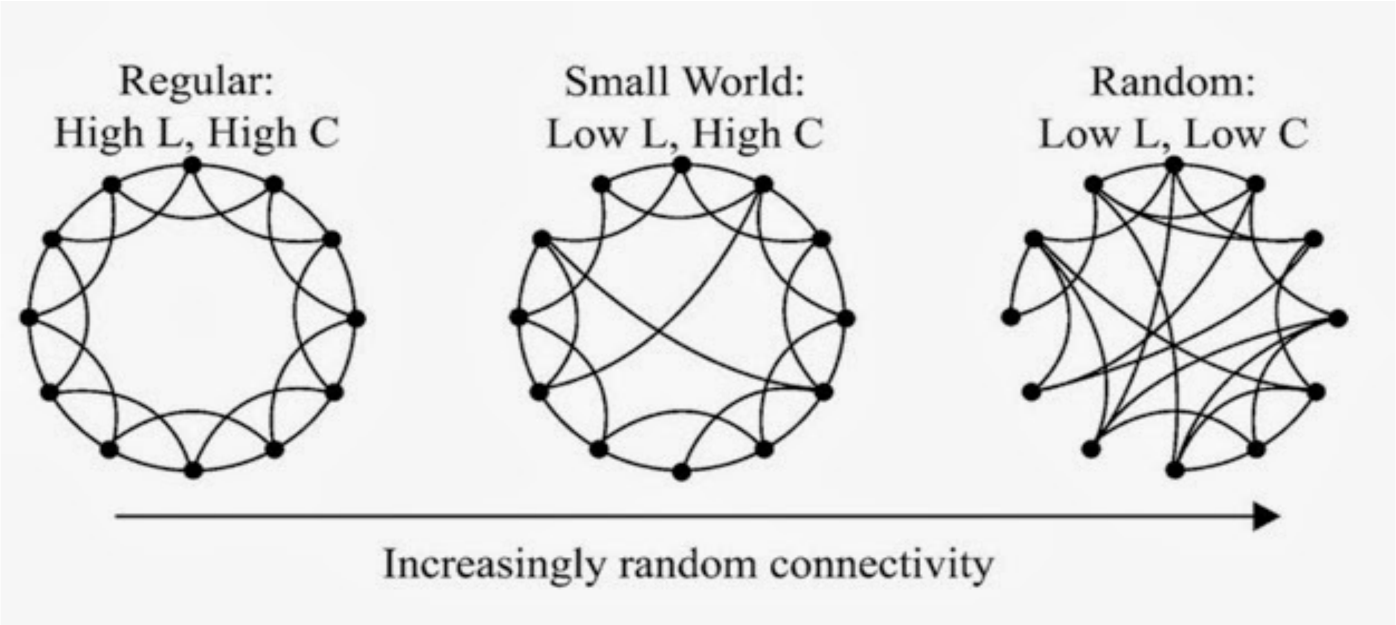
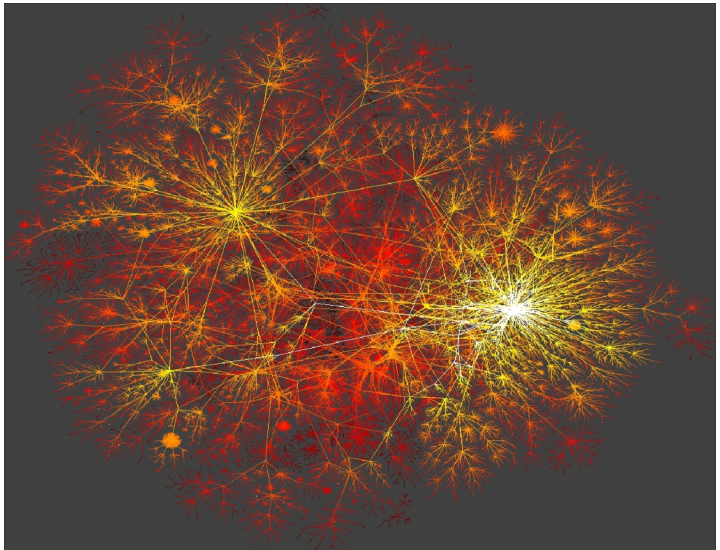
Current Problems in Complex Networks

- Prediction
- Control
- Network Resilience
- Tipping point

Complex Networks



- Random networks: Erdos and Renyi, 1959
- Small-world networks: Watts and Strogatz, 1998



