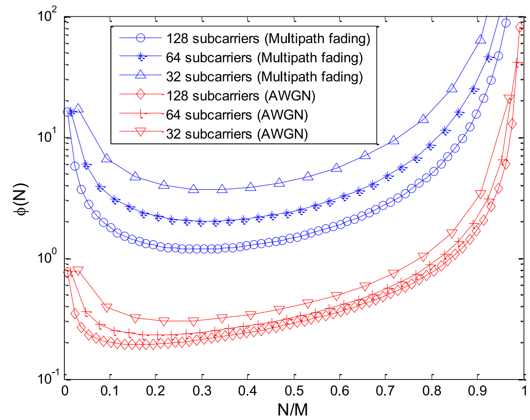


Fig. 3. Function $\Phi(N)$ with $\beta = 64$ and $E_b/N_0 = 9dB$ over an AWGN channel and a two-path Rayleigh fading channel with equal average gain powers.

In the performed analysis, the largest multipath time delay is assumed to be much shorter than the bit duration, i.e., $\tau_l << \beta$. This assumption is widely adopted and is valid in most practical applications [19]. In this case, inter-symbol interference (ISI) is negligible in comparison to the interference within each symbol due to multipath time delays [15]. By ignoring ISI, the decision variable for bit a_i transmitted over the (i+N-1)-th subcarrier is represented by

$$Z_i \approx \sum_{k=1}^{\beta}\left(\sum_{l=1}^{L}\alpha_l x_{k-\tau_l} + \varepsilon_k\right)\cdot\left(\sum_{l=1}^{L}\alpha_l a_i x_{k-\tau_l} + n_{i+N-1,k}\right) \tag{9}$$

For large spreading factors, one has

$$\sum_{k=1}^{\beta} x_{k-\tau_p} x_{k-\tau_q} \approx 0 \quad \text{for} \quad p \neq q \tag{10}$$

Equation (9) can then be simplified as

$$Z_i \approx \sum_{k=1}^{\beta}\left(a_i\sum_{l=1}^{L}\alpha_l^2 x_{k-\tau_l}^2 + \sum_{l=1}^{L}\alpha_l x_{k-\tau_l} n_{i+N-1,k} + \sum_{l=1}^{L}\alpha_l a_i x_{k-\tau_l}\varepsilon_k + \varepsilon_k n_{i+N-1,k}\right) \tag{11}$$

According to the analysis in [15]–[17], decision variables in Eq. (11) are effectively Gaussian distributed for high values of the spreading factor. If the transmitted bits are equally probable, the BER of SA-MCDCSK can be derived, as

$$\begin{aligned} P &= \frac{1}{2}erfc\left(\frac{E[Z_i]}{\sqrt{2var[Z_i]}}\right) \\ &= \frac{1}{2}erfc\left(\left(\frac{M(N+1)}{N(M-N)}\gamma_b^{-1} + \frac{\beta M^2}{2N(M-N)^2}\gamma_b^{-2}\right)^{-\frac{1}{2}}\right) \end{aligned} \tag{12}$$

where $E[\cdot]$ and $var[\cdot]$ are the expectation operator and variance operator, respectively, and $erfc(\cdot)$ denotes the complementary error function. Here, $\gamma_b = \sum_{l=1}^{L}\alpha_l^2 E_b/N_0$ and the PDF of γ_b is available in [20], while the bit energy of transmitted signal is given by $E_b = M\sum_{i=1}^{\beta} x_i^2/2(M-N)$.

Note that, with $L = 1$ and $\gamma_b = E_b/N_0$, Eq. (12) can be used to evaluate the BER performance of SA-MCDCSK over AWGN channels.

