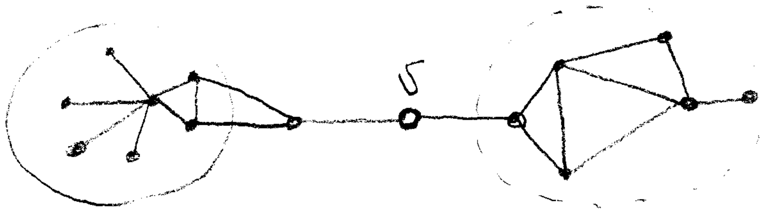
Lecture 11 Betweenness Centrality



S — important for info. flow on net.

Say, (1) every pair of nodes exchanges a message with equal probability / unit time

(2) Messages travel on shortest path

$\Rightarrow$ "Information" through node / per unit time

$\sim$ # of shortest paths through the node

$$x_i \sim \sum_{s,t} n^i_{st} / g_{st}$$

n^i_{st}: # of shortest paths from s to t through i.

g_{st}: total # of shortest paths from s to t

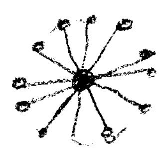
Remarks:

① if $g_{st} = 0 \Rightarrow n^i_{st} = 0$ convention. $\frac{n^i_{st}}{g_{st}} = 0$

② $s = t$ not excluded

③ Wide range in the value of betweenness centra.

e.g., star, n nodes

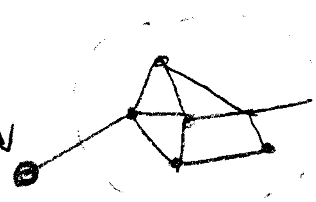


Central node

$$x_c = \underbrace{n^2}_{\substack{\text{total \# of} \\ \text{paths } (n^c_{st} = g_{st})}} - \underbrace{(n-1)}_{\text{\# of peripheral nodes}}$$

— upper bound

Leaf node l

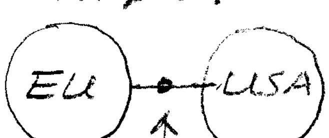


$n-1$	from l to others
$n-1$	opposite direct.
$+\ 1$	l to itself
$2n-1$	

Lower bound $2n-1$

$$\frac{\text{Upper bound}}{\text{Lower bound}} = \frac{n^2-n+1}{2n-1} \approx \frac{1}{2}n$$

Example:



Fernando Rey

FR : 7.5×10^8

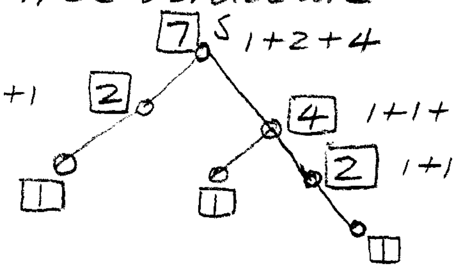
Lowest : 8.9×10^5 $\Big\} \sim 10^3 \sim n$

(1971 film: The French Connection with Gene Hackman)

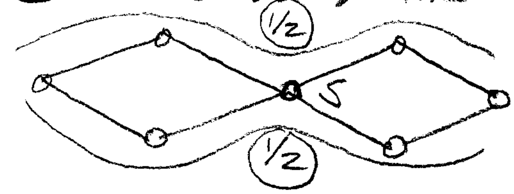
Algorithm — score for each node: # of shortest paths through the node but ending at S

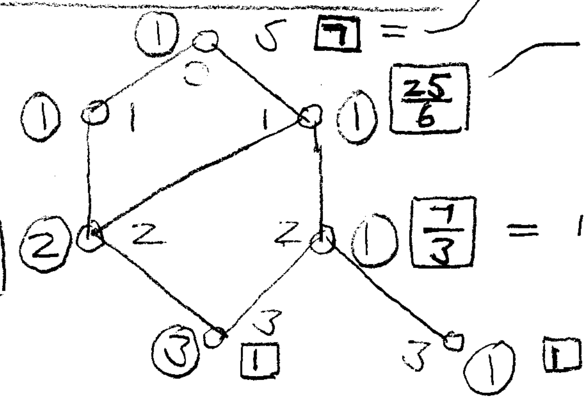
① Tree structure

no multiple path



② Multiple paths





S: $\boxed{7} = 1 + \frac{11}{6}\cdot\frac{1}{1} + \frac{25}{6}\,\frac{1}{1}$

$\boxed{\frac{25}{6}} = 1 + \frac{5}{3}\cdot\frac{1}{2} + \frac{7}{3}\,\frac{1}{1}$

d

$1 + \frac{5}{3}\cdot\frac{1}{2}\ \boxed{\frac{11}{6}}$

$1 + 1\cdot\frac{2}{3} = \boxed{\frac{5}{3}}$

$\boxed{\frac{7}{3}} = 1 + 1\cdot\frac{1}{3} + 1\cdot\frac{1}{1}$

(i) Total # of shortest paths from each node to S

S: $d = 0 \qquad w_S = 1$

$i \leftarrow d$

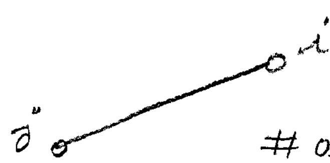
$j \leftarrow d+1$ if j has not been assigned a distance
$w_j = w_i$

