# MAT 274, Section E, Fall 2002

## Midterm Exam 1

10:40am-11:55am, Thursday, 10/3/2002

**Name**

**ASU ID**

**I. Multiple-choice problems** (8 points - 1 point each)

1. The following differential equation describes the motion of a pendulum, subject to external periodic forcing and friction:

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \sin x = \cos(\omega t),$$

where $\alpha > 0$ and $\omega > 0$ are constants. The equation is

**(a)** first-order and linear.

**(b)** first-order and nonlinear.

**(c)** second-order and nonlinear.

**(d)** second-order and linear.

2. Consider two first-order equations: $dx/dt = x^p$, and $dy/dt = y^q$, where $p$ and $q$ are positive constants. With the initial conditions $x(0) = y(0) = 1$, one finds that $x(t) > y(t)$ for $t > 0$. It can be concluded that,

**(a)** $p > q$.

**(b)** $p = q$.

**(c)** $p < q$.

**(d)** $p = q + 1$.

3. Consider the equation: $(3xy + y^2)dx + (x^2 + xy)dy = 0$. Multiplying an integrating factor $\mu(x, y)$ yields

$$\mu(x, y)[(3xy + y^2)dx + (x^2 + xy)dy] = 0.$$

With what choise of $\mu(x, y)$ will the new equation be exact?

**(a)** $\mu(x, y) = constant$

**(b)** $\mu(x, y) = x$

**(c)** $\mu(x, y) = xy$

**(d)** $\mu(x, y) = x^2$.

4. The logistic equation: $dx/dt = x(1-2x)$ describes the dynamics of a simple population. Assume $x(0) > 0$. Which of the following is true?

   **(a)** For any $x(0) > 0$, $x(t) \to 0$ as $t \to \infty$.

   **(b)** For any $x(0) > 0$, $x(t) \to 1/2$ as $t \to \infty$.

   **(c)** If $x(0)$ is sufficiently large, then $x(t) \to \infty$ as $t \to \infty$.

   **(d)** If $x(0)$ is sufficiently small, then $x(t) \to 0$ as $t \to \infty$.

5. Consider the IVP for $t \geq 0$: $dy/dt = y^{1/3}$, $y(0) = 0$. Which of the following is true?

   **(a)** There exists no solution.

   **(b)** There exists a unique solution.

   **(c)** There are two distinct solutions.

   **(d)** There are three distinct solutions.

6. Eric and Mary want to solve the following first-order equation: $dx/dt = f(t, x)$ on computer by using the Euler's method. Eric uses a small step size $h$ and Mary uses the step size $h/4$. After solving the equation over an identical time interval, Eric and Mary find that their computer-generated solutions have errors $\epsilon_{eric}$ and $\epsilon_{mary}$, respectively. If they compare their errors, they find, roughly,

   **(a)** $\epsilon_{eric} \approx \epsilon_{mary}$.

   **(b)** $\epsilon_{eric} \approx 2\epsilon_{mary}$.

   **(c)** $\epsilon_{eric} \approx 4\epsilon_{mary}$.

   **(d)** $\epsilon_{eric} \approx 8\epsilon_{mary}$.

7. Look at the following first-order equation: $dx/dt + 2x = \cos(3t)$. At large $t$, which of the following best describes the qualitative behavior of the solution?

   **(a)** $x(t)$ is a periodic function of $t$.

   **(b)** $x(t)$ grows exponentially as $t$ increases.

   **(c)** $x(t)$ decreases exponentially as $t$ increases.

   **(d)** $x(t)$ is a linear function of $t$.

8. Given the following IVP: $dx/dt = \sqrt{x}$ and $x(0) = 1$. Let $S : (-0.5 \leq t \leq 0.5, 0.5 \leq x \leq 1.5)$ be a region in the $(t, x)$ plane. Which of the following is true?

   **(a)** There is no solution in $S$.

   **(b)** There are many distinct solutions in $S$.

   **(c)** There is a unique solution in $S$.

   **(d)** The information given is not sufficient for me to decide whether there is a unique solution in $S$.

II. (4 points) Find the solution for the following IVP: $dx/dt + x = e^{-t}$, $x(0) = 1$. What is the limit $\lim_{t\to\infty} x(t)$?

III. (8 points) Experimental observations of falling objects in the presence of air resistance suggest that in some cases, the force due to air resistance is proportional to the *square* of the velocity. Thus, the velocity of a falling object in this situation can be modeled as,

$$m\frac{dv}{dt} = -mg + kv^2,$$

where $k > 0$ is a constant of proportionality.

1. (2 points) Assume that in the time of interest, the object never hits the ground. Argue that the velocity of the object tends to a constant value as $t$ becomes large. Without actually solving the differential equation, write down an expression for this constant velocity (terminal velocity).

2. (3 points) Find an implicit solution for $v(t)$.

3. (3 points) Find an explicit solution for $v(t)$, from which write down an expression for the terminal velocity. (*Hint*: how to get rid of the absolute sign in part 2?)