Scale-free networks
Barabasi and Albert, 1999

Modern Network Science and Engineering

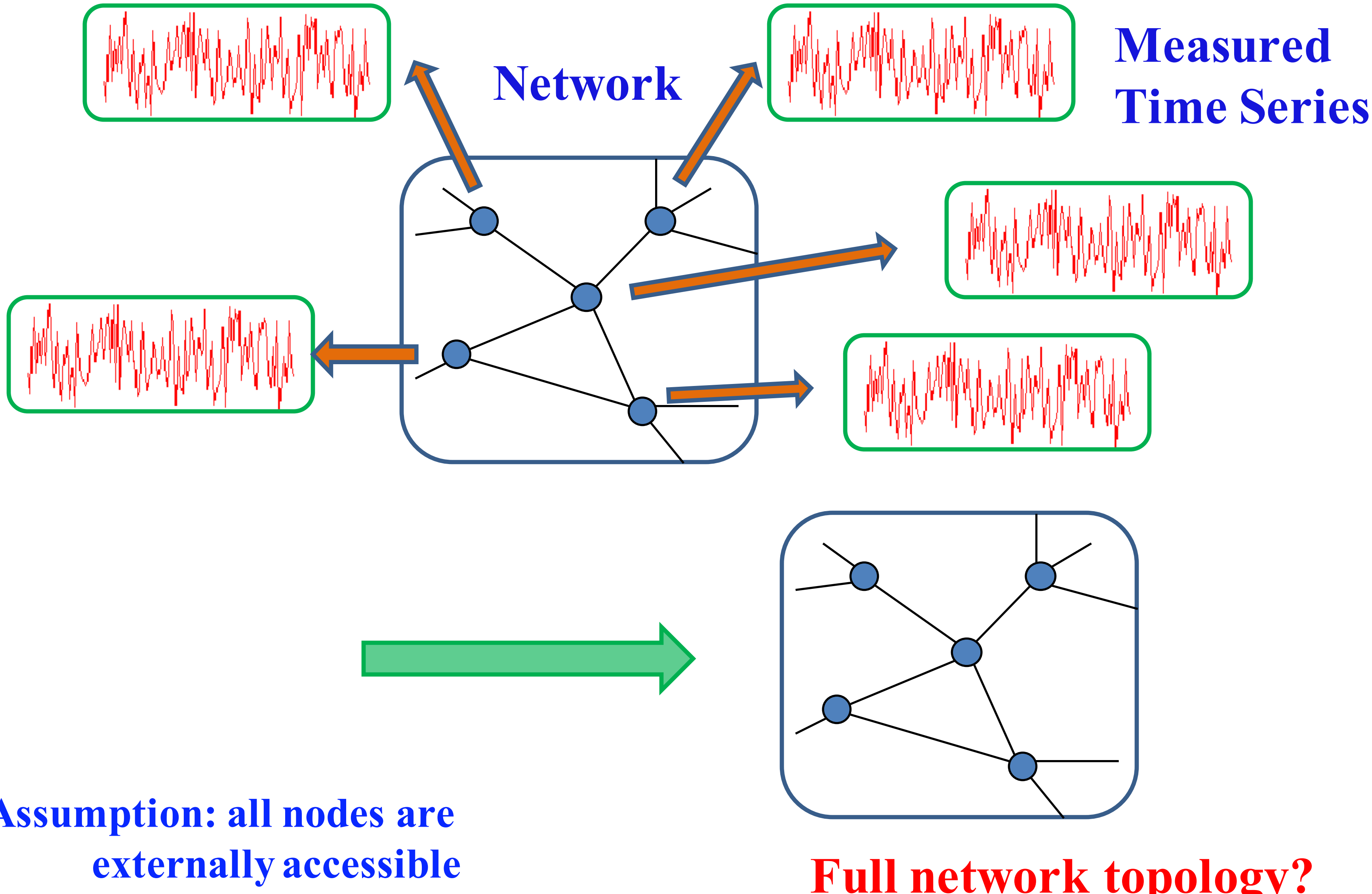
Before 2010

- Complex network topologies and the responsible mechanisms
- Dynamics on complex networks – synchronization, virus spreading, traffic flows, information propagation, percolation, cascading dynamics (network security), etc.
- Dynamics of networks – evolution of network structures with time and how dynamics on networks are affected

After 2010

- **Prediction**
- **Control**
- **Network Resilience**
- **Tipping point**
- **…**

Reverse Engineering of Complex Networks



Assumption: all nodes are externally accessible

A Recent Review Article

Physics Reports 644 (2016) 1–76

Contents lists available at ScienceDirect

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journal homepage: www.elsevier.com/locate/physrep

Data based identification and prediction of nonlinear and complex dynamical systems

