Lecture 1 ppt – Intro. to Complex Net. (1/9/2018)

Lecture 2 Network Basics (1/11/2018)

1. Basic network characterization

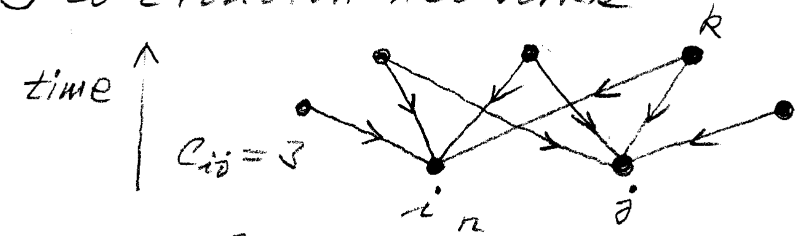
Weighted? / Directed?	N	Y
N	$A_{ij} = A_{ji} = 1$ i •——• j	$W_{ij} = W_{ji} \neq 0$ i ●━━● j
Y	$A_{ij} = 1,\ A_{ji} = 0$ i ←——• j	$W_{ij} \neq 0,\ W_{ji} = 0$ i ⟵━━● j

$\begin{cases} A_{ij} = 0,\ A_{ji} = 0 \\ A_{ii} = 0 \end{cases}$ → no link between i & j

$A_{n \times n}$ – adjacency matrix for N/N case

Examples:

① Co-Citation networks



i & j both cited by a node, say k

then ⇒ $\underbrace{A_{ik}}_{1} \underbrace{A_{jk}}_{1} = 1$ simultaneously

$$C_{ij} = \sum_{k=1}^{n} A_{ik} A_{jk} = \sum_{k=1}^{n} A_{ik} A^{T}_{kj}$$

$$\Rightarrow \quad C = A \cdot A^{T} \quad \text{– cocitation matrix (network)}$$

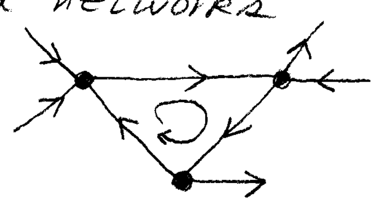
$$C^{T} = C \quad \text{– } C \text{ – symmetric}$$

– weighted / undirected

$$C_{ii} = \sum_{k=1}^{n} A_{ik}^{2} = \sum_{k=1}^{n} A_{ik}$$ – total # of edges pointing to i

② Acyclic directed networks

Cyclic

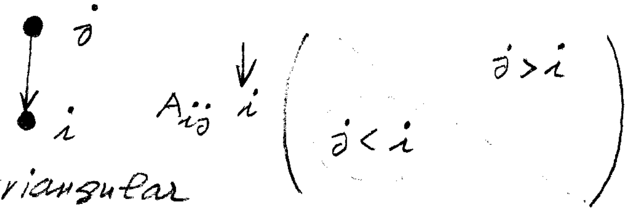


Acyclic

arbitrary path from any node must end at a node w/o outgoing link

Reordering $A_{ij} = 0$ for $j < i$

nodal index ↑



A — upper triangular
— All eigenvalues = 0 →

So, acyclic network → eigenvalues of $A = 0$
Opposite, ? ← nilpotent A

For a cyclic network, must have at least one non-zero eigenvalue — true or false?

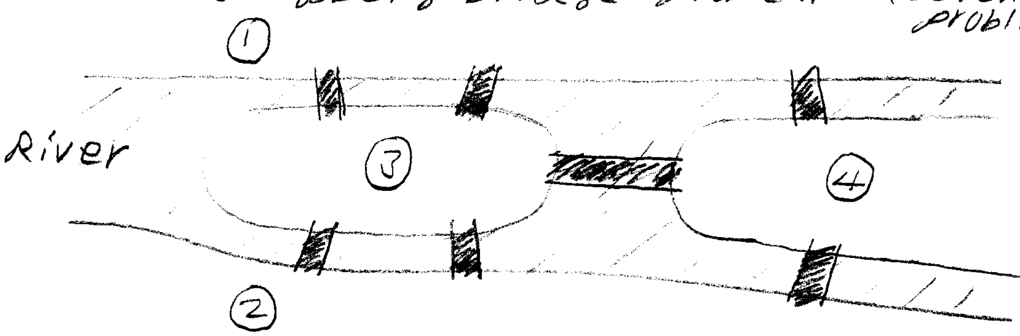
A result from Sec. 6.10

$$L_r = \sum_{i=1}^{n} \kappa_i^{\,r} \qquad \kappa_i \text{ — } i\text{th eigenvalue of } A$$

