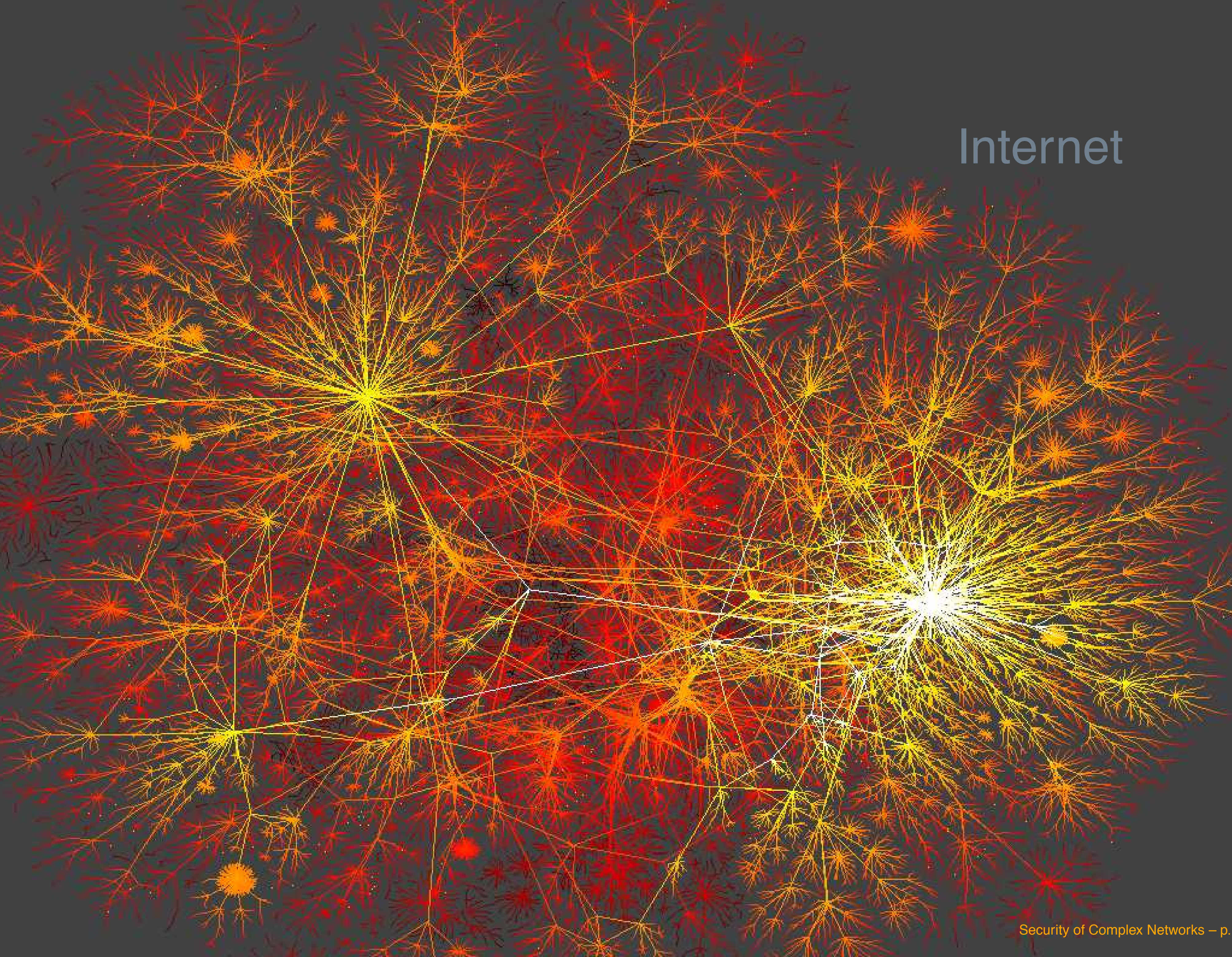
Internet



Early Model

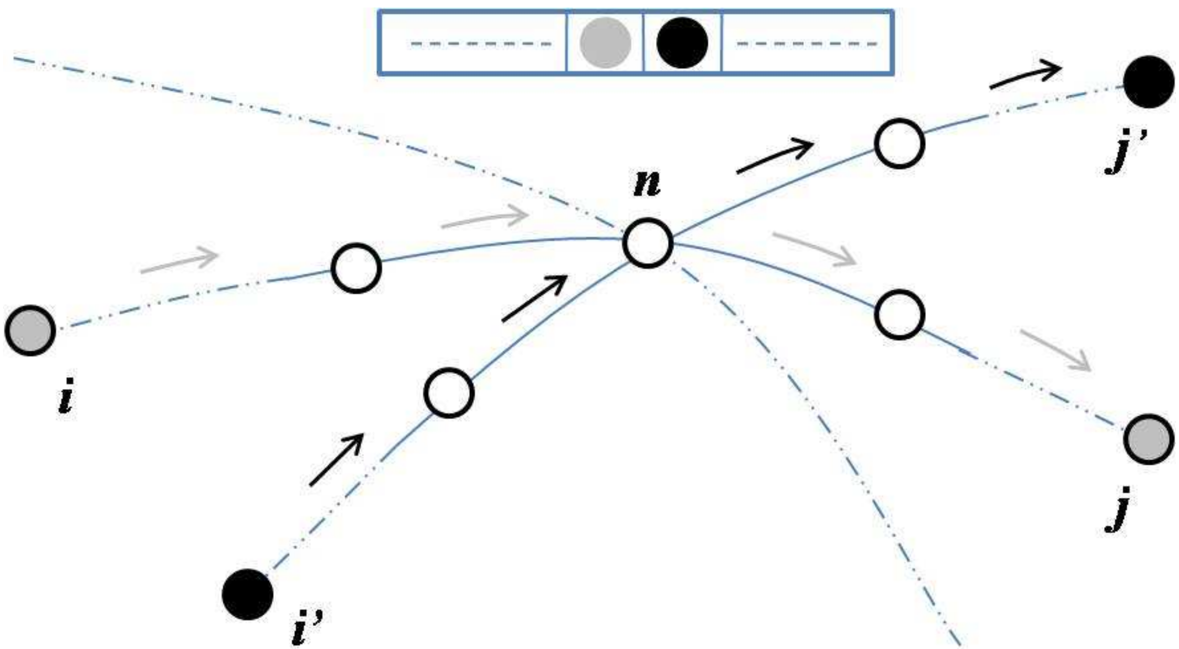
- Capacity of node: maximum load that it can handle.
- In reality, capacity is limited by cost.
- Capacity C_j of node j is assumed to be proportional to its initial load L_j,

$$C_j = (1+\alpha)L_j, \quad j = 1, 2, ...N,$$

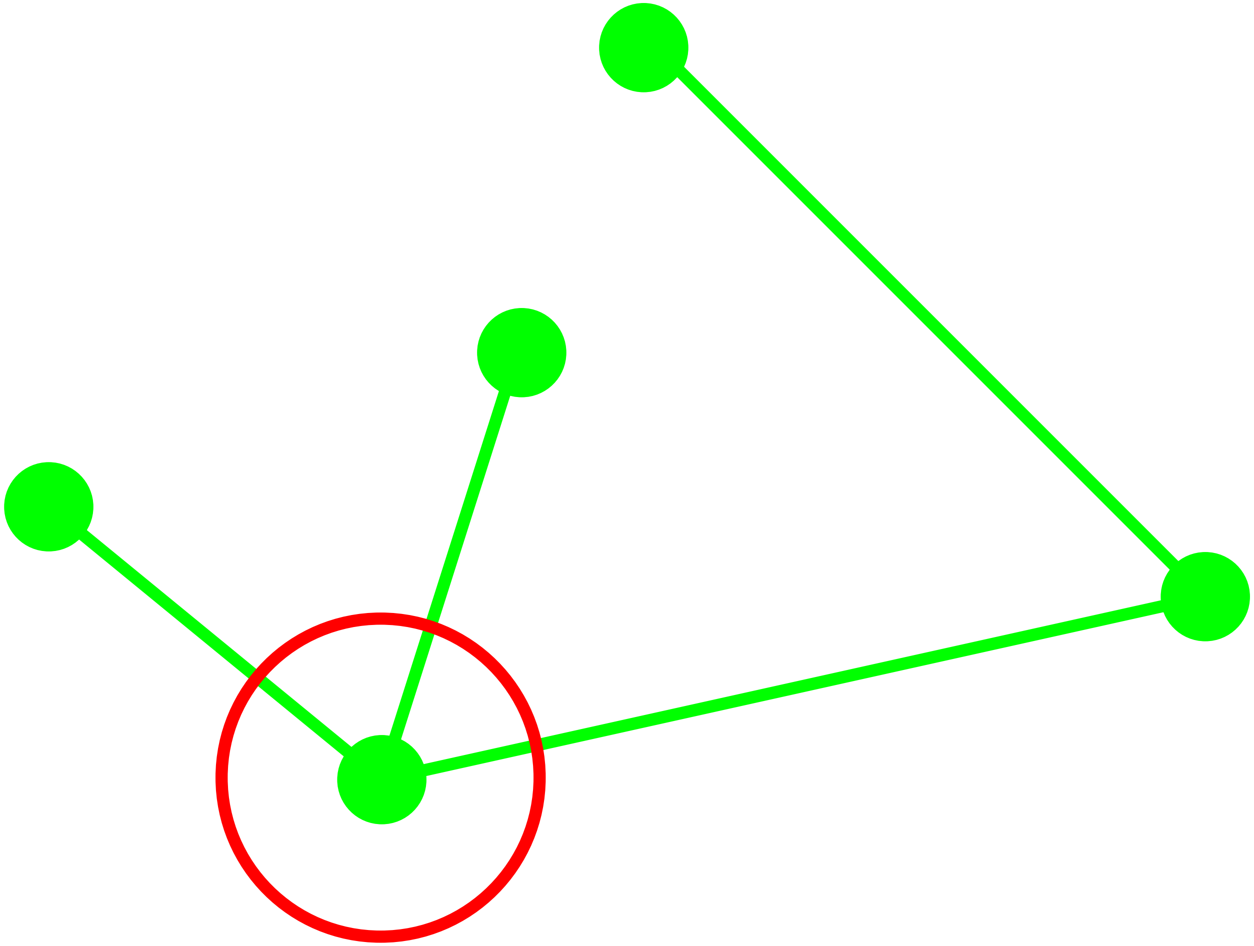
  where $\alpha \geq 0$ is the tolerance parameter.
- A node fails if its load $> C$.
- Cascading failure: nodes fail (due to attack or random failure) $\rightarrow$ load redistribution $\rightarrow$ more nodes fail $\rightarrow$ load redistribution $\rightarrow \ldots$
- A. E. Motter and Y.-C. Lai, Phys. Rev E 66, 065102(R) (2002).

Load (Idealized)

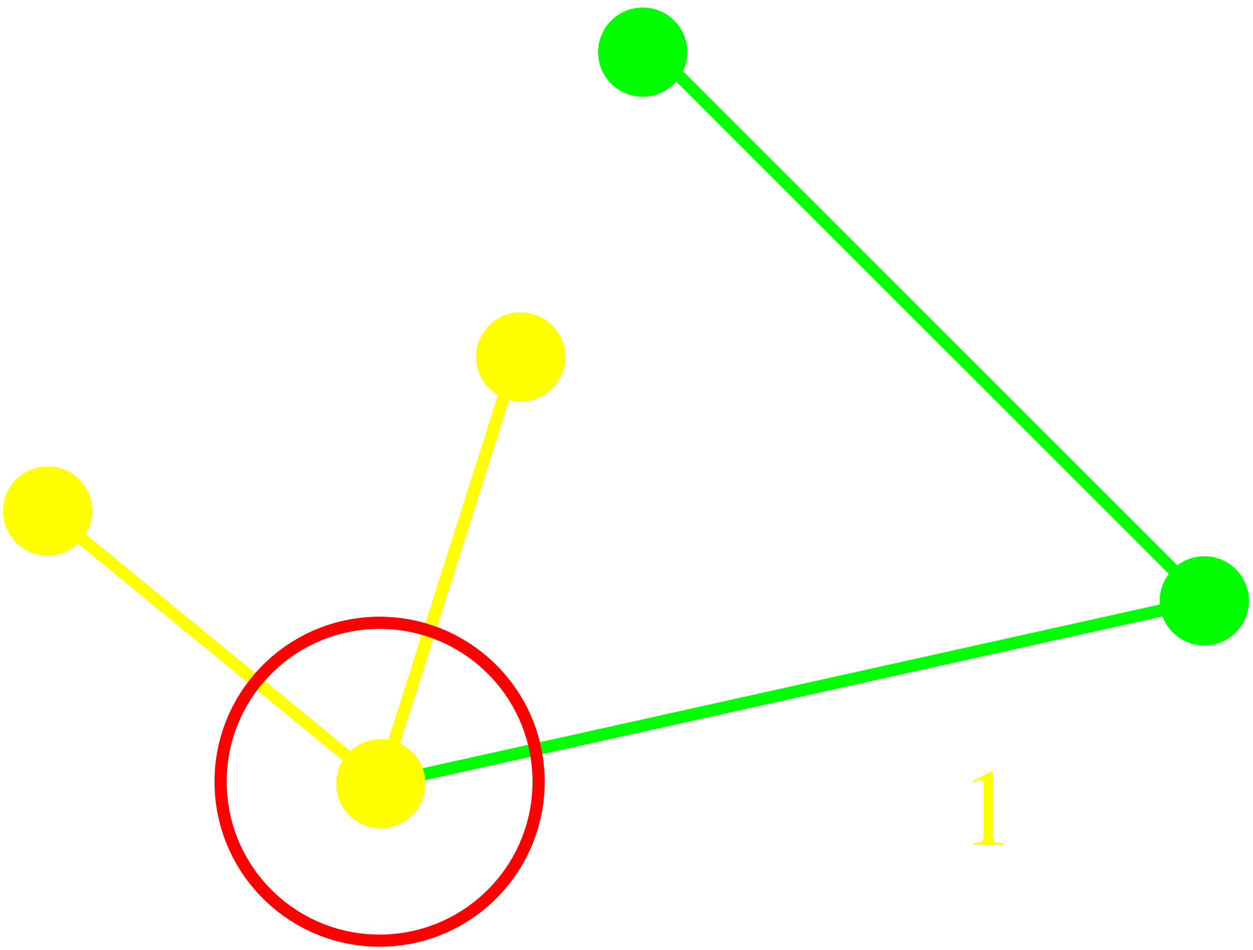
Load on node n (or link) is defined as the number of shortest paths between all pairs of nodes passing through n.