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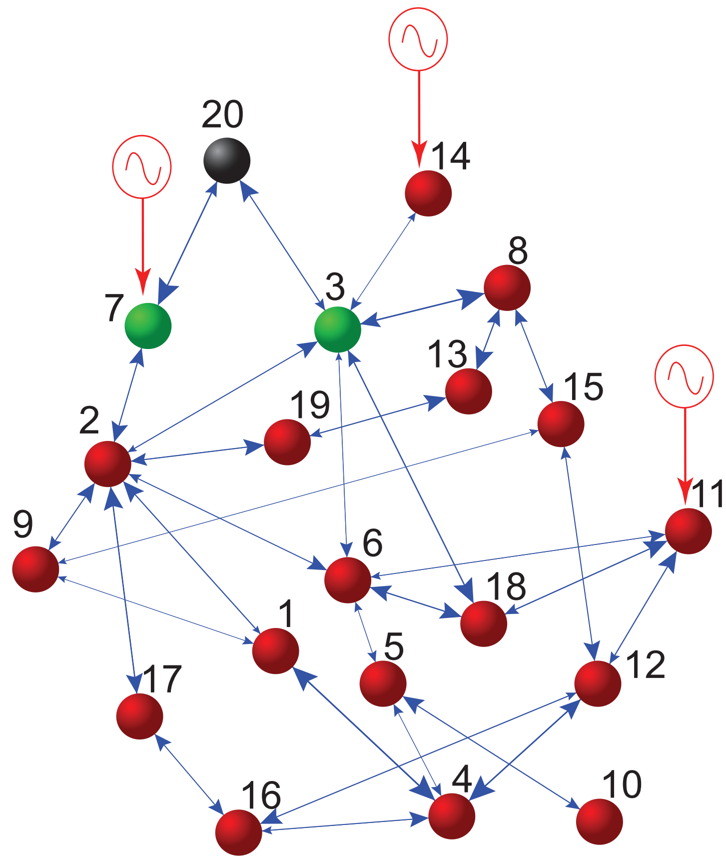
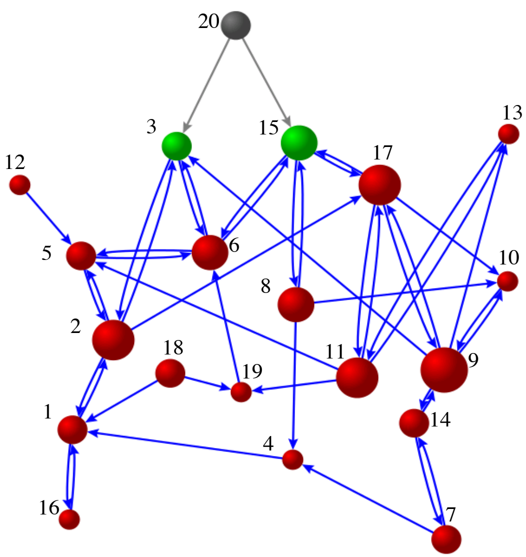
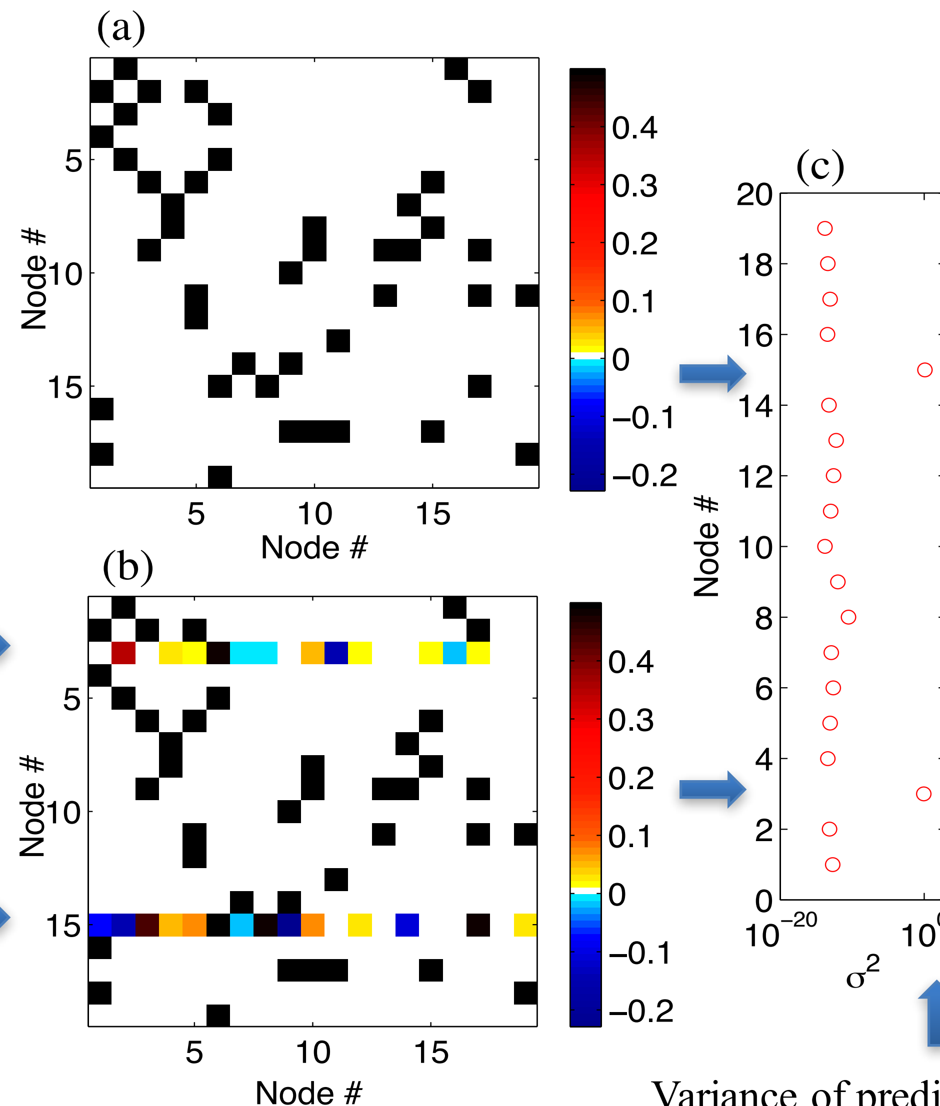
Detecting Hidden Nodes