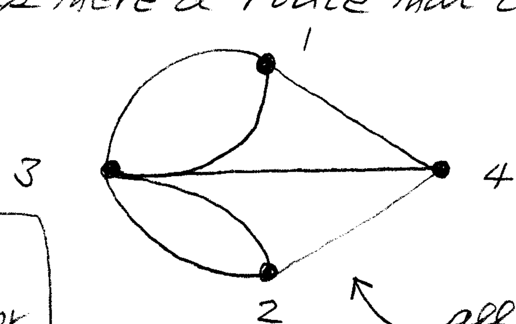
↑ total # of cycles of length r

Say the network is cyclic ⇒ $L_r > 0$
⇒ at least one term in $L_r \neq 0$
⇒ some $\kappa_i \neq 0$ → TRUE

③ Euler's graph
— Königsberg bridge problem (seven-bridge problem)



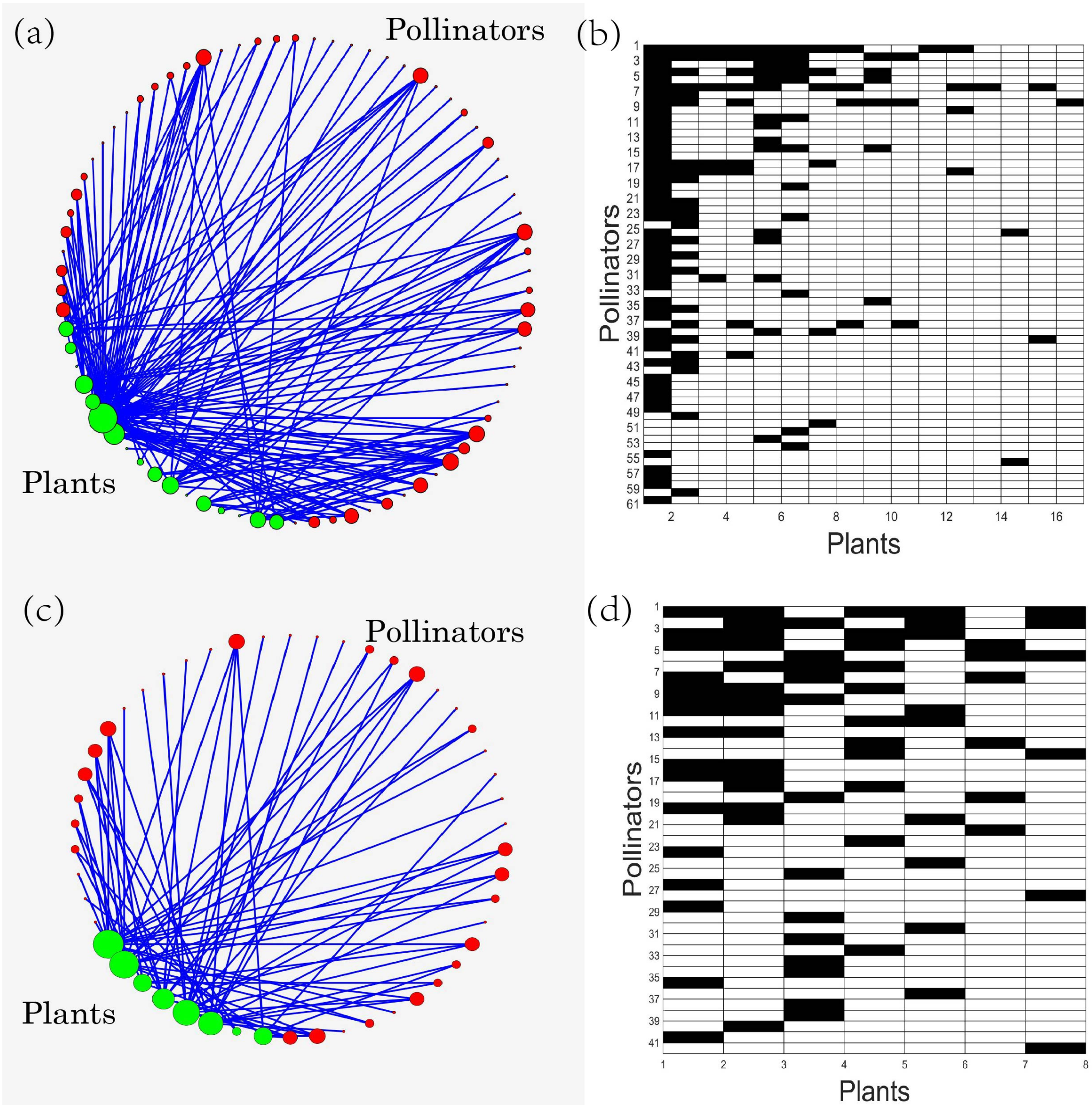
Is there a route that crosses all seven bridges exactly once? ← Eulerian path
— must both enter and leave a node except the first & last ones
— at most two nodes with odd degree



all four nodes have odd degrees ⇒ no Eulerian path exists!