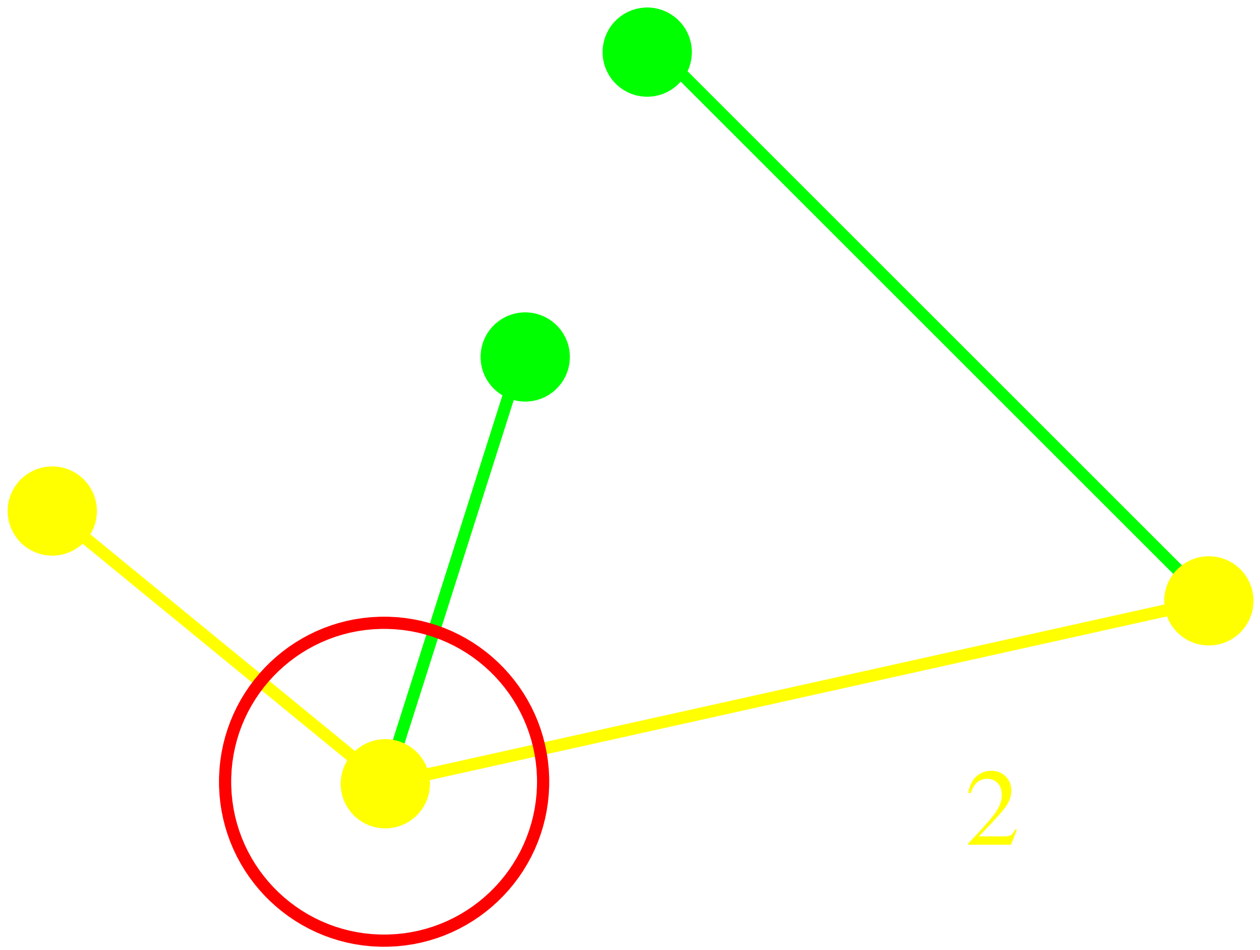
Load: A Simple Example



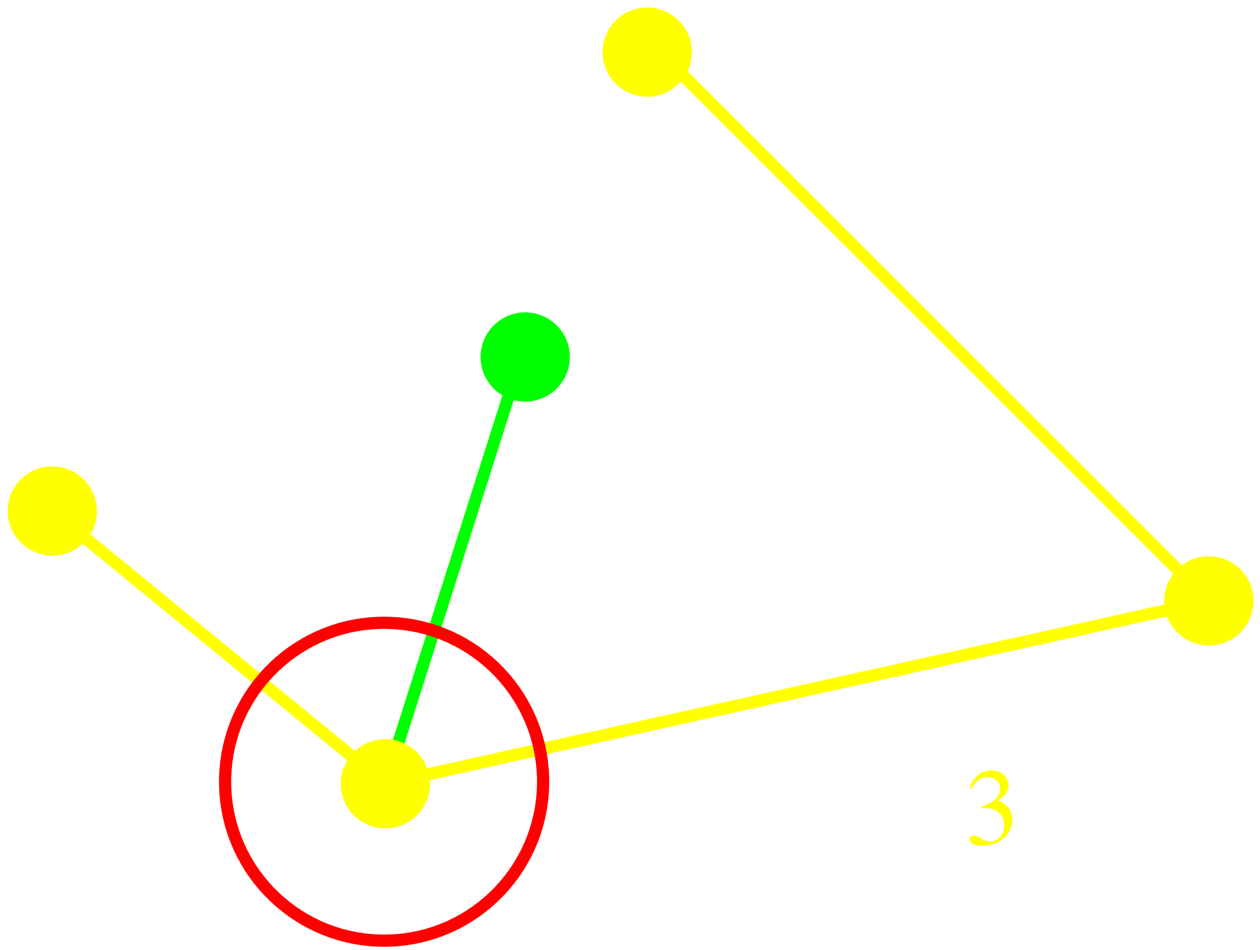
Load: A Simple Example



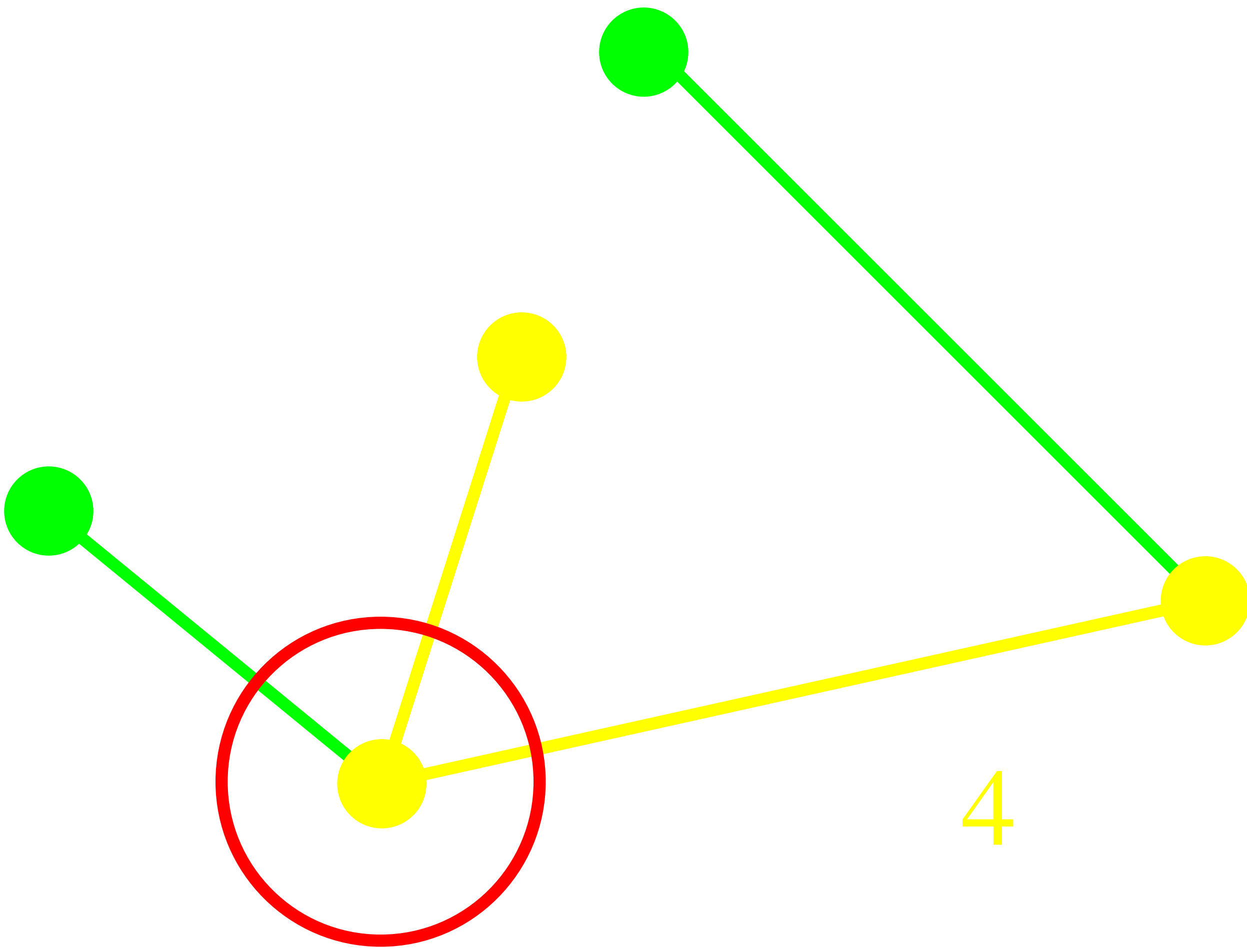
Load: A Simple Example



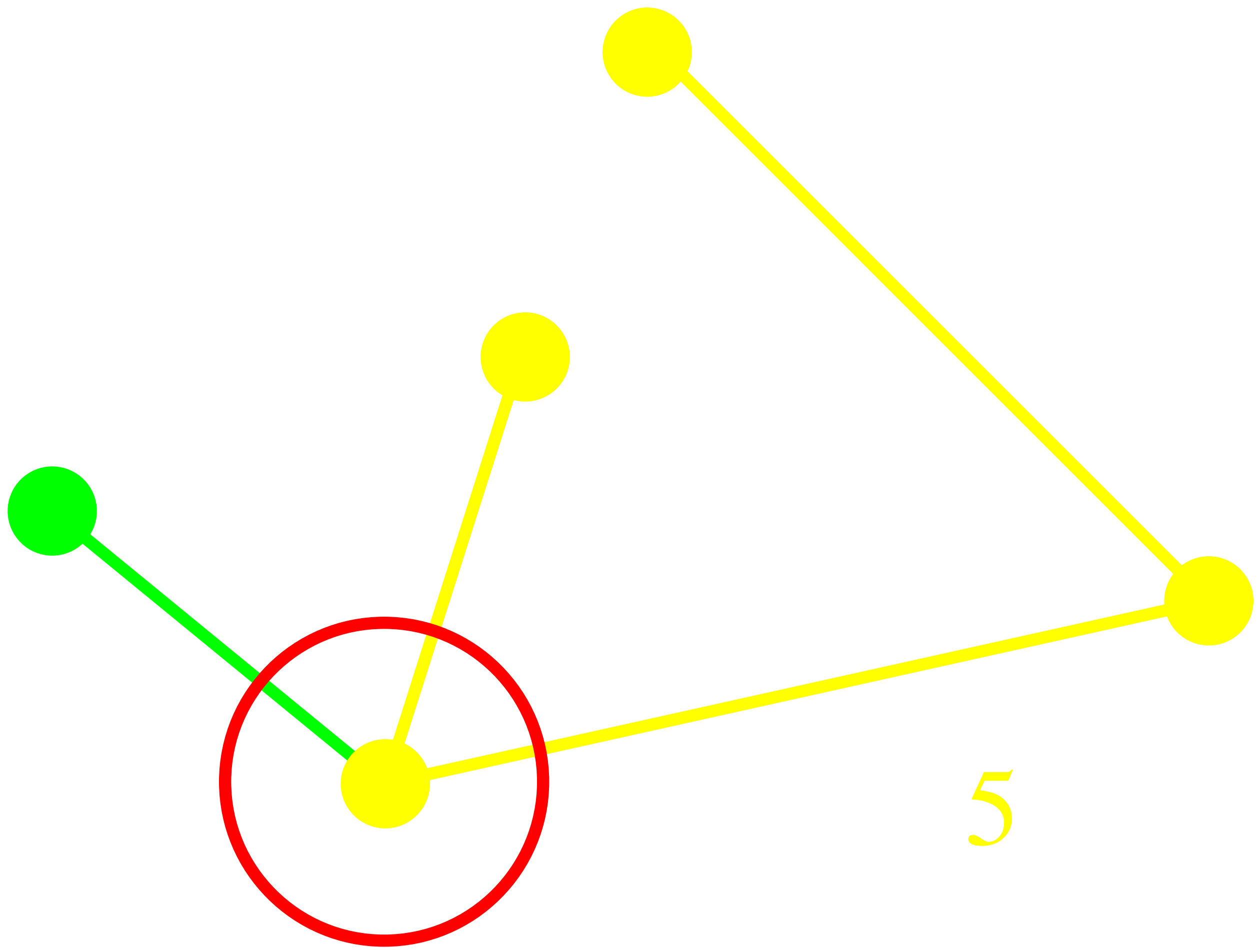
Load: A Simple Example



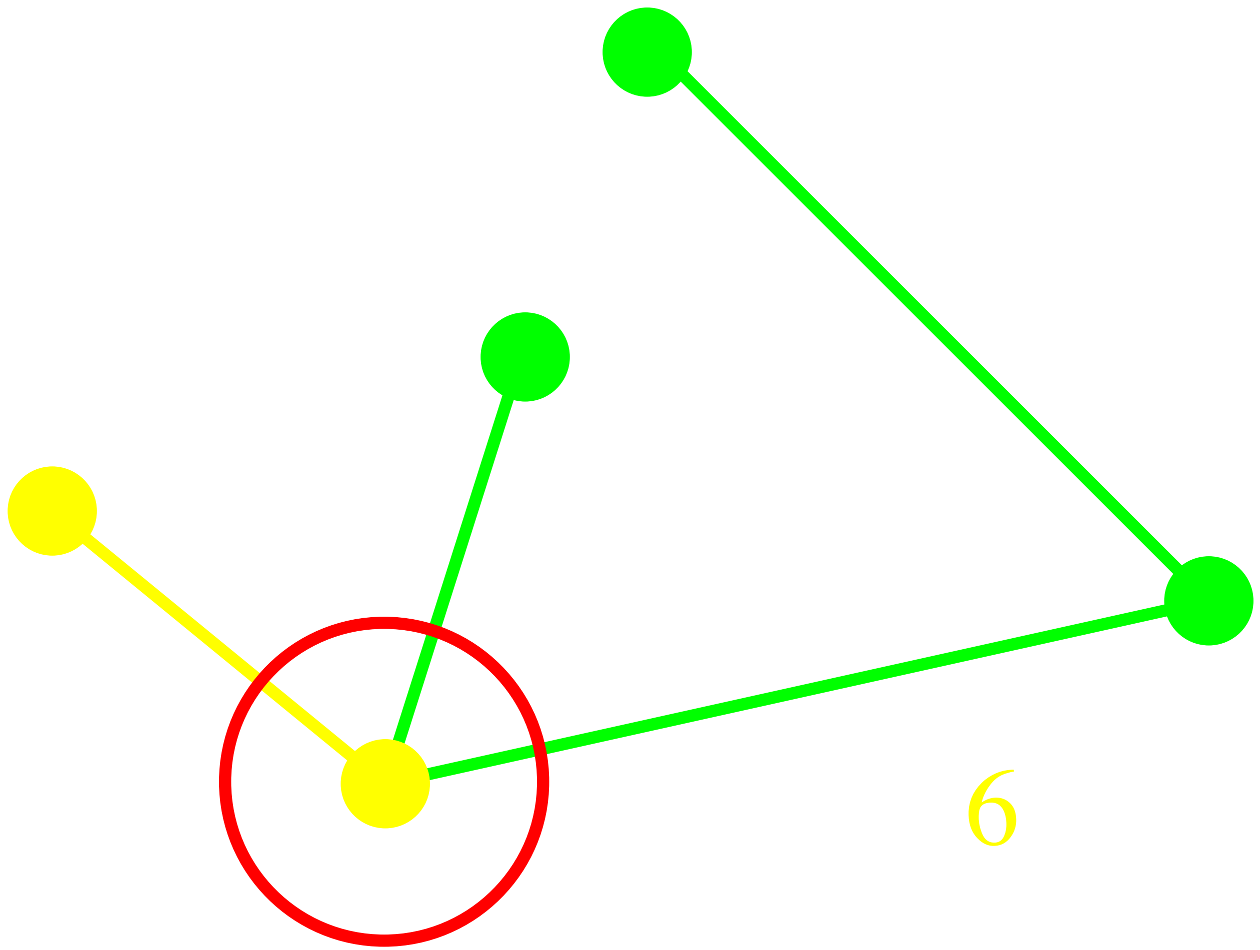
Load: A Simple Example