$w_j \leftarrow w_j + w_i$ if j has already been assigned distance $d+1$

w_j unchanged if j has distance d

$d+1 \to d$
& Repeat

(ii)



$$\frac{\#\text{ of paths from } j \text{ to } S \text{ through } i}{\#\text{ of paths from } j \text{ to } S} = \frac{w_i}{w_j}$$

(iii) Leaf nodes $x_t = 1$

From bottom of tree

$$x_i = 1 + \sum_{j} x_j \frac{w_i}{w_j}$$

all neighbors of i immediately below i

$\Rightarrow \boxed{x_S}$ — do this for every node in the network

Scaling of Betweenness Centrality in Weighted Complex Networks

$$\boxed{B_K(k) \sim k^{\alpha}}$$

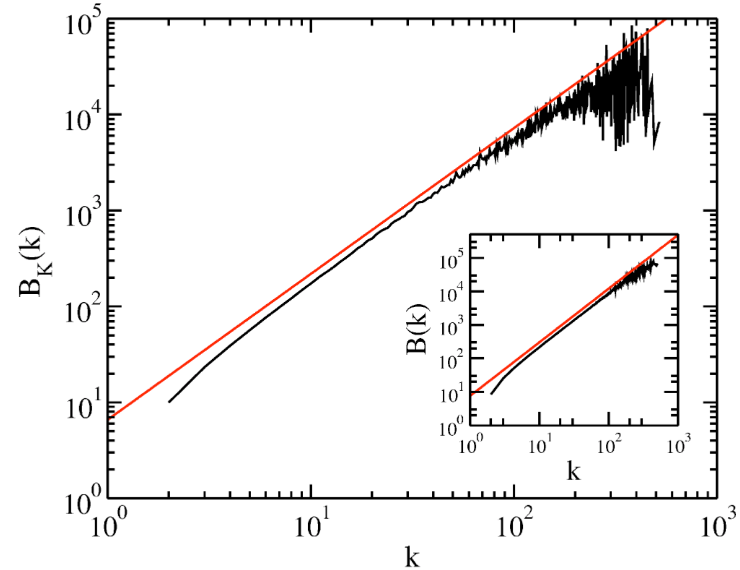


FIG. 4. Algebraic scaling between $B_K(k)$ and k for a weighted scale-free network with $m=2$ and $N=10\,000$. The value of the scaling exponent is $\alpha \approx 1.5$. A similar scaling relation is obtained for the corresponding nonweighted network, where $\alpha \approx 1.6$, as shown in the inset.

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Characterization of weighted complex networks

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To account for possible distinct functional roles played by different nodes and links in complex networks, we introduce and analyze a class of weighted scale-free networks. The weight of a node is assigned as a random number, based on which the weights of links are defined. We utilize the concept of *betweenness* to characterize the weighted networks and obtain the scaling laws governing the betweenness as functions both of the weight and of the degree. The scaling results may be useful for identifying influential nodes in terms of physical functions in complex networks.

Synchronization in Complex Networks

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Heterogeneity in Oscillator Networks: Are Smaller Worlds Easier to Synchronize?

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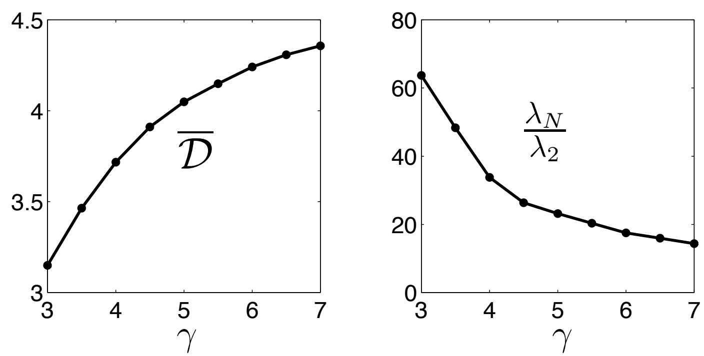
Small-world and scale-free networks are known to be more easily synchronized than regular lattices, which is usually attributed to the smaller network distance between oscillators. Surprisingly, we find that networks with a homogeneous distribution of connectivity are more synchronizable than heterogeneous ones, even though the average network distance is larger. We present numerical computations and analytical estimates on synchronizability of the network in terms of its heterogeneity parameters. Our results suggest that some degree of homogeneity is expected in naturally evolved structures, such as neural networks, where synchronizability is desirable.