IV. Optimal Subcarrier Number for the Reference

As shown in Eq. (12), the BER depends on the number of subcarriers, N, allocated to the reference. To obtain the optimal value of N, the relationship between BER and N is explored here. For fixed β, M and γ_b, define the following function $\Phi(N)$:

$$\Phi(N) = \frac{M(N+1)}{N(M-N)}\gamma_b^{-1} + \frac{\beta M^2}{2N(M-N)^2}\gamma_b^{-2} \tag{13}$$

Figure 3 depicts the curves of the function $\Phi(N)$ over both AWGN and multipath Rayleigh fading channels. When N increases from zero, $\Phi(N)$ decreases first, reaches a minimum, and then increases, showing a resonance-like phenomenon as a consequence of the interplay between noise reduction and the energy efficiency decrement caused by duplicated reference transmission. As $erfc(\cdot)$ is a monotonically decreasing function, the BER in Eq. (12) increases with function $\Phi(N)$ in Eq. (13). As a result, given β, M and γ_b, the BER of the proposed system can be optimized when the function $\Phi(N)$ reaches its minimum. The design of the proposed system is then equivalent to the following optimization problem:

$$\textit{minimize } \Phi(N) \textit{ subject to } N < M \textit{ and } N \textit{ is a positive integer} \tag{14}$$

For example, with $\beta = 64$ and $E_b/N_0 = 9dB$, optimal values of N are 20, 13 and 8 for 128, 64 and 32 subcarriers, respectively, over the AWGN channel. The corresponding minima of $\Phi(N)$ are 0.1923, 0.2316 and 0.3015, respectively.

V. Simulation Results and Performance Comparison

In this section, SA-MCDCSK and MC-DCSK systems are simulated and compared over an AWGN channel and a two-path Rayleigh fading channel, with $\tau_1 = 0$, $\tau_2 = 2$ and $E[\alpha_1^2] = E[\alpha_2^2] = 0.5$. In both systems, chaotic sequences are generated from the logistic map $x_{k+1} = 1 - 2x_k^2$ as in [11]. The roll-off factor α is set to 0.25 and the total bandwidth of all subcarriers is 4MHz. In addition, both NR-DCSK and DCSK systems with the same logistic map are also simulated for performance comparison.

In Figure 4, the simulated performances of SA-MCDCSK are compared to the analytical ones calculated by Eq. (12) over AWGN and multipath Rayleigh fading channels, respectively. There is a good agreement between theoretical predictions and

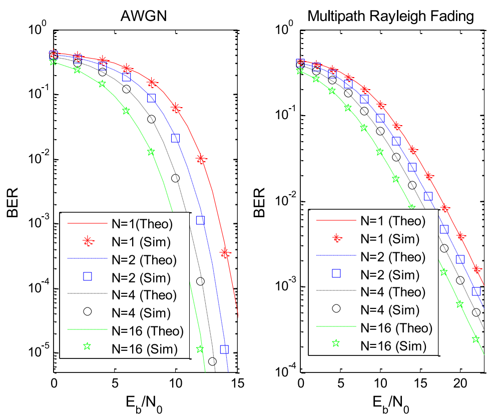


Fig. 4. Analytical and simulated BER performances of SA-MCDCSK with $\beta = 128$ and $M = 64$.

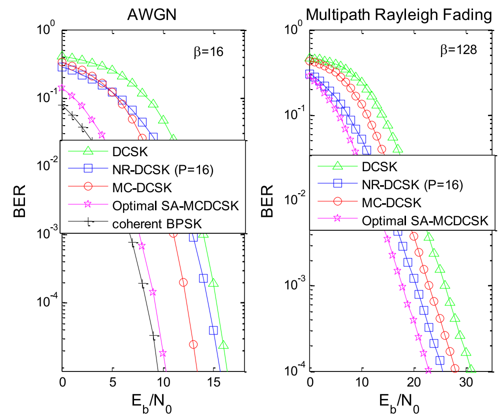


Fig. 5. BER comparison among DCSK, NR-DCSK, MC-DCSK and SA-MCDCSK with optimal N ($M = 128$).

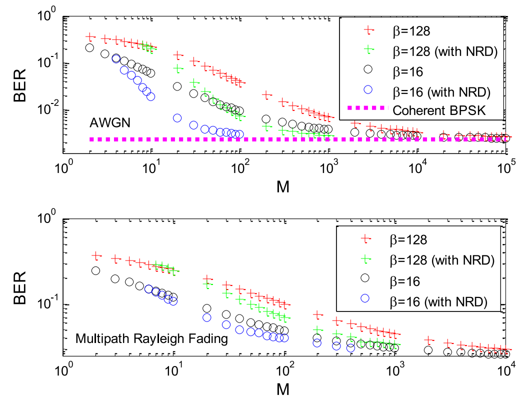
simulated results for both channels, confirming the analysis in Section III.