No information is available from the black node. How can we ascertain its existence and its location in the network? How can we distinguish hidden node from local noise sources?

Detecting Hidden Node





<u>**Idea**</u>

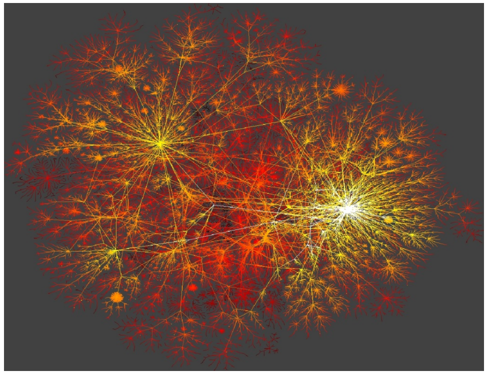
- Two green nodes: immediate neighbors of hidden node
- Information from green nodes is not complete
- Anomalies in the prediction of connections of green nodes

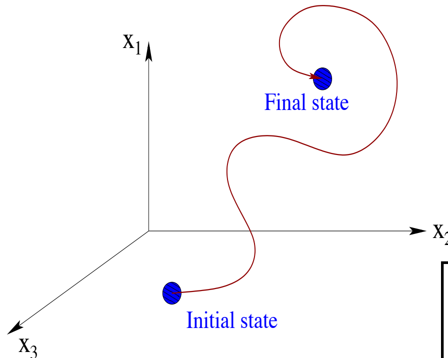
Variance of predicted coefficients

Data Based Prediction of Complex Networks: Outstanding Problems

- Locating multiple hidden nodes
- Reconstructing full network structure with incomplete data
- Detecting sources of spreading (diffusion) based on binary data
- Network analysis based on big data
- …

Linear Network Control: Well Done





A dynamical system is controllable if it can be driven from **any** initial state to **any** desired final state **in finite time** by suitable choice of input control signals.

General Mathematical framework: **Kalman's Controllability Rank Condition**

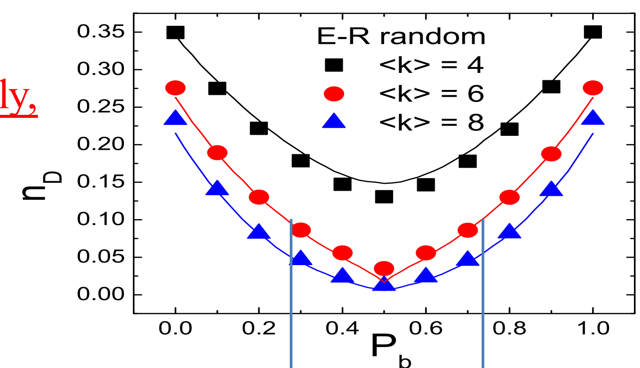
Focus of existing works: **minimal number of signals** required to control the network

e.g., Structural controllability: Y.-Y. Liu, J.-J. Slotine, and A.-L. Barabasi, "Controllability of complex networks," *Nature* **473**, 167 (2011)

Exact controllability: Z.-Z. Yuan, C. Zhao, Z.-R. Di, W.-X. Wang, and Y.-C. Lai, "Exact controllability of complex networks," *Nat. Commun.* **4**, 2447 (2013)

Issue: controllability is mathematically well defined but physically, control may be difficult

In terms of **ENERGY**



High probability of energy divergence

Energy bounds: G. Yan, J. Ren, Y.-C. Lai, C. H. Lai, B. Li, "Controlling complex networks: how much energy is needed?" *PRL* **108,** 218703 (2012).

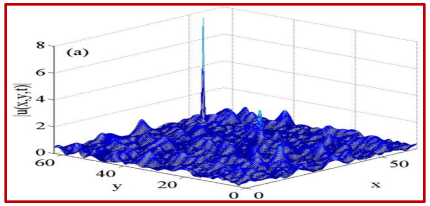
Energy scaling: Y.-Z. Chen, L.-Z. Wang, W.-X. Wang, and Y.-C. Lai, "Energy scaling and reduction in controlling complex networks," *Roy. Soc. Open Sci.* **3**, 160064 (2016).