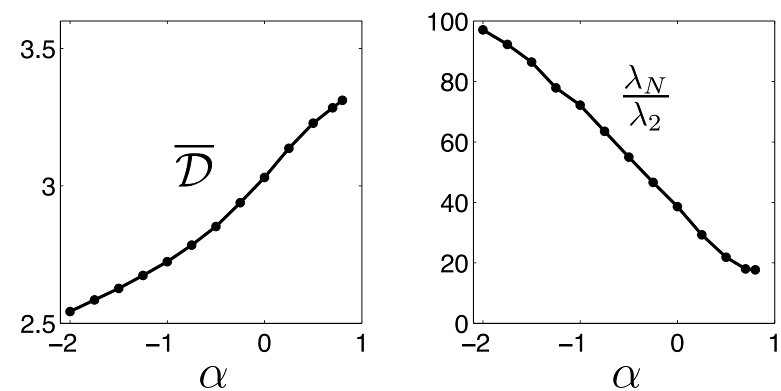
Semi-random scale-free network model

Smaller $\gamma \longleftrightarrow$ More heterogeneous degree distribution



Growing scale-free network model

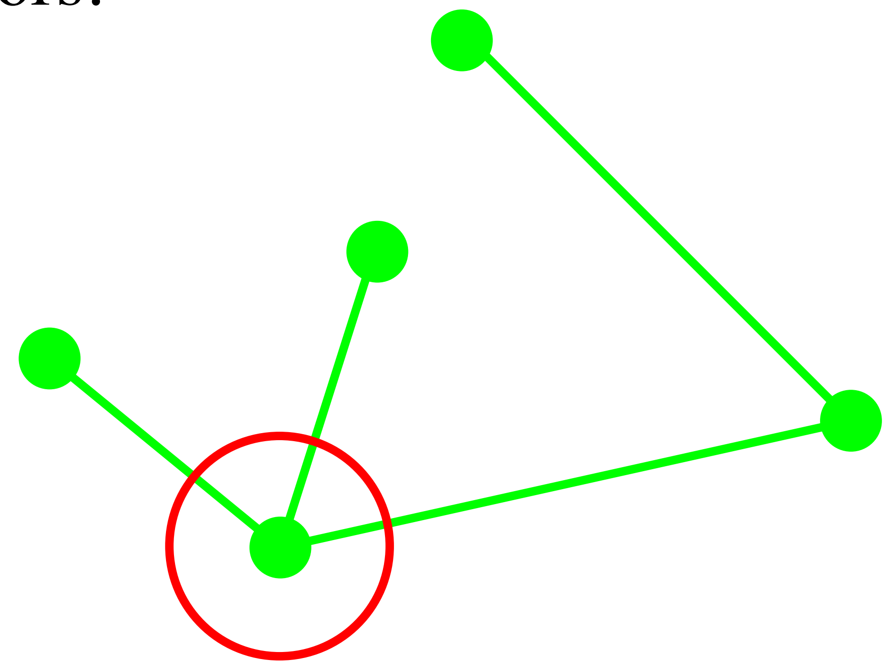
Smaller $\alpha \longleftrightarrow$ More heterogeneous degree distribution



$$\textit{Heterogeneity} \Longrightarrow \begin{cases} \textit{Smaller } \overline{\mathcal{D}}\text{, but} \\ \textit{More difficult to synchronize} \end{cases}$$

Load

Load on node i (or link) is defined as the number of shortest paths between two nodes which pass through i, and is a *measure of congestion of communication traffic* between oscillators.