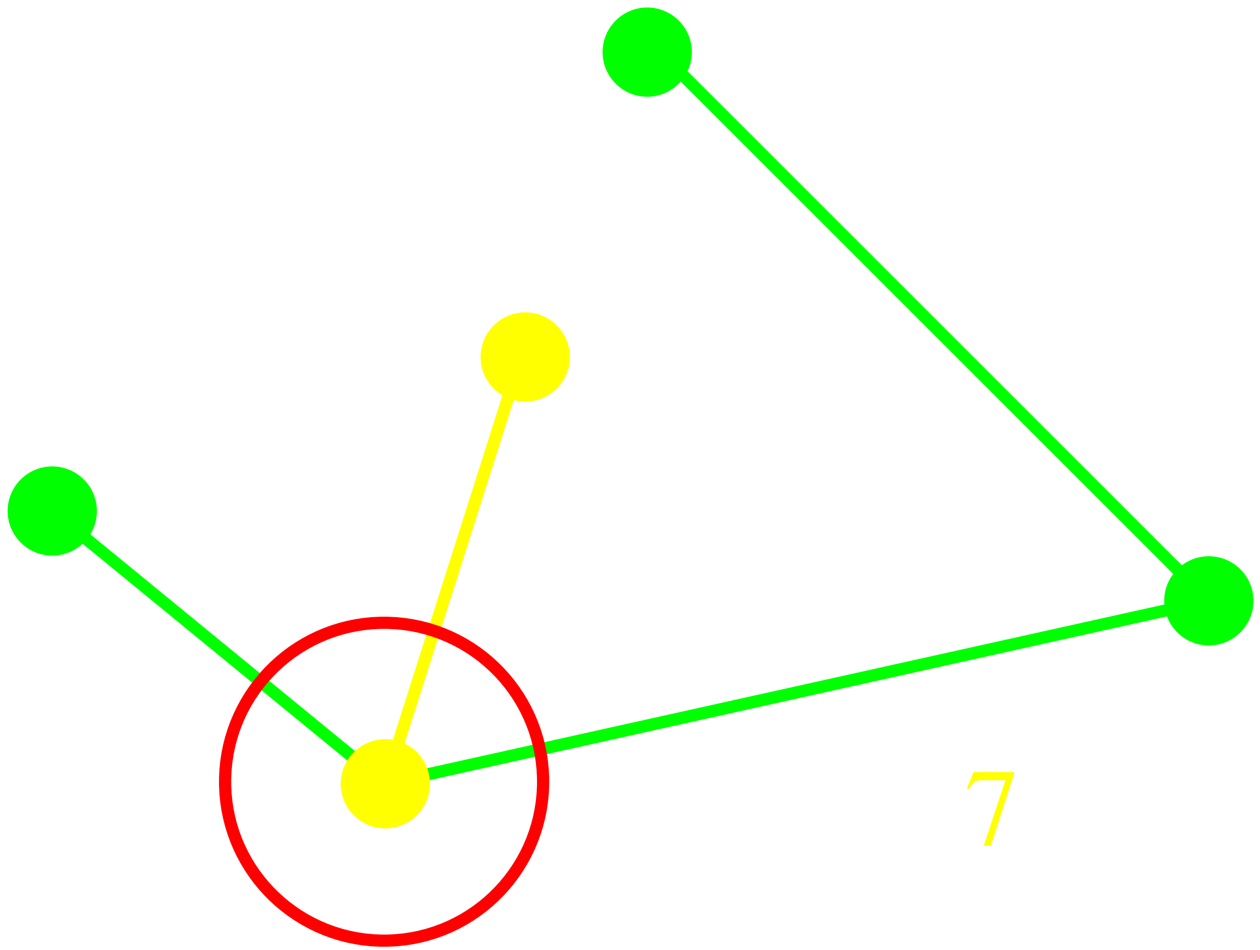
Load: A Simple Example



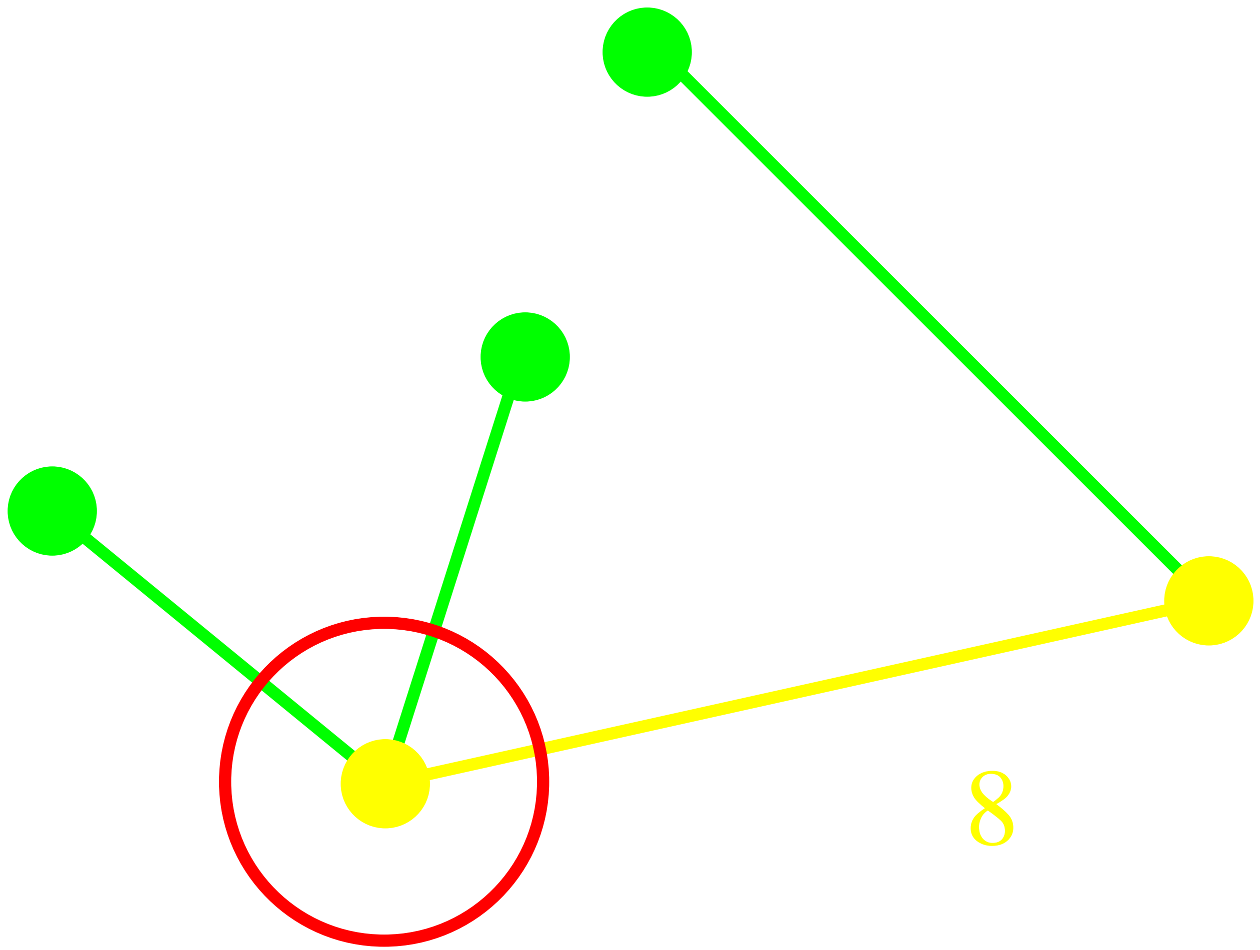
Load: A Simple Example



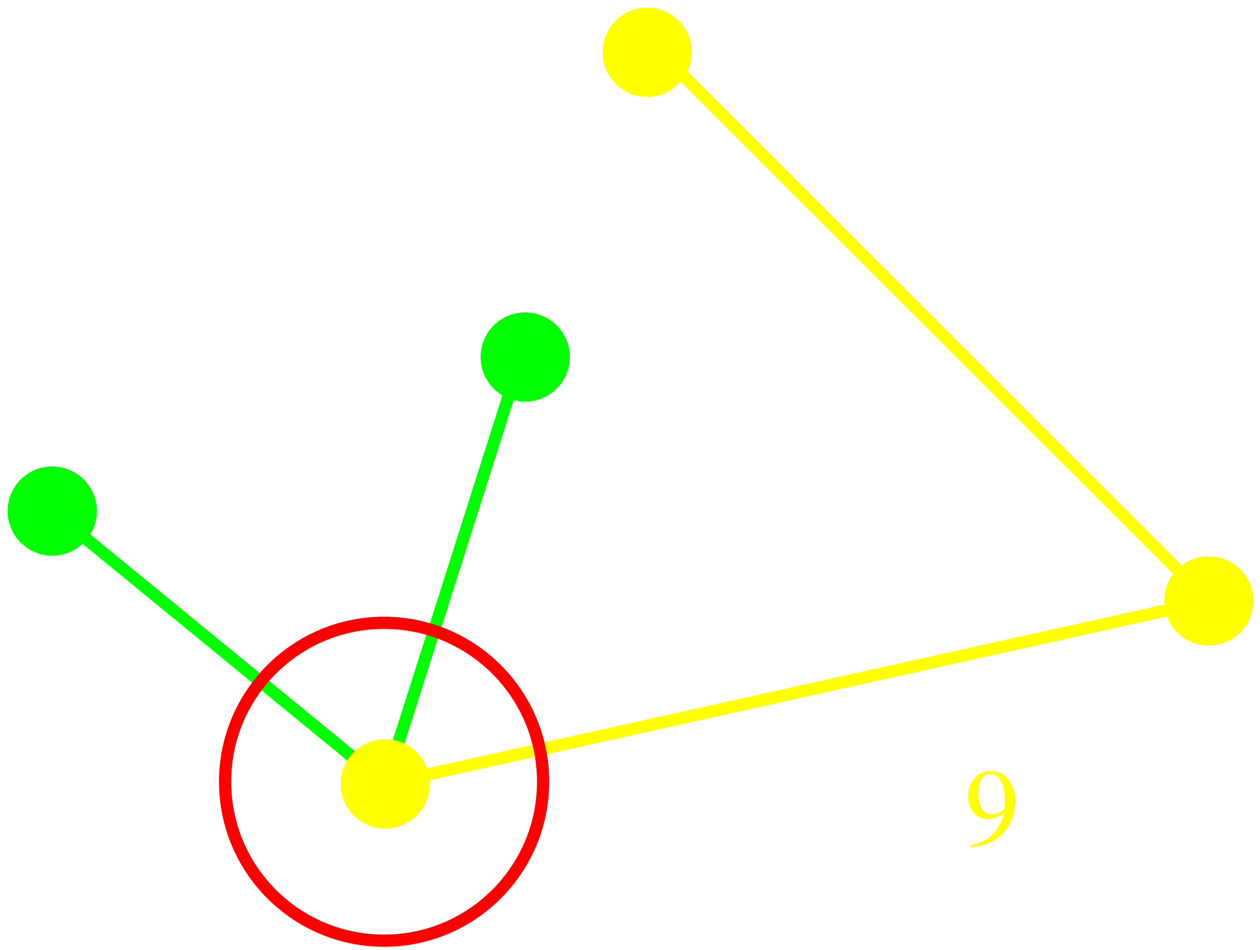
Load: A Simple Example



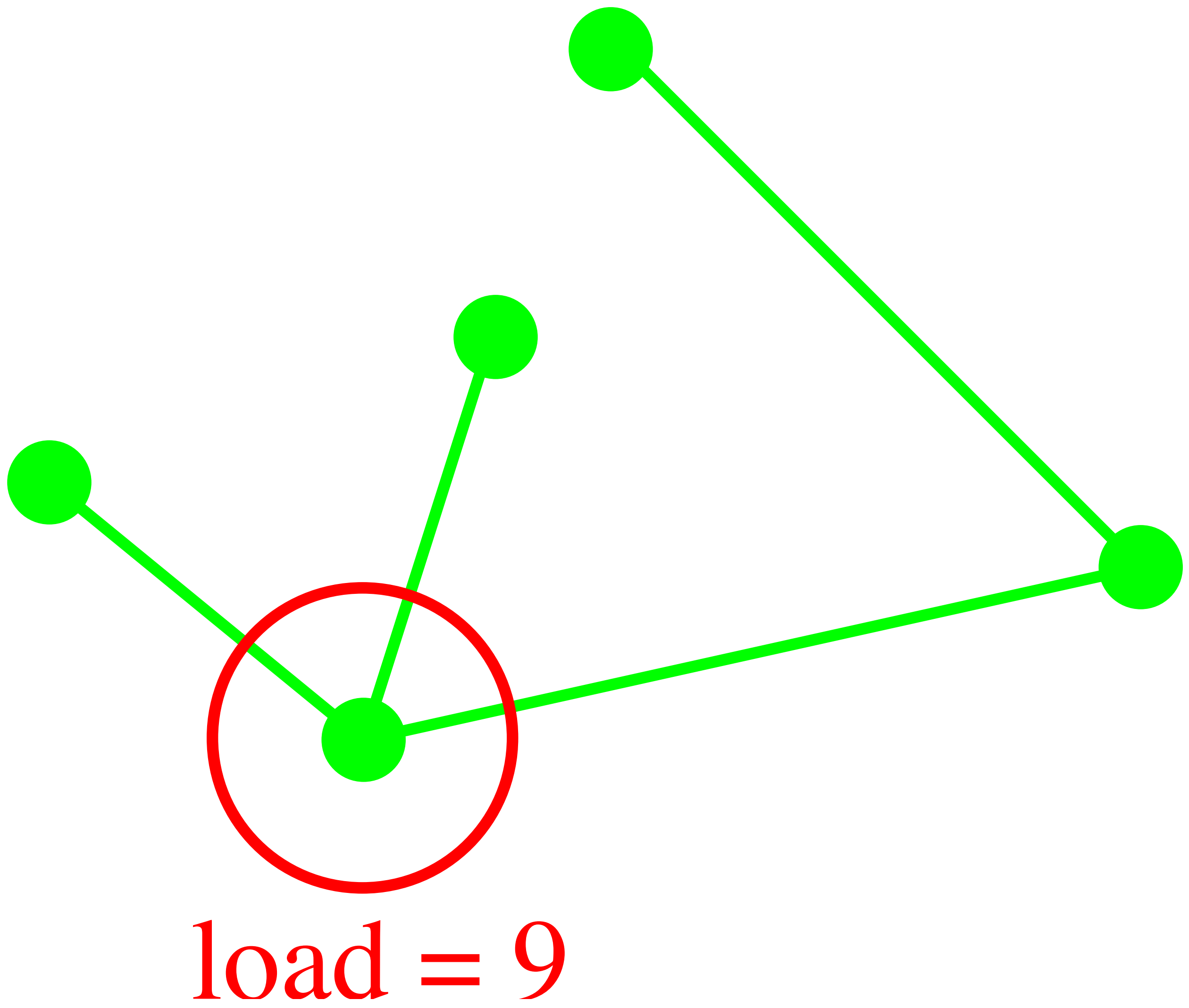
Load: A Simple Example



Load: A Simple Example



Load: A Simple Example



Damage Assessment

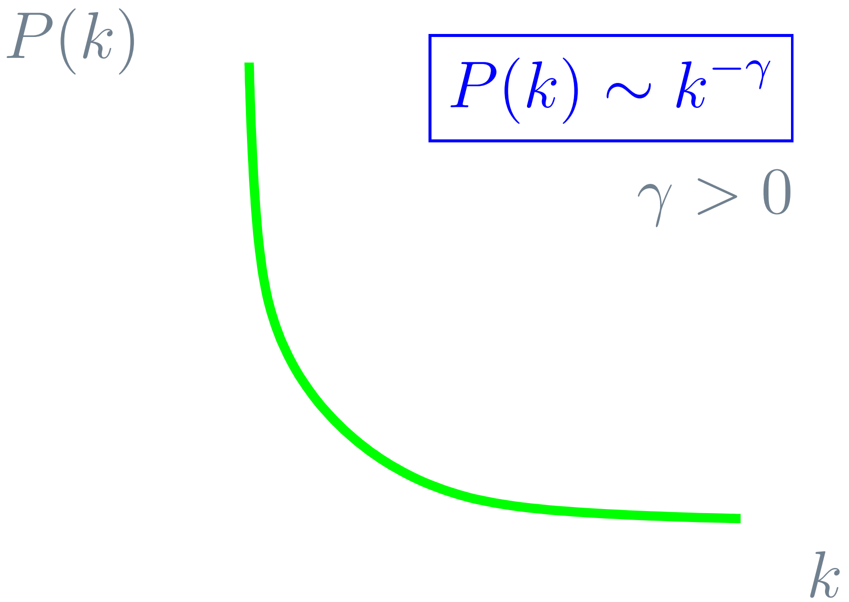
- Relative size G of the largest connected component,

$$G = N'/N,$$

  where N and N' are the numbers of nodes in the largest component before and after the cascade, respectively.

- If α is large, G is close to one. As α is decreased, G should decrease.
- If G is significantly less than unity, the network is effectively disintegrated.