Physical controllability: L.-Z. Wang, Y.-Z. Chen, W.-X. Wang, and Y.-C. Lai, "Physical controllability of complex networks," *SREP* **7**, 40198 (2017).

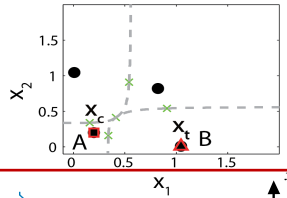
Controlling Complex Nonlinear Dynamical Networks: DIVERSITY

1. Lack of a general mathematical control/controllability framework
2. Extremely **diverse** nonlinear dynamical behaviors require a diverse array of control methodologies:

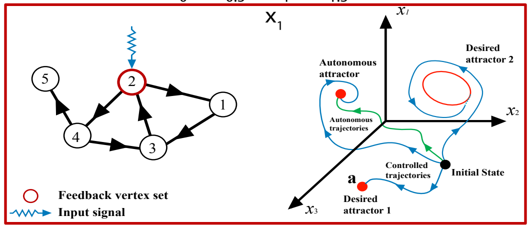
- Controlling **collective dynamics**, e.g., Y.-Z. Chen, Z.-G. Huang, and Y.-C. Lai, "Controlling extreme events on complex networks," *SREP* **4**, 6121 (2014)



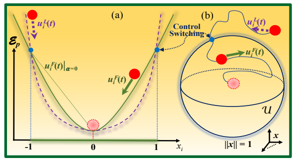
- Controlling **destinations** (attractors), e.g., L.-Z. Wang, R.-Q. Su, Z.-G. Huang, X. Wang, W.-X. Wang, C. Grebogi, and Y.-C. Lai, "A geometrical approach to control and controllability of complex nonlinear dynamical networks," *Nat. Commun.* **7**, 11323 (2016).



- Control principle based on feedback vertex set – ongoing work

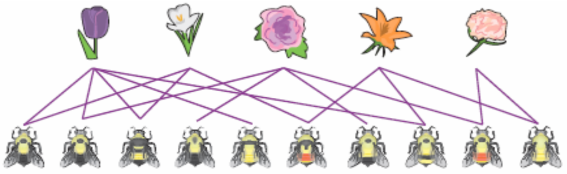


- Closed-loop control, Y.-Z. Sun, S.-Y. Leng, Y.-C. Lai, C. Grebogi, and W. Lin, *Phys. Reve. Lett.* **119**, 198301 (2017)



- Predicting and controlling tipping point in complex mutualistic networks

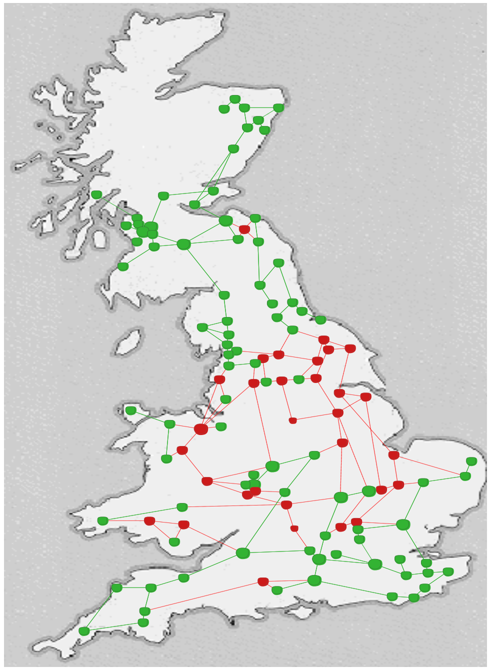
1. J.-J. Jiang, Z.-G. Huang, T. P. Seager, W. Lin, C. Grebogi, A. Hastings and Y.-C. Lai, *PNAS (Plus),* in press
2. J.-J. Jiang, A. Hastings, and Y.-C. Lai, "Controlling tipping point in complex systems," preprint (2017)



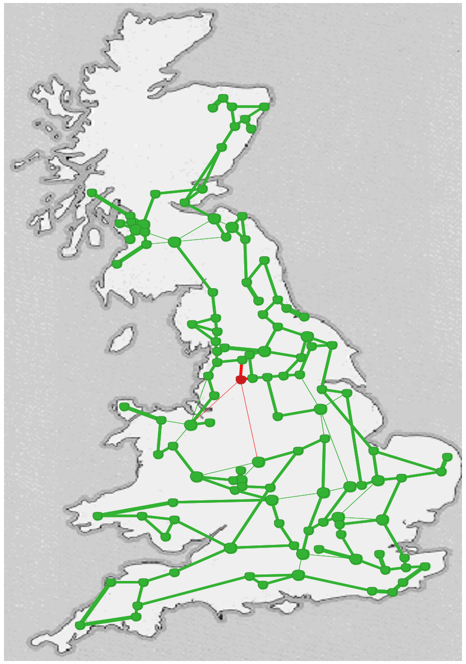
Controlling Complex Networks: Outstanding Problems

- General framework of controllability for nonlinear networks – the role of network topology?
- Data based control of nonlinear networks
- Control of time varying, nonlinear dynamical networks
- Optimizing flows on complex networks through control
- Control of tipping point in complex networks

Cascading Failures in Complex Networks – Control?