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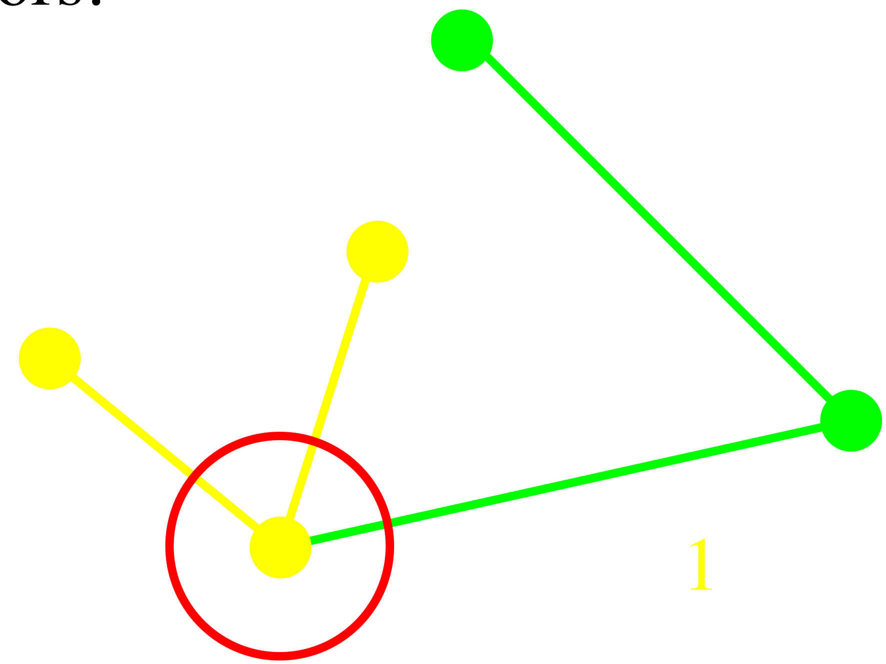

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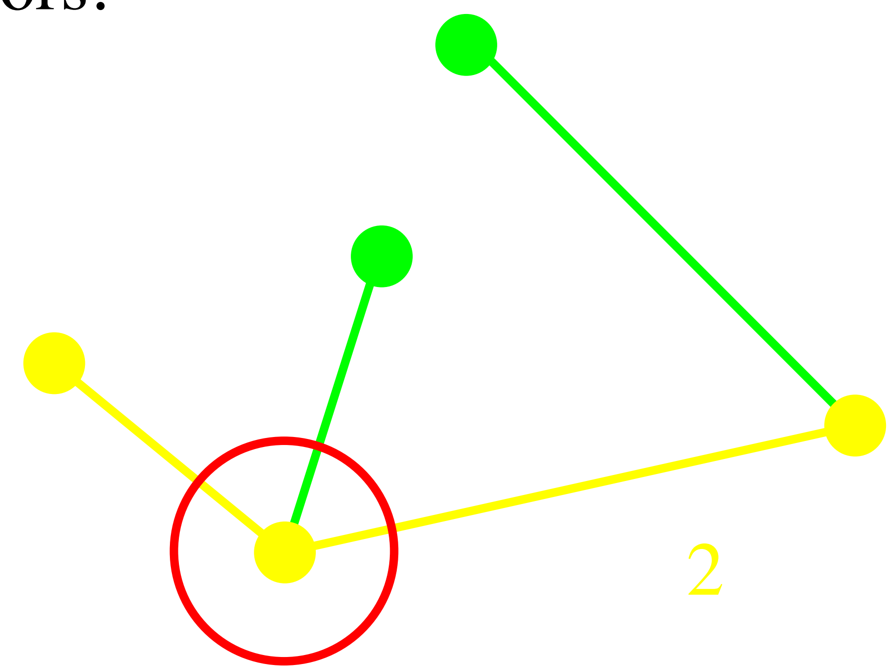

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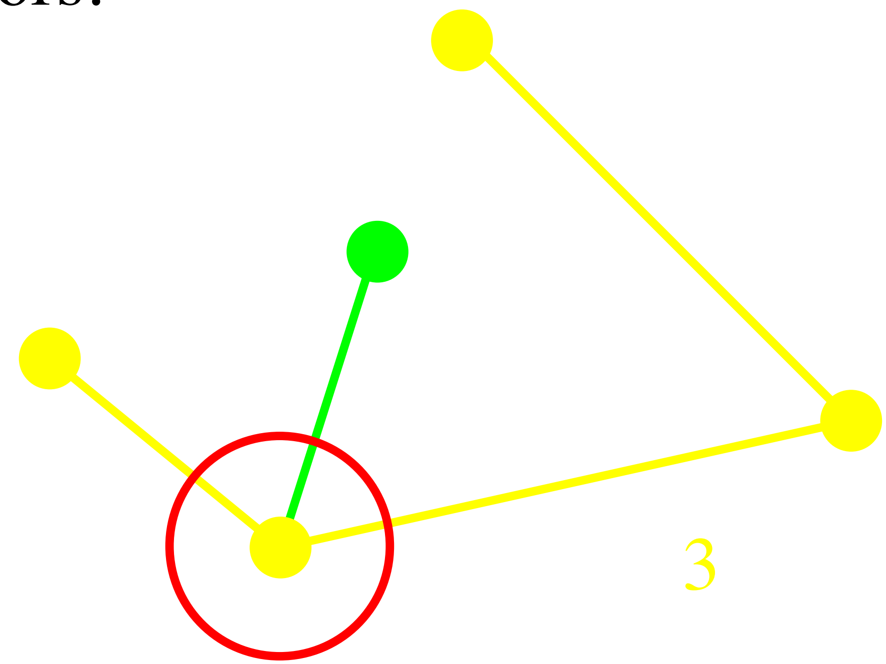