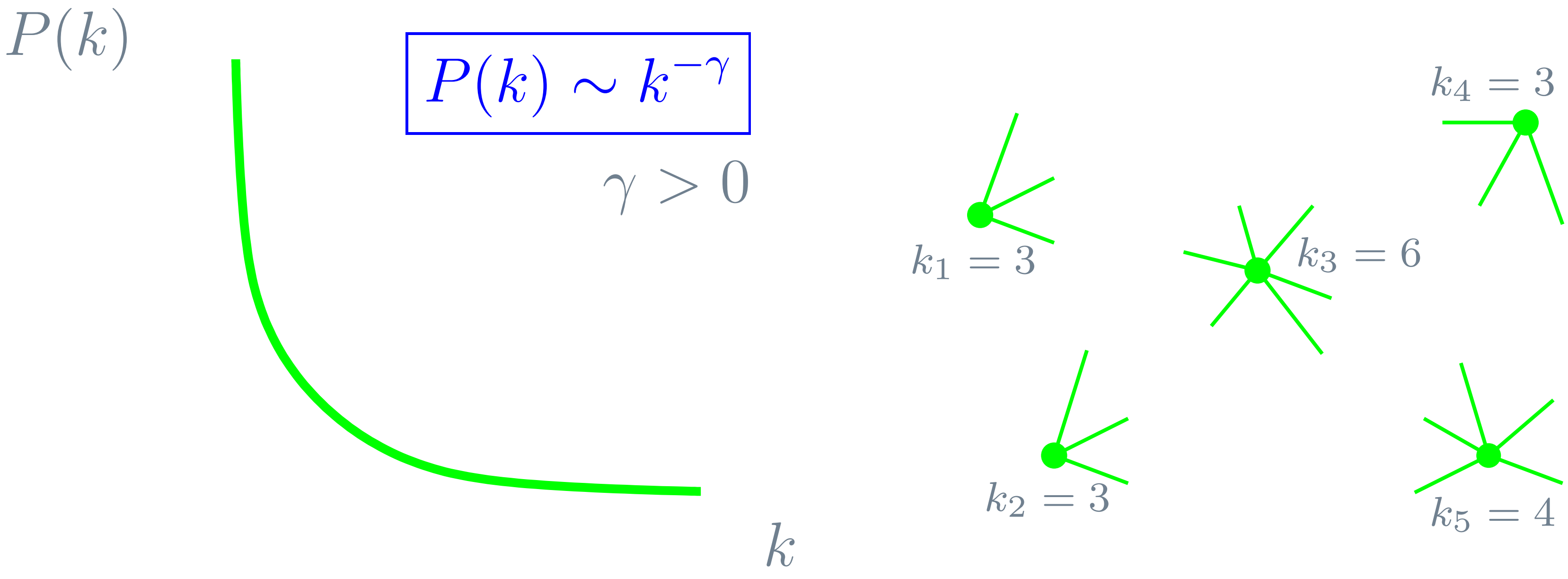
Example: Scale-Free Network



$$P(k) \sim k^{-\gamma}$$

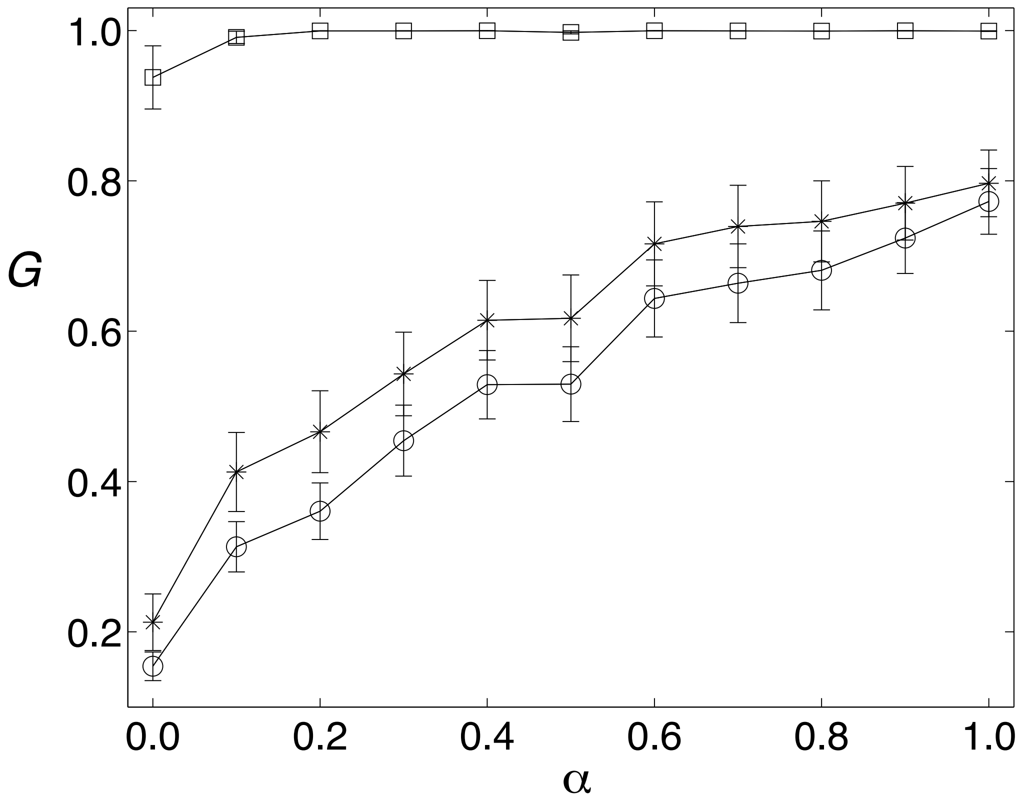
$\gamma > 0$

Example: Scale-Free Network



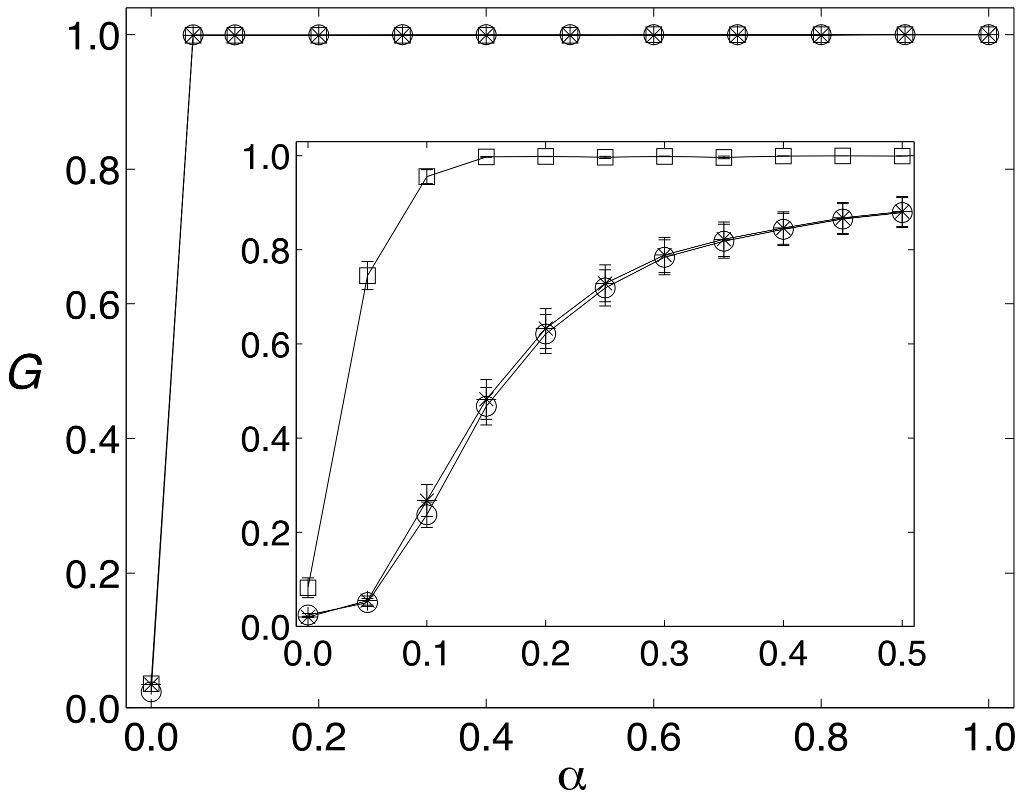
- Degree k_i (number of links) for node i is chosen at random according to $P(k) \sim k^{-\gamma}$;
- Nodes are connected randomly.
- Newman et al., Phys. Rev. E 64, 026118 (2001).

Example of Cascading Failure



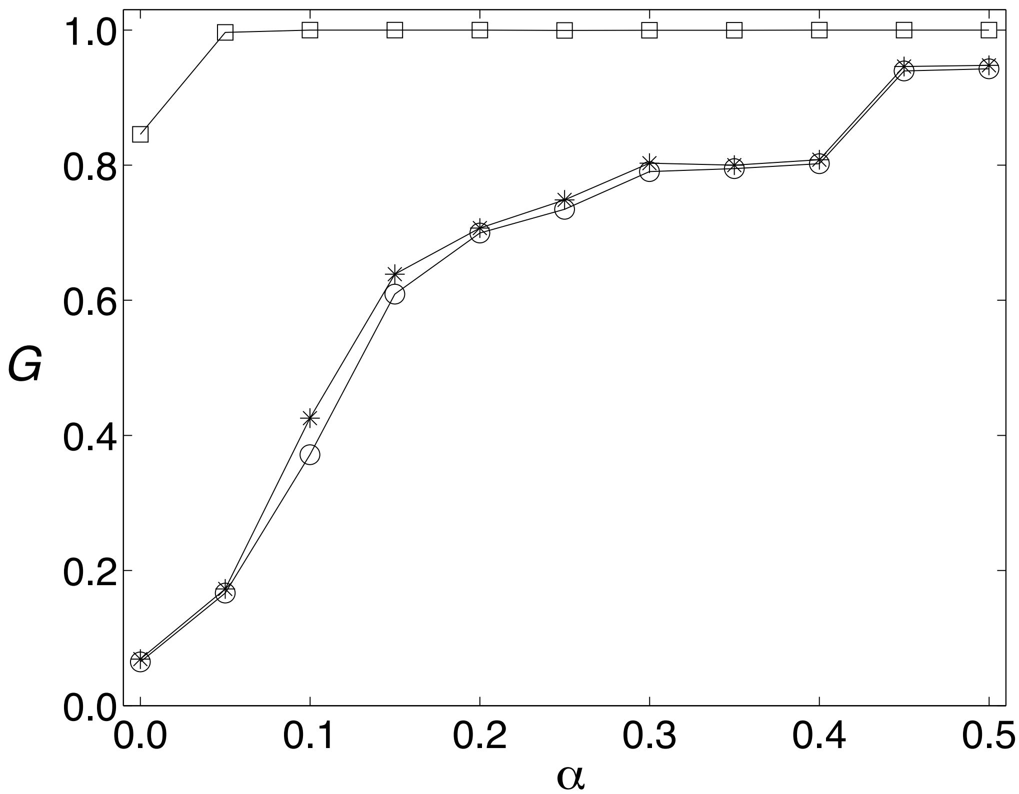
- Scale-free network - $N \approx 5000$ and $\langle k \rangle = 2$;
- Squares, asterisks, circles - removal of a single node at random, with highest degree, and with highest load, respectively.

Homogeneous Networks Are Safe



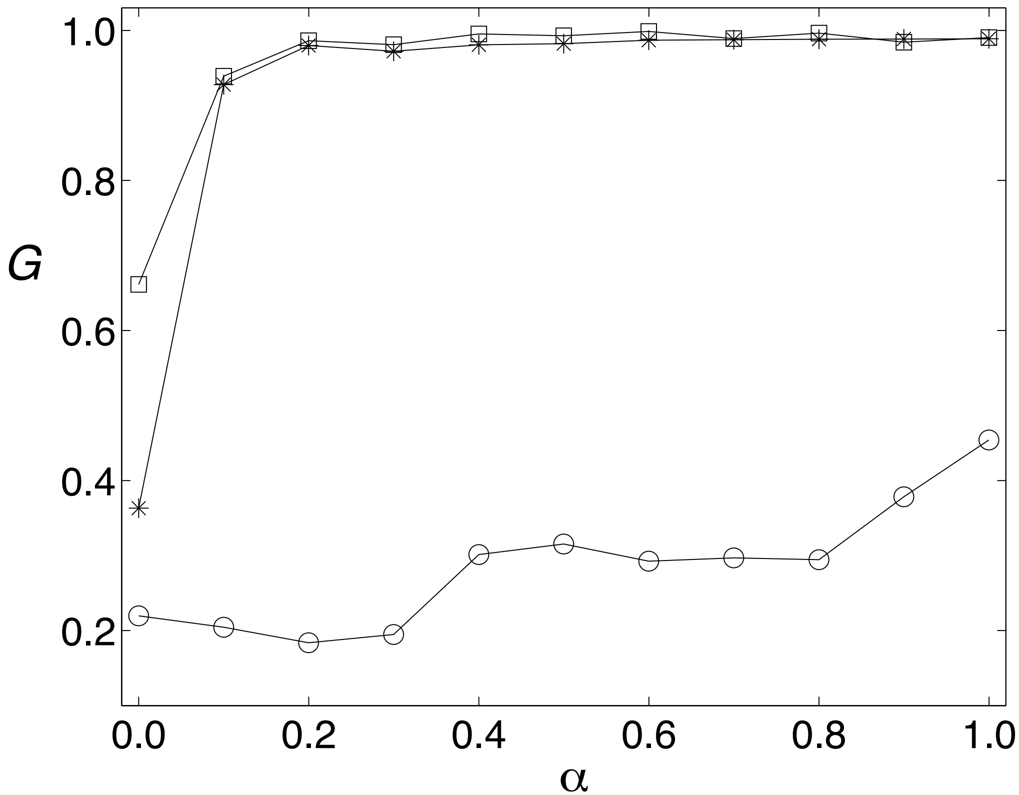
- Homogeneous network - $N = 5000$ and $k = 3$ for each node.
- Inset: scale-free network with $N \approx 5000$ and $\langle k \rangle \approx 3$.