Without Control



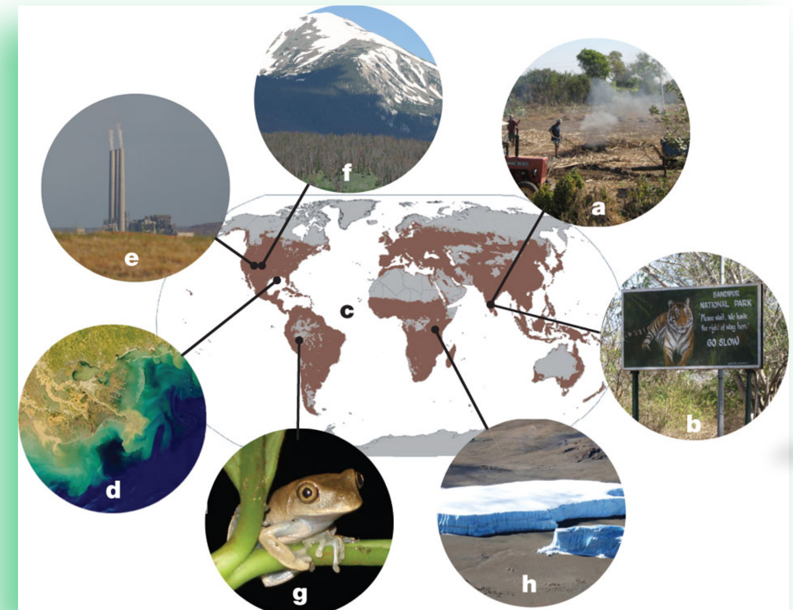
With Control

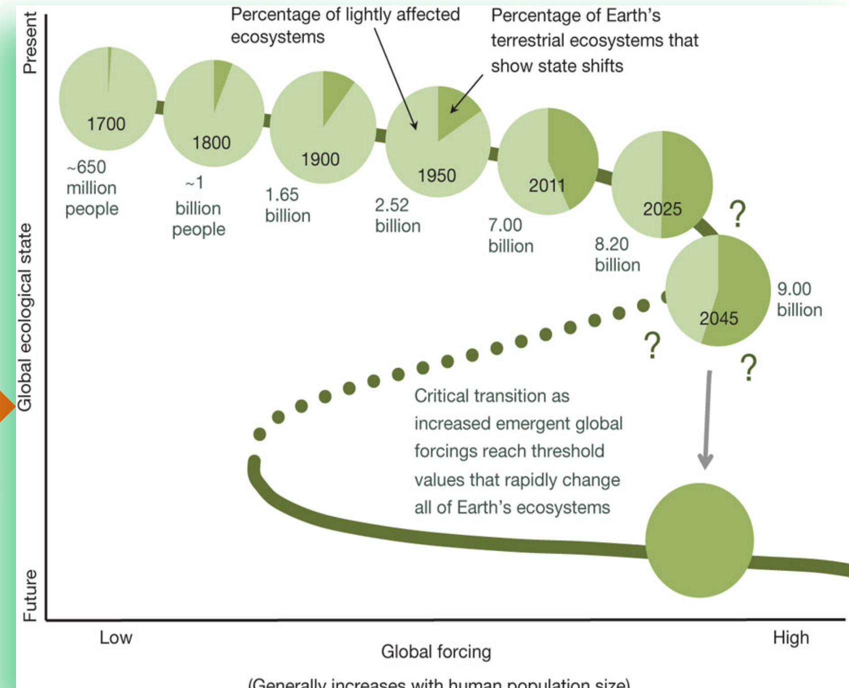
A simple weight-based control scheme:
R. Yang, W.-X. Wang, Y.-C. Lai, G.-R. Chen, Phys. Rev. E 79, 026112 (2009).

Cascading Dynamics in Complex Networks: Outstanding Problems

- Data based prediction of cascading failures
- "Ranking" of nodes or clusters of nodes in terms of their vulnerability to cascading failures
- Cascading dynamics in time varying networks
- Control strategies
- …

Tipping point: Prediction & Control?