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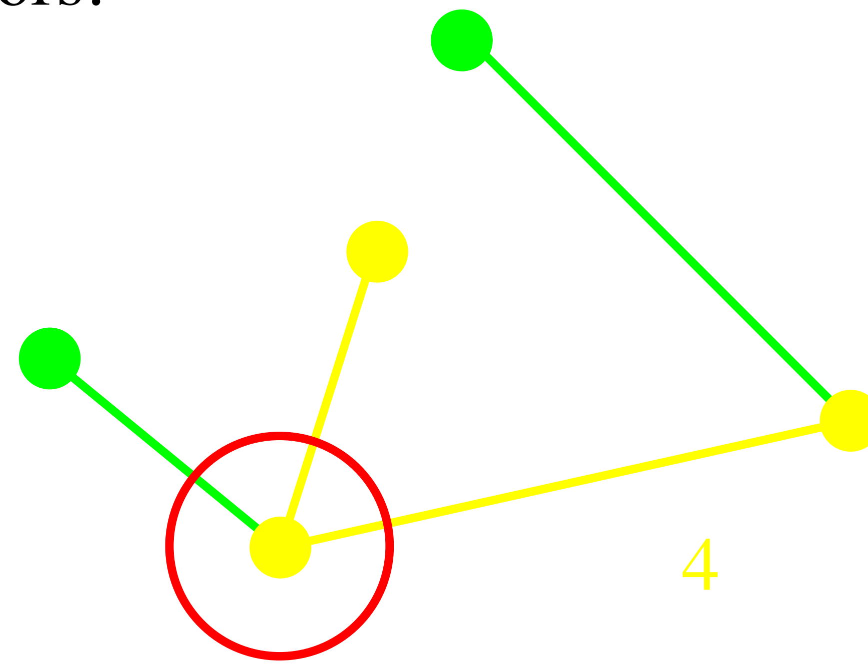

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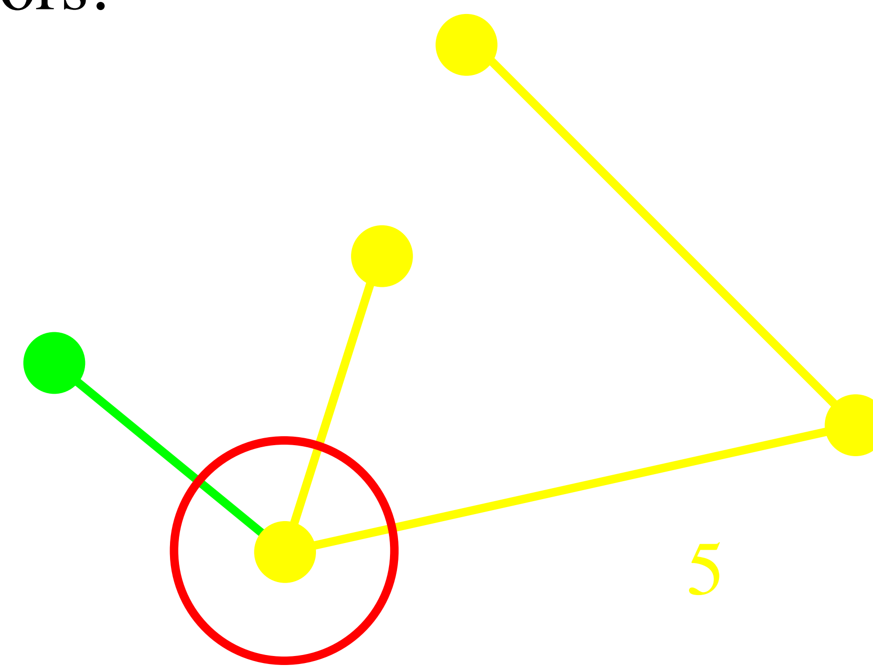