Figure 5 shows the simulated BER curves of SA-MCDCSK with optimal N (labeled as 'Optimal SA-MCDCSK'), where N varies with the E_b/N_0 level and is obtained by solving the optimization problem in Eq. (14) for each E_b/N_0 level. The corresponding BER curves associated with DCSK, NR-DCSK, MC-DCSK and coherent BPSK are also shown for comparison. One can see that the SA-MCDCSK system with optimal N performs much better than the other three DCSK-based systems over both AWGN and multipath Rayleigh fading channels, and its BER is only slightly higher (about 1dB) than that of the coherent BPSK over the AWGN channel. Compared to NR-DCSK that uses time reference diversity for noise reduction, SA-MCDCSK is superior not only in the BER performance but also in the following three aspects. Firstly, time reference diversity leads to a low bit rate in NR-DCSK as additional time is used to send redundant samples. Secondly, time reference diversity introduces a strong similarity in the reference and data signals, which degrades the communication security in NR-DCSK. Thirdly, with a fixed spreading factor, time reference diversity greatly reduces the correlation duration for demodulation in NR-DCSK, thus making the system quite sensitive to ISI caused by multipath time delays. Compared to NR-DCSK, therefore, the proposed SA-MCDCSK system is more competitive in high bit rate wireless communications.

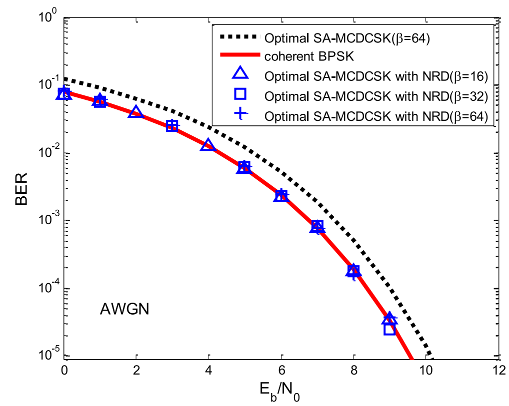
Further improvements of the BER performance are possible in SA-MCDCSK through noise reduction in the received data signals. Since the transmission of each data signal, either in its identical or inverted version, occurs multiple times in SA-MCDCSK, one can average all received data signals for smoothing with the modulations removed [17]. As a result, Noise Reduction in Data (NRD) can be achieved by substituting each element $b_{i,k}$ in matrix B with $b^*_{i,k}$ given by

$$b^*_{i,k} = \frac{\sum_{\substack{j=1\\j\neq i}}^{M-N}\left[\sum_{\kappa=1}^{\beta} b_{i,\kappa} b_{j,\kappa}\right] b_{j,k}}{\sum_{\substack{j=1\\j\neq i}}^{M-N}\left|\sum_{\kappa=1}^{\beta} b_{i,\kappa} b_{j,\kappa}\right|} \tag{15}$$

Figure 6 demonstrates the effect of NRD on the optimal BER performance for the SA-MCDCSK system in both AWGN and multipath Rayleigh fading channels, while Figures 7 and 8 show the influence of NRD on the values of optimal N in the multipath fading channel. In Figure 6, one can observe that NRD leads to a much better optimal BER performance. Figure 6 also indicates that optimal performances



(a) E_b/N_0=6dB.



(b) M=1024

Fig. 6. Simulated BER performances of the optimal SA-MCDCSK systems with and without NRD for various β.

of SA-MCDCSK without NRD are close to that of the coherent BPSK over an AWGN channel if the number of subcarriers is sufficiently large. With weaker noise in received data signals, the BER performance of SA-MCDCSK with NRD can be further optimized to approach that of the coherent BPSK, yet using significantly fewer subcarriers.