Cascades on Internet



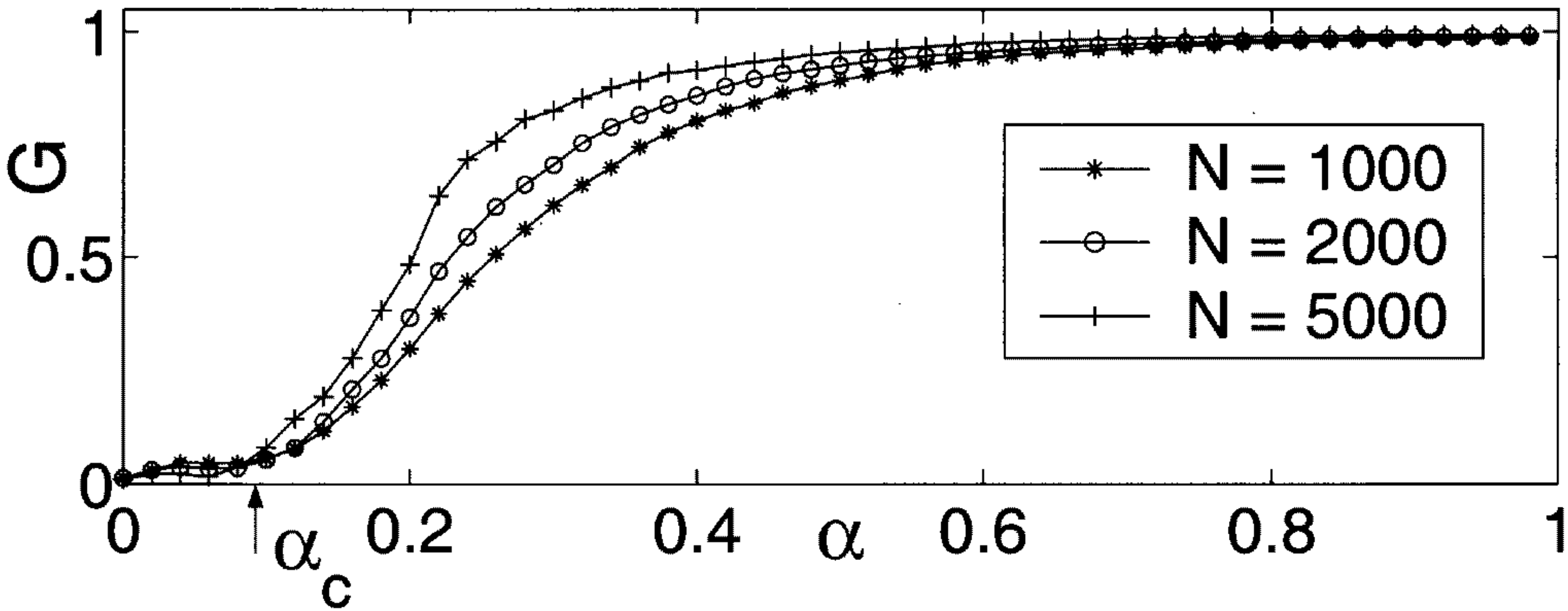
- Internet at autonomous system level; $N = 6474$ and $\langle k \rangle \approx 3.88$.

Cascades in Electrical Power Grid



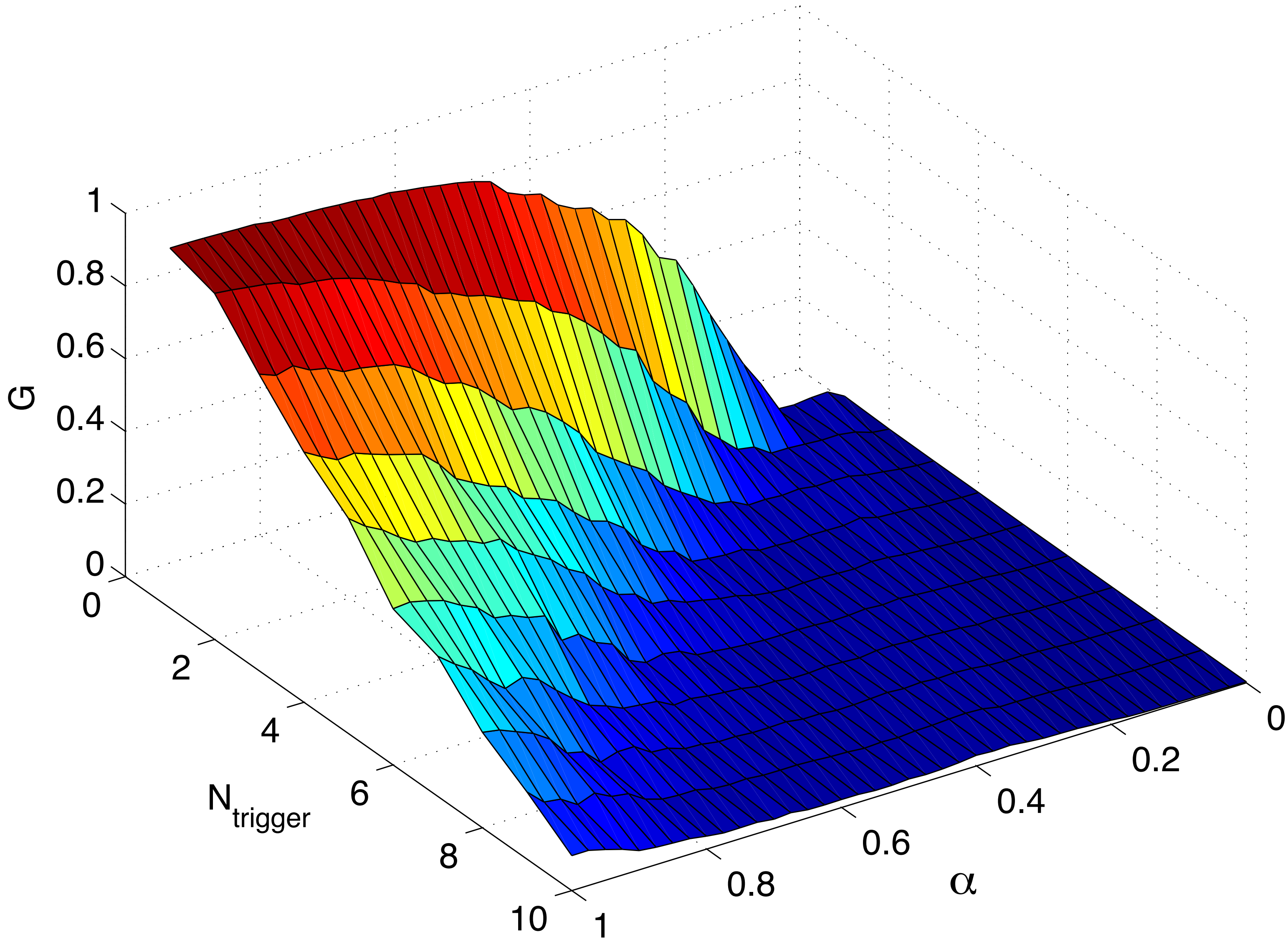
- $N = 4941$ and $\langle k \rangle \approx 2.67$.

Phase Transition



- Phase transition point $\alpha_c \approx 0.1$, below which attack on a single node can disintegrate the network totally.
- For sufficiently large α, network is robust against cascading breakdown.

Multiple Attacks



Theoretical Issues

- Focus on single attack to disable the most influential node.
- How to determine α_c and α_s?

Theoretical Estimate of α_c (1)

- Degree and load distribution
  $P(k) = ak^{-\gamma}$ and $L(k) = bk^{\eta}$,
  K.-I. Goh, B. Kahng, and D. Kim, Phys. Rev. Lett. 87, 278701 (2001).
- Say N - total number of nodes and S - total load

$$\int_1^{k_{max}} P(k)dk = N \quad \text{and} \quad \int_1^{k_{max}} P(k)L(k)dk = S.$$

- We obtain

$$a = \frac{(1-\gamma)N}{[k_{max}^{1-\gamma} - 1]} \quad \text{and} \quad b = \frac{\beta S}{a(1-k_{max})^{-\beta}},$$

where $\beta \equiv \gamma - \eta - 1$.

Theoretical Estimate of α_c (2)

- Say the highest-degree node has been removed. We have

$$P'(k) = a'k^{-\gamma'} \approx a'k^{-\gamma} \text{ and } L'(k) = b'k^{\eta'} \approx b'k^{\eta}.$$

and similarly

$$a' = \frac{(1-\gamma)(N-1)}{k_{max}^{1-\gamma} - 1} \text{ and } b' = \frac{S'}{a'(1-k_{max})^{-\beta}},$$

where S' is the new total load.

- Change in the load

$$\Delta L(k) \approx (b' - b)k^{\eta} = (\frac{b'}{b} - 1)L(k).$$

Theoretical Estimate of α_c (3)

- Change in the load

$$\Delta L(k) \approx (b' - b)k^{\eta} = (b'/b - 1)L(k).$$

- Maximum load increase that the node can handle

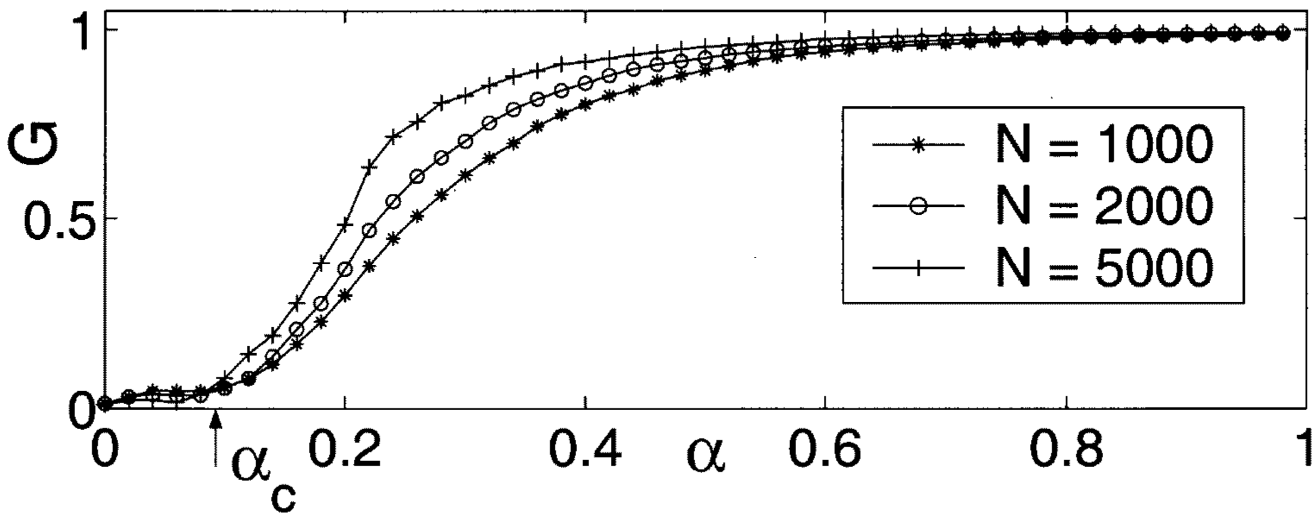
$$C(k) - L(k) = \alpha L(k)$$

- Thus, if $(b'/b - 1) < \alpha$, the node still functions. It fails if $(b'/b - 1) > \alpha$. This gives

$$\begin{aligned}\alpha_c &= b'/b - 1 \\ &\approx \left\{1 - k_{max'}^{-\beta}\left[-1 + \left(\frac{k_{max}}{k_{max'}}\right)^{-\beta}\right]\right\}\left(\frac{S'}{S}\right) - 1.\end{aligned}$$

Theoretical Estimate of α_c (4)

- α_c is independent of network size N, insofar as it is large.
- Example: scale-free network with $N = 2000$, $k_{max} = 81$, $k'_{max} = 60$, $S \approx 1.86 \times 10^7$, and $S' \approx 1.91 \times 10^7$ theoretical estimate gives $\alpha_c \approx 0.1$.



- L. Zhao, K. Park, and Y.-C. Lai, Phys. Rev. E 70, 035101(R) (2004).