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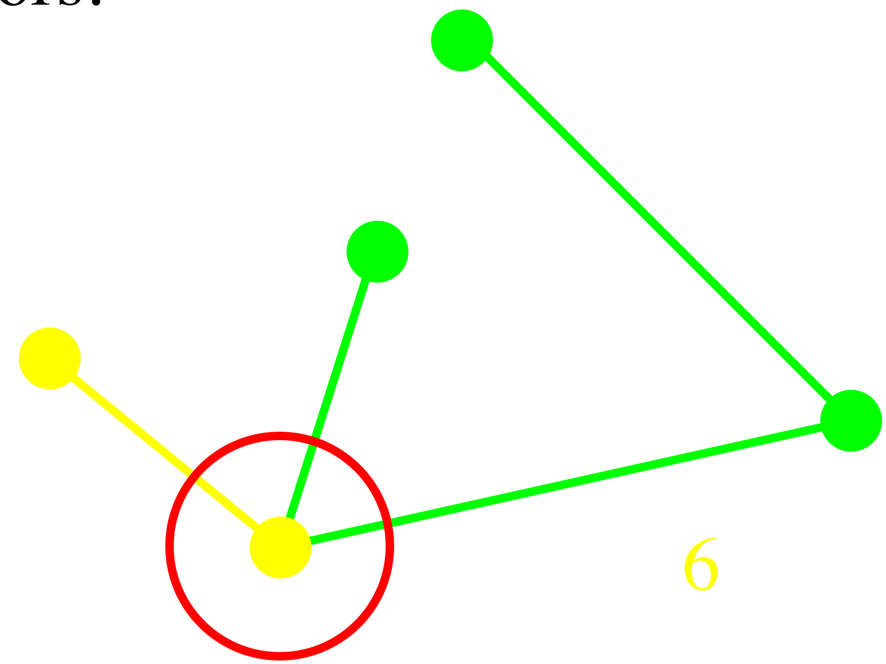

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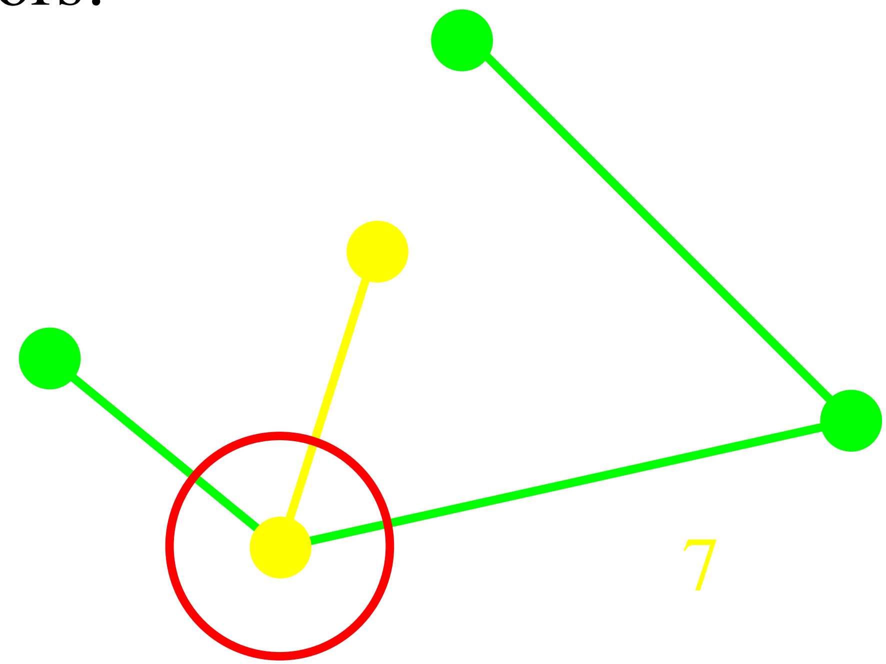

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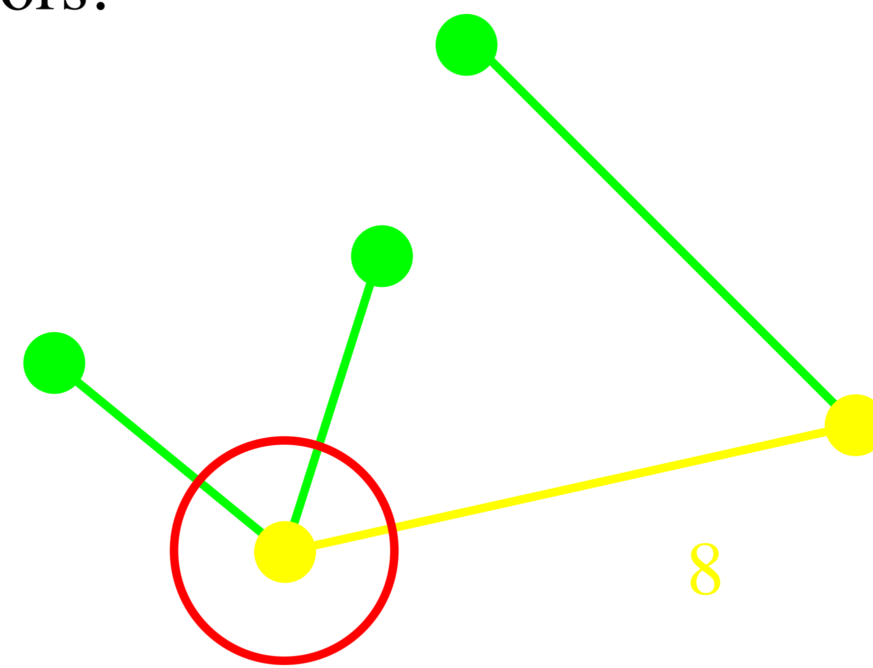