④ Bipartite network
plant-pollinator network
— ppt

Real Bipartite Networks: Pollinator-Plant Networks of Mutualistic Interactions



Network A: Data from Hicking, Norfold, UK - 61 Pollinators, 17 plants, and 146 mutualistic interactions [L. Dicks, S. Corbet, and R. Pywell, "Compartmentalization in plant-insect flower visitor web," *J. Anim. Ecol.* **71**, 32-43 (2002)]

Network B: Data from Hestehaven, Denmark – 42 pollinators, 8 plants, and 79 mutualistic connections [A. C. Montero, "The ecology of three pollinator network," Master thesis, Aarhus University, Denmark (2005)]

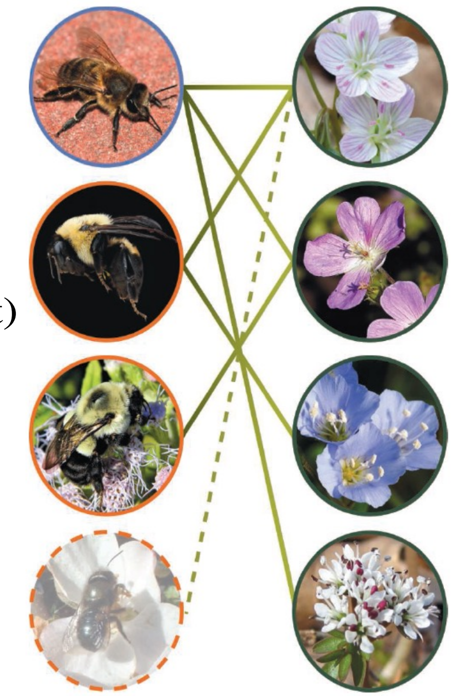
Data from 59 such networks are currently available: ***http://www.web-of-life.es***

Nonlinear Dynamical Network

$$\frac{dP_i}{dt} = P_i\left(\alpha_i^{(P)} - \sum_{j=1}^{S_P}\beta_{ij}^{(P)}P_j + \frac{\sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j}{1 + h\sum_{j=1}^{S_A}\gamma_{ij}^{(P)}A_j}\right)_i + \mu_P,$$

Holling type-II dynamics

$$\frac{dA_i}{dt} = A_i\left(\alpha_i^{(A)} - \kappa_i - \sum_{j=1}^{S_A}\beta_{ij}^{(A)}A_j + \frac{\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j}{1 + h\sum_{j=1}^{S_P}\gamma_{ij}^{(A)}P_j}\right) + \mu_A,$$



$\gamma_{ij} = \varepsilon_{ij}\dfrac{\gamma_0}{(k_i)^t}$, $0 \le t \le 1$ ($t = 0$: structure has no effect; $t = 1$: structure is important)

$\varepsilon_{ij} = 1$ if plant/pollinator i and pollinator/plant j are connected; 0 otherwise;

P_i, A_i – Abundance of ith plant and ith pollinator;

S_P, S_A – numbers of plants and pollinators;

$\alpha_i^{(P)}, \alpha_i^{(A)}$ – intrinsic growth rates of ith plant and ith pollinator;

β_{ii}, β_{ij} – intraspecific and interspecific competition strength ($\beta_{ii} >> \beta_{ij}$);

μ_P, μ_A – immigration of plants and pollinators;

γ_0 – strength of mutualistic interaction;

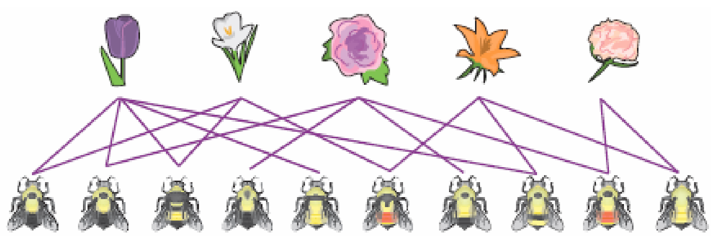
κ_i – pollinator decay rate - bifurcation parameter

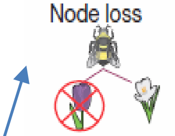
- Lever, Nes, Scheffer, and Bascompte, "The sudden collapse of pollinator communities," *Ecol. Lett.* **17**, 350-359 (2014)
- Rohr, Saavedra, and Bascompte, "On the structural stability of mutualistic systems," *Science* **345**, 1253497 (2014).
- J.-J. Jiang, Z.-G. Huang, T. P. Seager, W. Lin, C. Grebogi, A. Hastings, and Y.-C. Lai, "Predicting tipping points in mutualistic networks through dimension reduction," *PNAS (Plus),* in press

Forcing due to Human Activities

Cause of perturbation: global warming caused climate change, excessive use of pesticides leading to death of pollinators, loss of habitats due to pollution, etc.

Bipartite mutualistic network





Parameter change