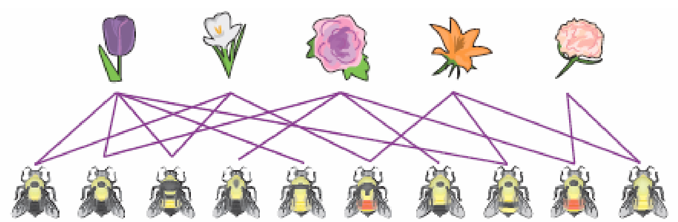


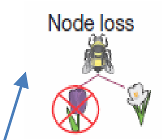
Barnosky, Anthony D., et al. *Nature* **486**, 52-58 (2012).

Perturbation Types

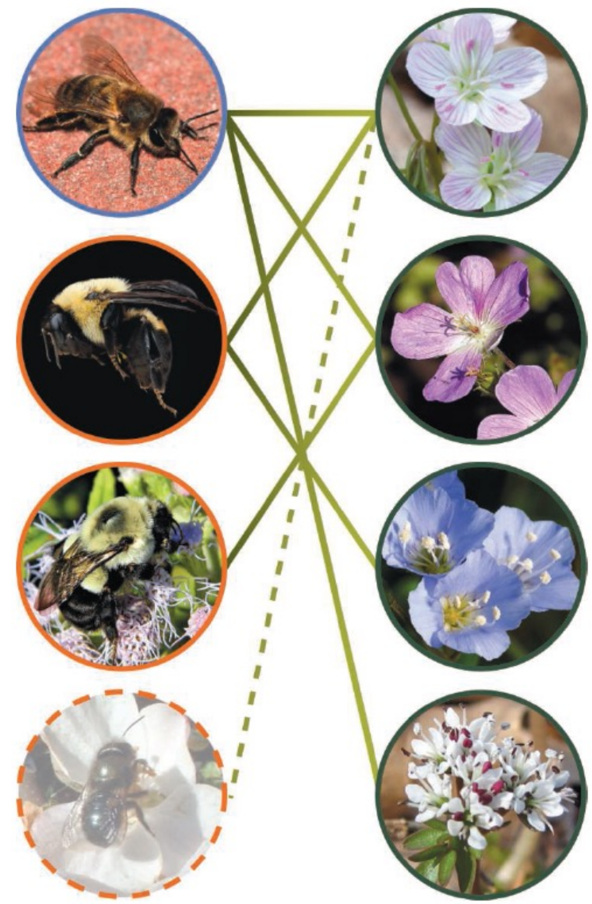
Cause of perturbation: global warming caused climate change, excessive use of pesticides leading to death of pollinators, loss of habitats due to pollution, etc.

Bipartite mutualistic network





Parameter change

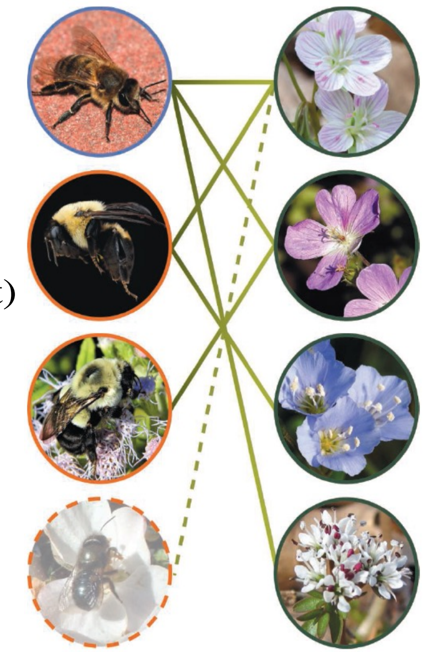


Nonlinear Network of Mutualistic Interactions

$$\frac{dP_i}{dt} = P_i \left(\alpha_i^{(P)} - \sum_{j=1}^{S_P} \beta_{ij}^{(P)} P_j + \frac{\sum_{j=1}^{S_A} \gamma_{ij}^{(P)} A_j}{1 + h \sum_{j=1}^{S_A} \gamma_{ij}^{(P)} A_j} \right)_i + \mu_P ,$$

$$\frac{dA_i}{dt} = A_i \left(\alpha_i^{(A)} - \kappa_i - \sum_{j=1}^{S_A} \beta_{ij}^{(A)} A_j + \frac{\sum_{j=1}^{S_P} \gamma_{ij}^{(A)} P_j}{1 + h \sum_{j=1}^{S_P} \gamma_{ij}^{(A)} P_j} \right) + \mu_A ,$$

Holling type-II dynamics



$\gamma_{ij} = \varepsilon_{ij} \dfrac{\gamma_0}{(k_i)^t}$, $0 \le t \le 1$ ($t = 0$: structure has no effect; $t = 1$: structure is important)

$\varepsilon_{ij} = 1$ if plant/pollinator i and pollinator/plant j are connected; 0 otherwise;

P_i, A_i – Abundance of ith plant and ith pollinator;

S_P, S_A – numbers of plants and pollinators;