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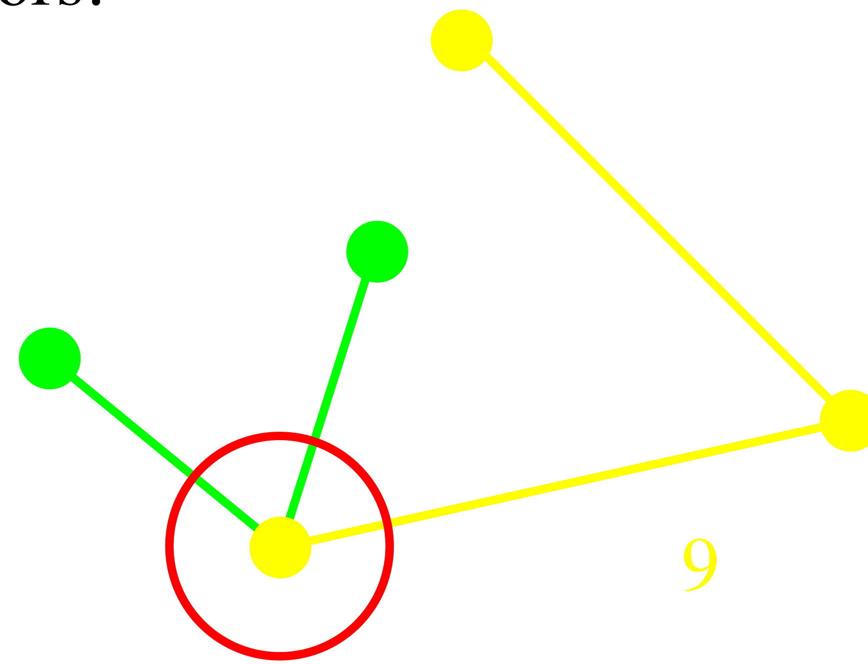

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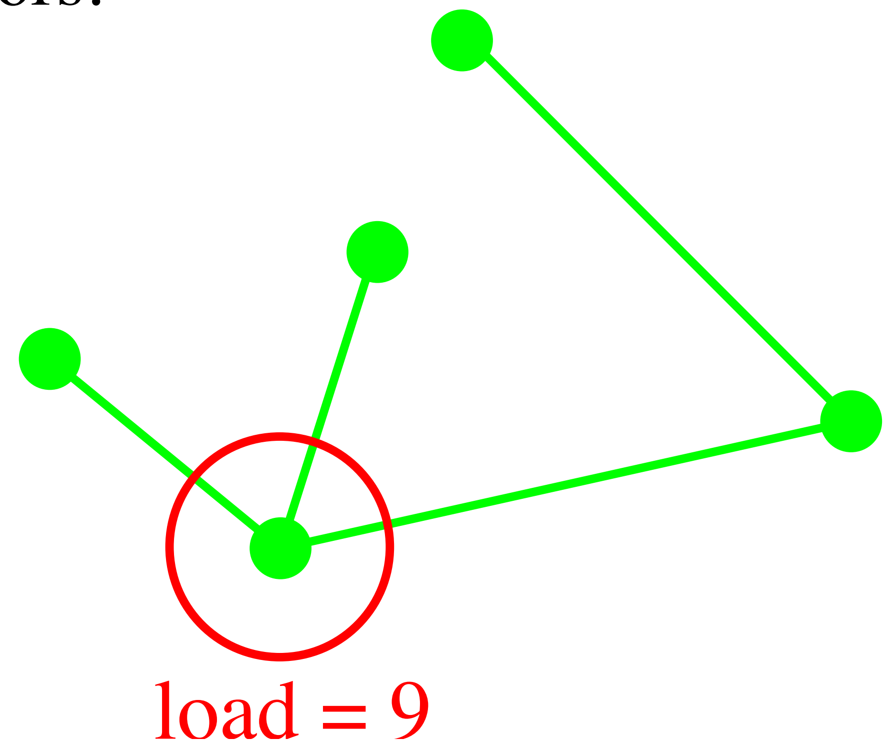