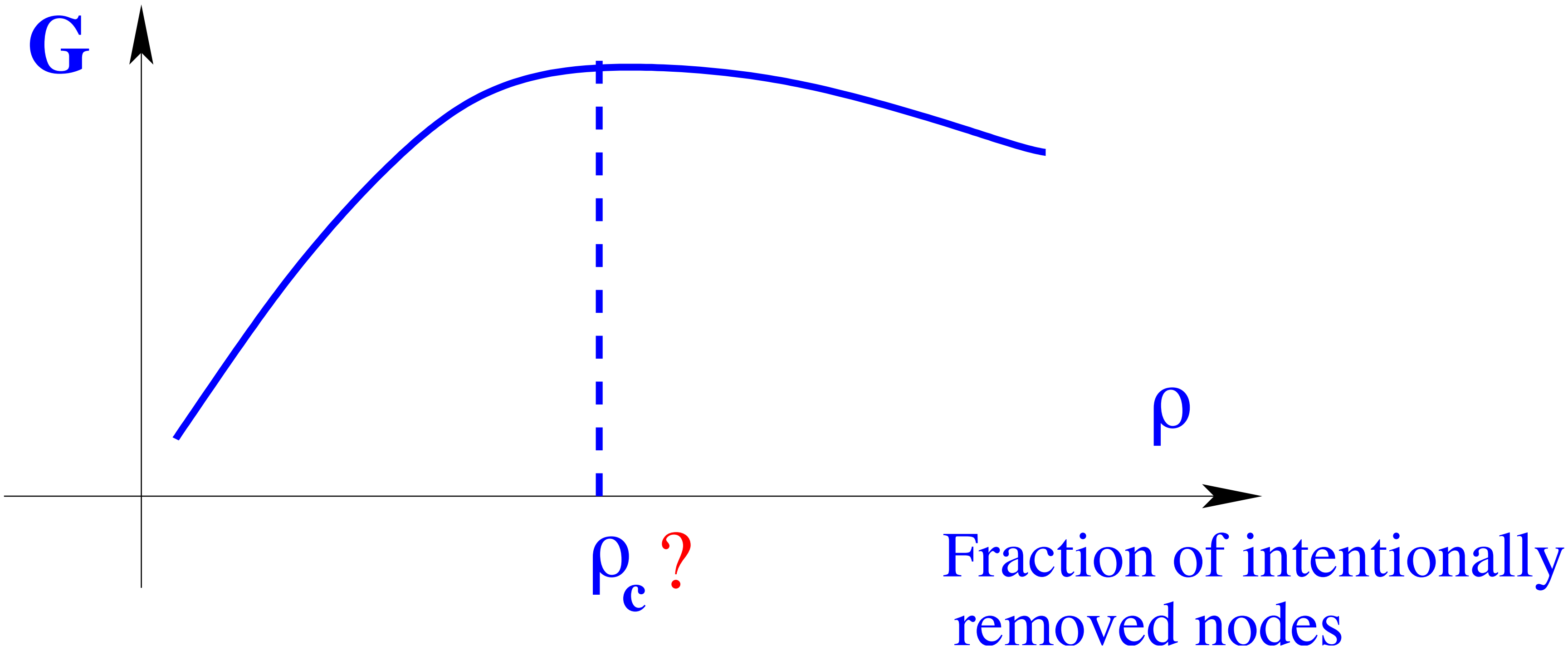
Theoretical Issues

- How to determine α_c and α_s?

Prevention of Cascades

- A closely related issue: How to prevent catastrophic cascades caused by attacks?
- Lowering the average loads in the network by removing a small set of nodes that contribute to the loads in the network but they themselves process little load.
- Cascades can be prevented or their sizes can be reduced significantly by intentionally removing a small, carefully selected set of "unimportant" nodes.
- A. E. Motter, Phys. Rev. Lett. 93, 098701 (2004).

A closely Related Problem



- How to estimate ρ_c?

Theoretical Estimate of λ_s and ρ_c (1)

- Capacity parameter: $\lambda = 1 + \alpha$.
- Total load can be written as

$$S = \sum_{i=1}^{(1-\rho)N} L_i + \sum_{i=N(1-\rho)+1}^{N} L_i \equiv S_0 + S_1,$$

where removed nodes are labeled by $(1-\rho)N + 1$ to N.

- After removing a ρ fraction of nodes

$$S' = \sum_{i=1}^{N(1-\rho)} L'_i \approx \sum_{i=1}^{N(1-\rho)} \sigma L_i,$$

where $0 < \sigma < 1$ is a shifting constant. What is σ?

Theoretical Estimate of λ_s and ρ_c (2)

- Note

$$S = N(N-1)D \approx N^2 D,$$
$$S' = N(1-\rho)[N(1-\rho)-1]D' \approx (1-\rho)^2 N^2 D',$$

  where $D \approx D'$ are network diameters before and after the removal.
- This gives $\sigma \approx (1-\rho)^2 \approx 1-2\rho$.
- On average, the difference between the loads of node i before and after the removal is $\Delta L_i = L_i - L_i' \approx 2\rho L_i$.
- This results in an extra amount of load tolerance $2\rho L_i$, or, $\lambda' = \lambda + 2\rho$.

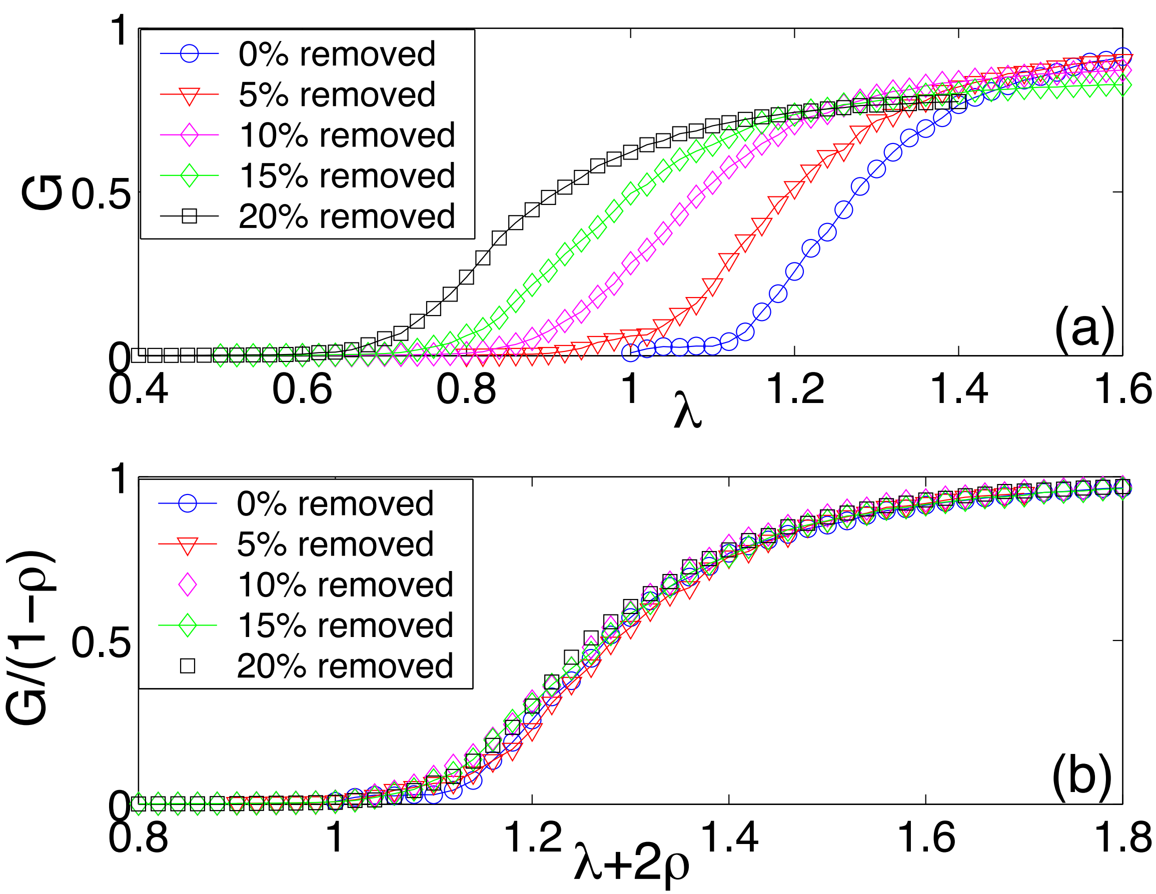
Theoretical Estimate of λ_s and ρ_c (3)

- $G(\lambda, \rho)$ - relative size of largest connected component in presence of both controlled removal and attack.
- $G(\lambda, 0) \equiv G^0(\lambda)$ - the size without controlled removal.
- We have

$$G(\lambda, \rho) \approx G^0(\lambda + 2\rho)(1 - \rho).$$

- Note: $G(\lambda, \rho)/(1 - \rho)$ versus $\lambda' \equiv \lambda + 2\rho$ is independent of ρ.

A Universal Relation in G



- Scale-free network of $N = 3000$ nodes.

Theoretical Estimate of λ_s and ρ_c (4)

- λ_s - critical capacity parameter value above which the network is resilient to global cascades even without any protection (i.e., $\rho = 0$).
- For $\lambda < \lambda_s$, in the event of attack, it is necessary to intentionally remove a small fraction of nodes to protect the network. For fixed λ, we have

$$\partial G/\partial \rho|_{\lambda<\lambda_s,\rho=0} > 0.$$

- For $\lambda > \lambda_s$, the network is secure against cascading breakdown. Removing a small fraction of nodes would simply reduce $G(\lambda, \rho)$ by a small amount. Thus,

$$\partial G/\partial \rho|_{\lambda>\lambda_s,\rho=0} < 0.$$

Theoretical Estimate of λ_s and ρ_c (5)

- Criterion for estimating λ_s: $\partial G/\partial\rho|_{\lambda=\lambda_s,\rho=0} = 0$.
- Utilizing $G(\lambda, \rho) \approx G^0(\lambda + 2\rho)(1 - \rho)$ gives

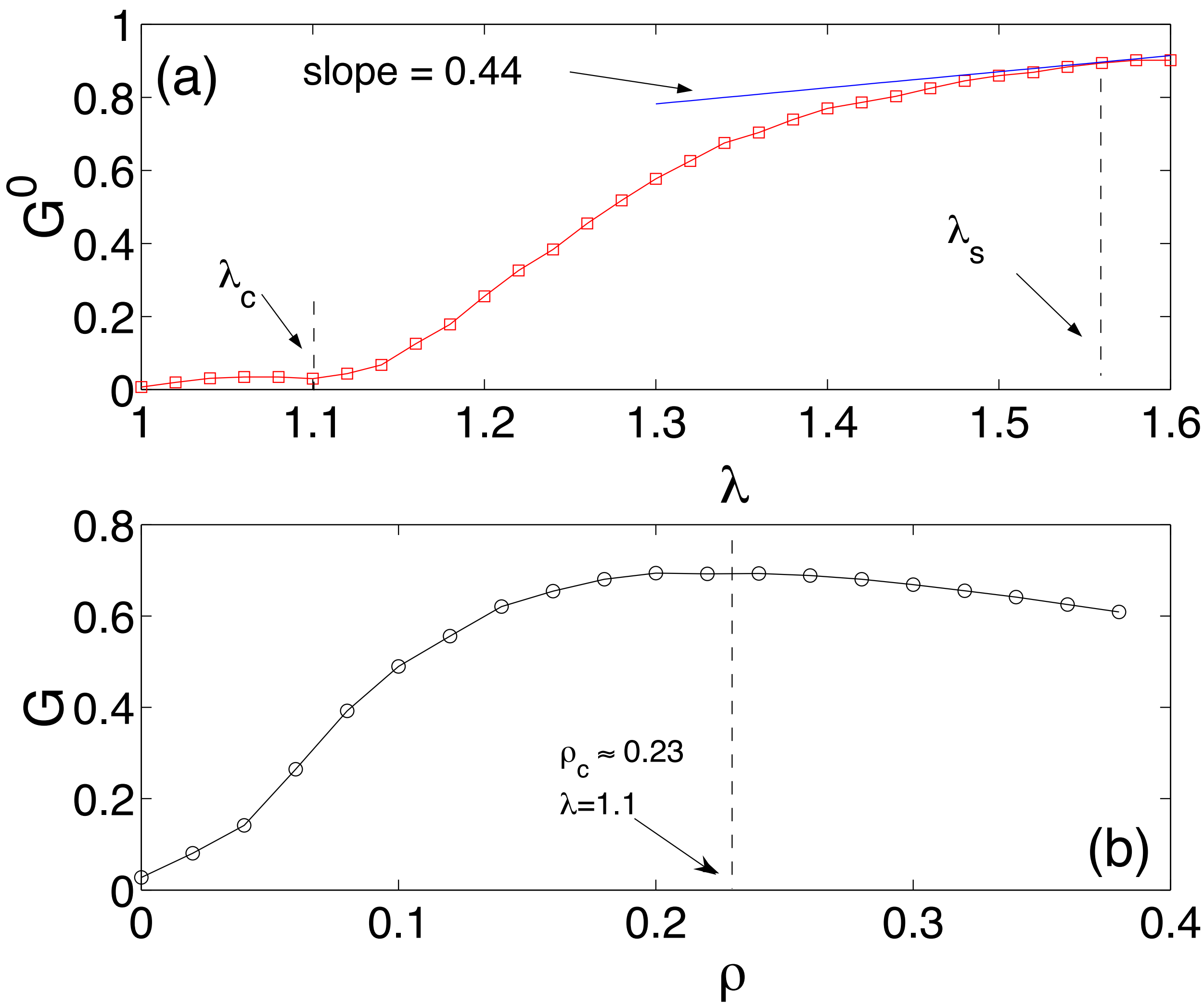
$$\frac{dG^0}{d\lambda}\bigg|_{\lambda=\lambda_s} \approx \frac{G^0(\lambda_s)}{2}.$$

- Say λ_0 - initial capacity. Controlled removal of a ρ_c fraction of low-degree nodes is equivalent to increasing λ_0 to λ_s with $\rho = 0$. Thus, $\lambda_s \approx \lambda_0 + 2\rho_c$ or

$$\rho_c \approx (\lambda_s - \lambda_0)/2.$$

- L. Zhao, K. Park, Y.-C. Lai, and N. Ye, Phys. Rev. E (Rapid Communications) 72, 025104 (2005).