Figure 7 shows that NRD can lead to an optimal BER performance with small values of optimal N. This is beneficial to alleviating the spectral efficiency and energy efficiency decrement caused by allocating more subcarriers to reference. Compared to MC-DCSK that sends only one copy of reference, SA-MCDCSK requires more consumption in both frequency and energy because of the need to send N copies

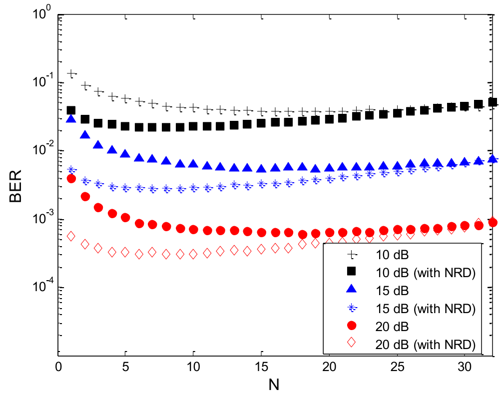


Fig. 7. Effect of NRD on the simulated BER of SA-MCDCSK system for $\beta = 128$ and $M = 64$ over a multipath Rayleigh fading channel.

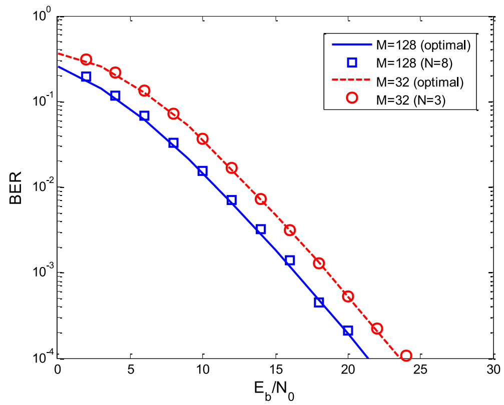


Fig. 8. BER comparison of the proposed system with fixed N and with optimal N over a multipath Rayleigh fading channel (β=128).

TABLE I
ENERGY EFFICIENCY COMPARISON BETWEEN MC-DCSK AND THE OPTIMAL SA-MCDCSK WITH 128 SUBCARRIERS

Spreading factor	E_b/N_0	SA-MCDCSK	SA-MCDCSK with NRD	MC-DCSK
16	5dB	85.9%	97.7%	99.2%
	10dB	89.8%	98.4%	99.2%
64	5dB	79.7%	96.1%	99.2%
	10dB	85.2%	98.4%	99.2%
128	5dB	75.8%	94.5%	99.2%
	10dB	82.0%	98.4%	99.2%

of the reference. To characterize the energy efficiency decrement, the Data energy to Bit energy Ratios (i.e., DBR defined in [15]) of MC-DCSK and of the optimal SA-MCDCSK system with and without NRD are compared in Table I for a subcarrier number of 128 in the AWGN channel. One can see that the optimal SA-MCDCSK system with NRD has a small energy efficiency decrement as compared with MC-DCSK. Similar conclusion can also be inferred for the spectral efficiency in the proposed system, as spectral efficiency is proportional to energy efficiency in SA-MCDCSK.

Interestingly, Figure 8 shows that, for SA-MCDCSK, NRD helps in making the optimal value of N insensitive to E_b/N_0. This indicates that SA-MCDCSK with NRD can yield the optimal BER performance without the need of altering the value of N under different SNR levels.

VI. CONCLUSION

We have proposed an improved MC-DCSK system, called SA-MCDCSK, in which we assign an optimal number of subcarriers to sending the reference signal, aiming to eliminate noise effectively and obtain better BER performance. The key idea is to transmit several copies of the reference signal over multiple subcarriers for reference diversity. By exploiting the redundancies in both the reference and the data signals, we succeeded in reducing noise drastically through a simple averaging of the received signals. Our proposed SA-MCDCSK system with the optimal number of reference-carrying subcarriers (an analytically predictable quantity) outperforms the existing MC-DCSK, NR-DCSK and DCSK systems over both AWGN and multipath fading channels. Our striking finding is that, with the noise reduction scheme so proposed, the BER performance is comparable to that of the coherent BPSK over AWGN channels, with only little decrease in energy and spectral efficiencies.

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