Role of Betweenness Centrality (Load) in Network Synchronization

Semi-random scale-free network model

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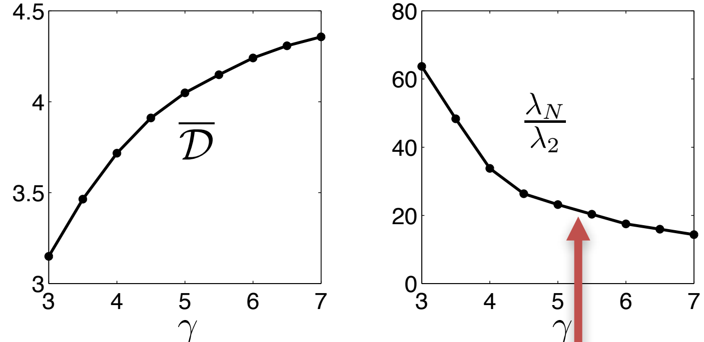

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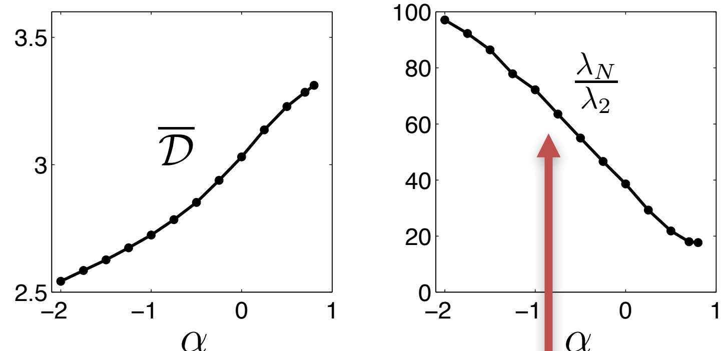

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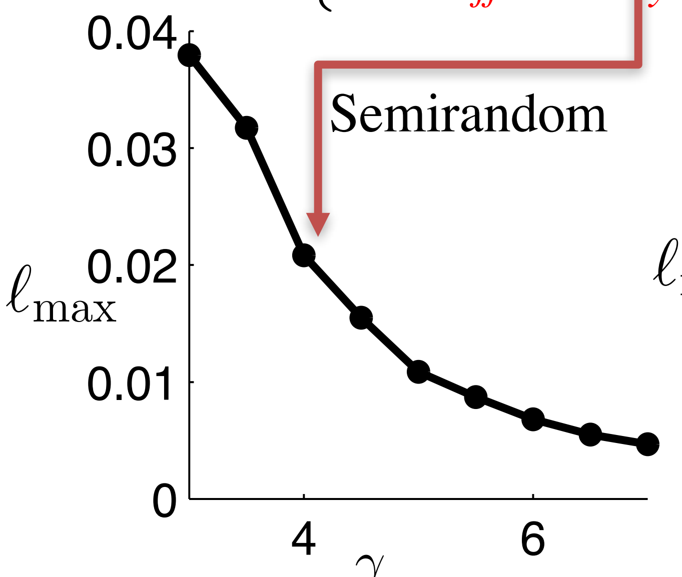

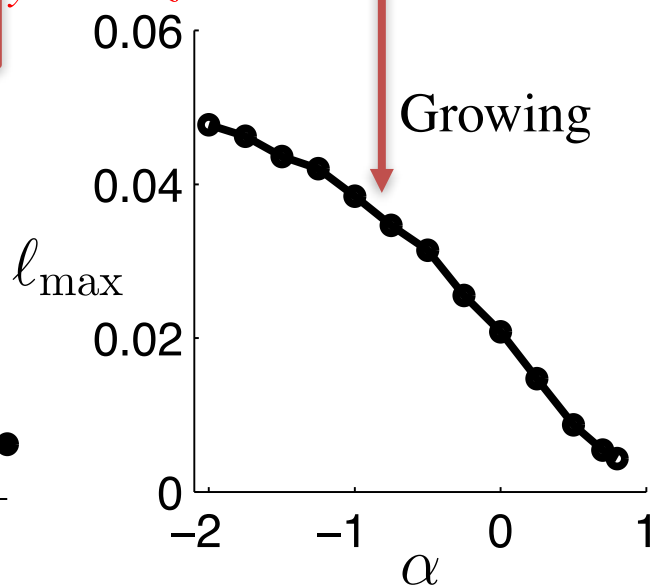

Heuristic explanation:

- The maximum load on nodes is a measure of the heterogeneity of load distribution
- Cancellation of "communication signals" on heavily loaded nodes (links)