$\alpha_i^{(P)}, \alpha_i^{(A)}$ – intrinsic growth rates of ith plant and ith pollinator;

β_{ii}, β_{ij} – intraspecific and interspecific competition strength ($\beta_{ii} >> \beta_{ij}$);

μ_P, μ_A – immigration of plants and pollinators;

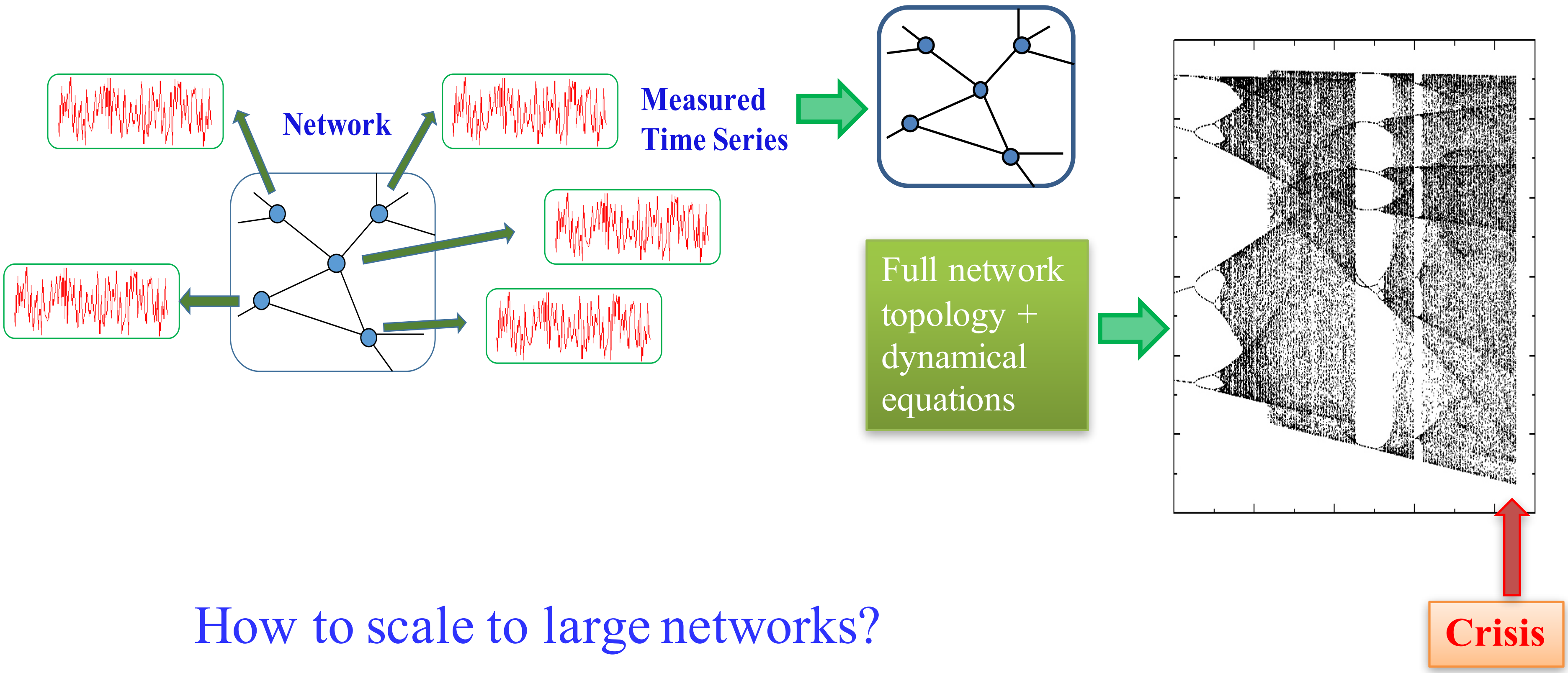
γ_0 – strength of mutualistic interaction;

κ_i – pollinator decay rate - bifurcation parameter

Possible control parameters

- Lever, Nes, Scheffer, and Bascompte, "The sudden collapse of pollinator communities," *Ecol. Lett.* **17**, 350-359 (2014)
- Rohr, Saavedra, and Bascompte, "On the structural stability of mutualistic systems," *Science* **345**, 1253497 (2014).
- J.-J. Jiang, Z.-G. Huang, T. P. Seager, W. Lin, C. Grebogi, A. Hastings, and Y.-C. Lai, "Predicting tipping points in mutualistic networks through dimension reduction," *PNAS (Plus),* in press

Predicting Tipping Point: Data-Driven Method?



How to scale to large networks?

Tipping Points in Complex Networked Systems: Outstanding Problems

- Unified theory for tipping point dynamics (e.g., phase transition in statistical physics)
- Control of tipping point in mutualistic networks
- Predicting tipping points in social networks
- Tipping point in dynamical power grids – prediction and control?
- …