MAT 274, Section E, Fall 2002

Midterm Exam 2

10:40am-11:55am, Thursday, 11/7/2002

Name

ASU ID

I. Multiple-choice problems (8 points - 1 point each)

1. The second-order differential equation

$$t^2 \frac{d^2x}{dt^2} + t\frac{dx}{dt} + 4x = 0.$$

is

(a) nonlinear.

(b) linear.

(c) quadratic because of the term involving t^2.

(d) none of the above.

2. The following equation has many applications in engineering:

$$\frac{d^2x}{dt^2} + a(t)\frac{dx}{dt} + b(t)x = f(t),$$

where $a(t)$, $b(t)$, and $f(t)$ are continuous functions of time t. Let $x_h(t)$ be the general solution of the corresponding homogeneous equation, and let $x_p(t)$ be a particular solution of the original equation. The general solution of the original equation can be written as (C is an arbitrary constant)

(a) $x(t) = x_h(t)$.

(b) $x(t) = x_h(t) + Cx_p(t)$.

(c) $x(t) = C[x_h(t) + x_p(t)]$.

(d) $x(t) = Cx_h(t) + x_p(t)$.

3. An ideal mass-spring system is described by the following second-order equation:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0,$$

where ω is the internal oscillating frequency of the system. Given the initial conditions $x(0) = x_0$ and $x'(0) = v_0$, one finds that the solution satisfying these initial conditions can be written as $x(t) = A \sin(\omega t + \phi)$. The amplitude of the oscillation is

(a) $A = x_0$.

(b) $A = \sqrt{x_0^2 + (v_0/\omega)^2}$.

(c) $A = v_0$.

(d) $A = v_0/\omega$.

4. The asymptotic behavior of the solution to the following differential equation:

$$\frac{d^2x}{dt^2} + 9x = \sin(3t),$$

can be described as,

(a) $|x(t)| \to 0$ as $t \to \infty$.

(b) $|x(t)| \sim t^2$ at large t.

(c) $|x(t)| \sim t$ at large t.

(d) $|x(t)| \to$ constant $\neq 0$ as $t \to \infty$.

5. A forced, damped, linear oscillator can be described by,

$$\frac{d^2x}{dt^2} + c\frac{dx}{dt} + \omega^2 x = f(t),$$

where $c > 0$, $\omega \neq 0$, and $c^2 - 4\omega^2 < 0$. Which of the following is true regarding the asymptotic behavior of the solution at very large t?

(a) The asymptotic solution does not depend on initial conditions.

(b) The asymptotic solution depends on initial conditions.

(c) The asymptotic solution does not depend on c and ω.

(d) The asymptotic solution does not depend on the form of $f(t)$.

6. A suitable trial form for a particular solution of the constant-coefficient differential equation:

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = \cos t,$$

where $a \neq 0$ and $b \neq 1$, is

(a) $x_p(t) = A\cos t$.

(b) $x_p(t) = A\sin t$.

(c) $x_p(t) = A\cos t + B\sin t$.

(d) $x_p(t) = A\cos(2t)$.

7. The interval of existence of the following two functions $x_1(t) = e^t \sin t$ and $x_2(t) = \sin t$ is $I : (-\infty, +\infty)$. Which of the following statements regarding $x_1(t)$, $x_2(t)$, or their Wronskian is correct?

(a) $W = 0$ for all t in I.

(b) $W \neq 0$ for all t in I.

(c) $x_1(t)$ and $x_2(t)$ can be solutions of the same homogeneous, second-order linear differential equations.

(d) $x_1(t)$ and $x_2(t)$ cannot be solutions of the same homogeneous, second-order linear differential equations.

8. The steady-state solution of the following driven equation,

$$\frac{d^2x}{dt^2} + 4x = F\sin t,$$

can be written as $x_{ss}(t) = A\sin(t + \phi)$. Which of the following combinations of A and ϕ is correct?

(a) $A = F/3$ and $\phi = 0$.

(b) $A = F/3$ and $\phi = \pi/2$.

(c) $A = F/2$ and $\phi = 0$.

(d) $A = F/2$ and $\phi = \pi/2$.

II. (5 points) Use the method of *variation of parameters* to obtain the general solution to the following differential equation:

$$y'' - y' + \frac{y}{4} = 4e^{t/2}.$$

(*This is problem 3 from HW7. You will receive no credit if you don't use the method of variation of parameters.*)

III. (7 points) Analysis of a system of two masses and two springs results in the following set of two coupled equations:

$$\begin{aligned} u_1'' + 5u_1 &= 2u_2, \\ u_2'' + 2u_2 &= 2u_1, \end{aligned} \tag{1}$$

where $u_1(t)$ and $u_2(t)$ are the displacements of the two masses.

1. (1 point) Obtain the fourth-order, homogeneous equation in u_1.

2. (2 points) Write down the corresponding characteristic equation for u_1 and obtain all roots.

3. (2 points) Write down the general solution $u_1(t)$. From this, obtain the general solution $u_2(t)$. (*Hint: there should be four arbitrary constants altogether in $u_1(t)$ and $u_2(t)$.*)

4. (2 points) Obtain the solutions $u_1(t)$ and $u_2(t)$ under the initial conditions $u_1(0) = 0$, $u_1'(0) = 0$, $u_2(0) = 2$, and $u_2'(0) = 0$.

(This problem was explained in detail in class on Oct. 22. I told you after the break that this problem would likely appear on Exam 2).