Numerical Verification

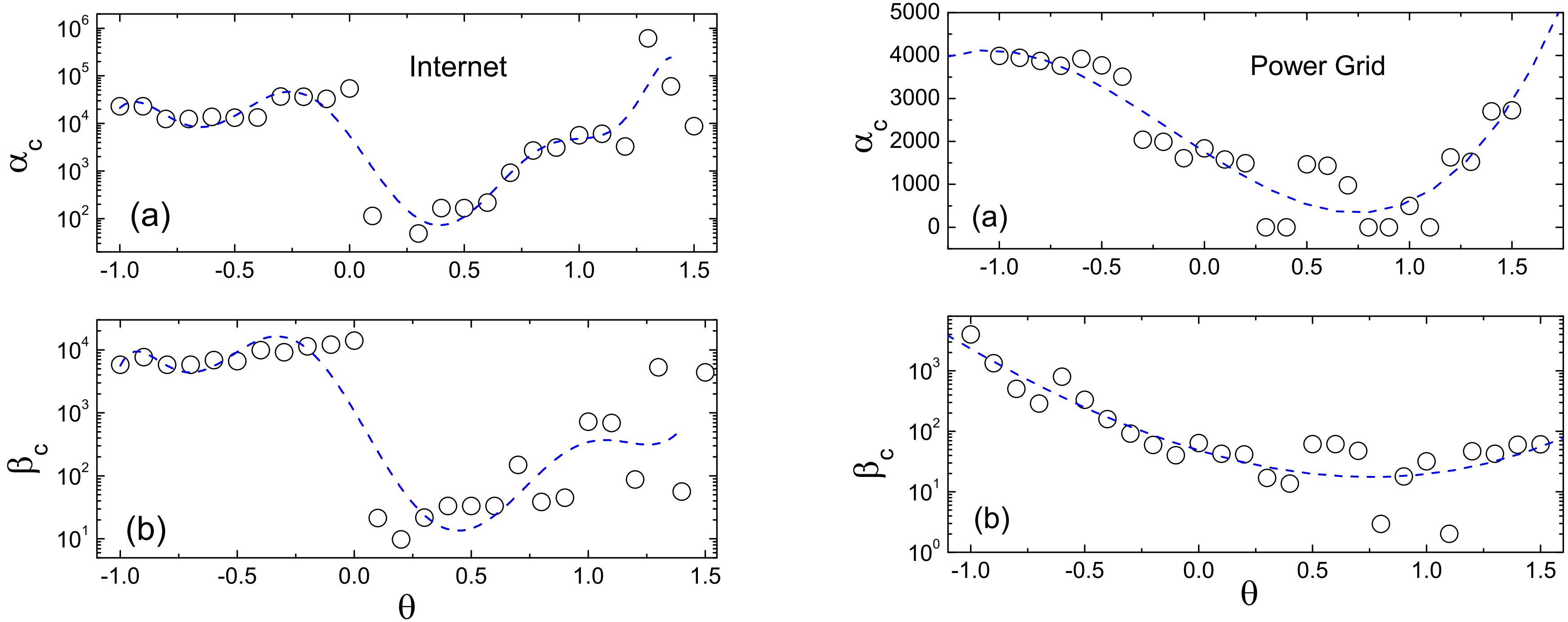


- Scale-free network of $N = 3000$ nodes.

Soft Control Strategy

- Control strategy without having to remove any nodes.
- Weighted networks: $W_{ij} = A_{ij}(k_i k_j)^\theta$.
- θ - Control parameter
- More realistic capacity-load relation:
  $C_i = \alpha + \beta L_i$
  [Kim and Motter, J. Phys. A: Math. Theor. 41, 224019 (2008)]

Soft Control Strategy - Examples



- R. Yang, W.-X. Wang, Y.-C. Lai, G.-R. Chen, “Optimal weighting scheme for suppressing cascades and traffic congestion in complex networks,” Physical Review E 79, 026112 (2009).

Complex Clustered Networks

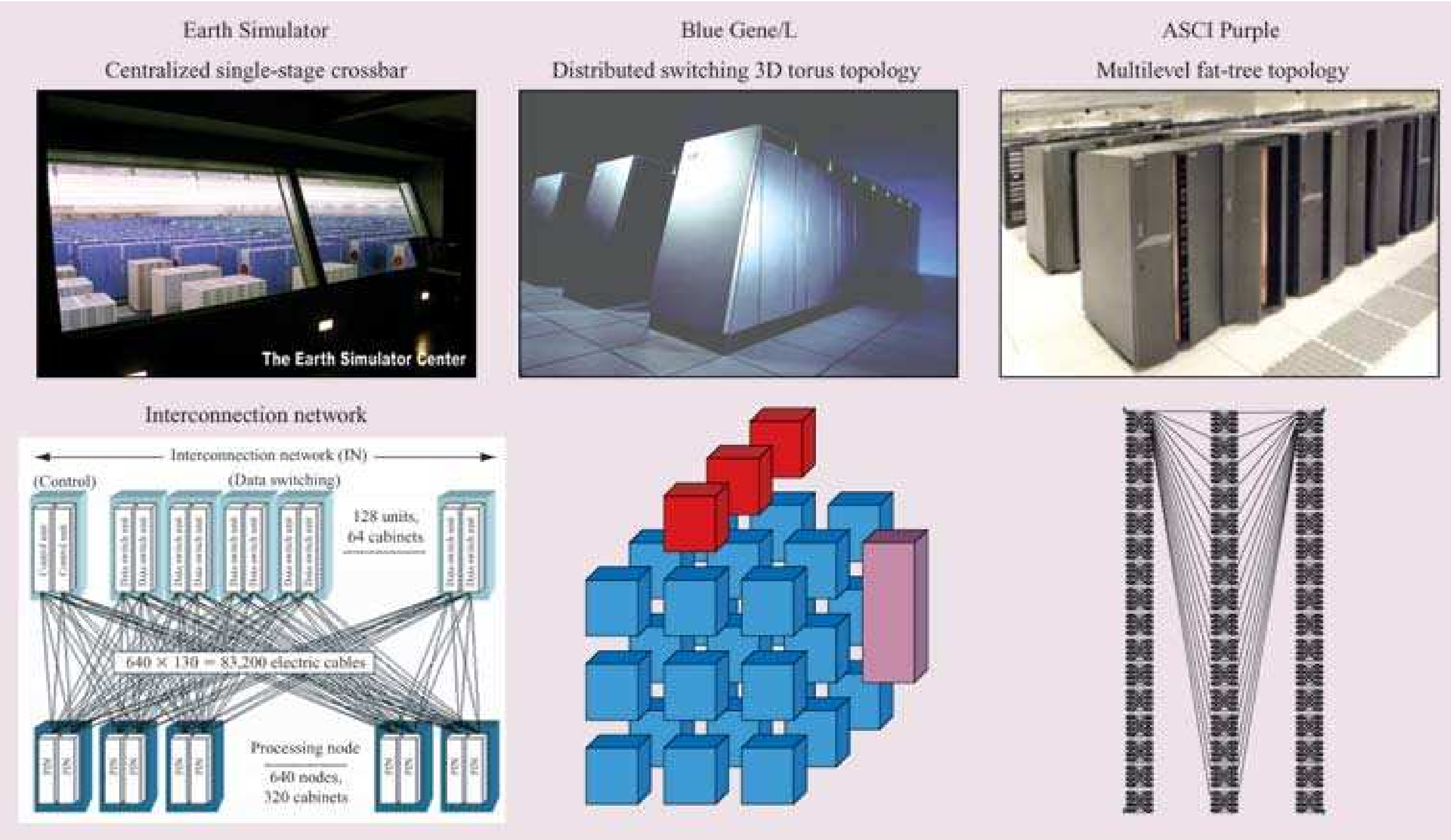
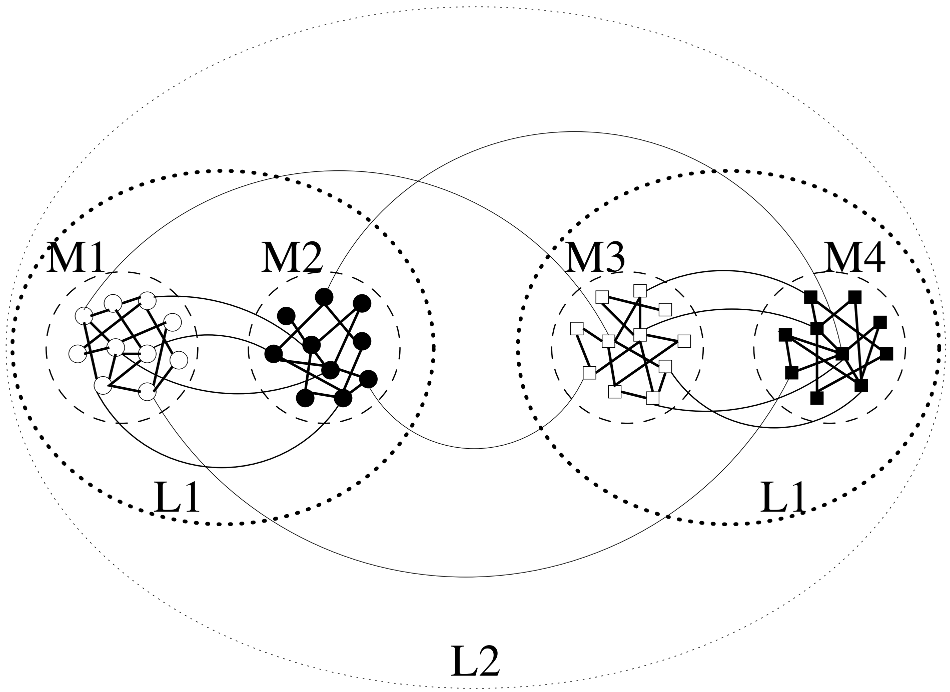
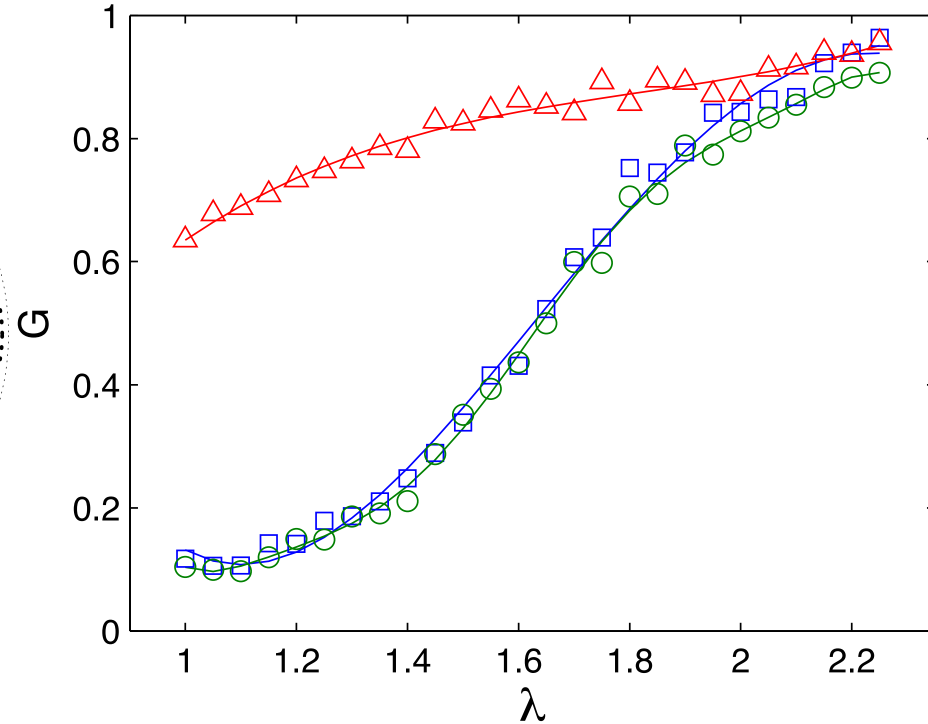


Figure 10

Clusters and massively parallel machines: Earth Simulator, Blue Gene/L, and ASCI Purple.

Cacades and Control





- L. Huang, Y.-C. Lai, and G.-R. Chen, “Understanding and preventing cascading breakdown in complex clustered networks,” Physical Review E 78, 036116(1-5) (2008).

Conclusions

- Cascading failures caused by intentional attack can be catastrophic for complex networks. Intentional removal of a small fraction of “unimportant” (low-degree) nodes can protect the network to some extent.
- Physical theory for cascading failures.
- Soft control strategy for preventing cascades and traffic congestions.
- Cascading dynamics associated with evolutionary games on complex networks.
- Work supported by AFOSR and NSF.